Classical Algebraic Geometry

Algebraic geometry has benefited enormously from the powerful general machinery developed in the latter half of the twentieth century. The cost has been that much of the research of previous generations is in a language unintelligible to modern workers, in particular, the rich legacy of classical algebraic geometry, such as plane algebraic curves of low degree, special algebraic surfaces, theta functions, Cremona transformations, the theory of apolarity and the geometry of lines in projective spaces.

The author's contemporary approach makes this legacy accessible to modern algebraic geometers and to others who are interested in applying classical results. The vast bibliography of over 600 references is complemented by historical notes and exercises that extend or exemplify results given in the book.

IGOR V. DOLGACHEV is Professor Emeritus in the Department of Mathematics at the University of Michigan.

Classical Algebraic Geometry

A Modern View

IGOR V. DOLGACHEV
University of Michigan

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town,
Singapore, São Paulo, Delhi, Tokyo, Mexico City

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9781107017658

First published 2012

Printed and Bound in the United Kingdom by the MPG Books Group

A catalog record for this publication is available from the British Library

Library of Congress Cataloging-in-Publication Data
Dolgachev, I. (Igor V.)
Classical algebraic geometry : a modern view / Igor V. Dolgachev.
p. cm.
ISBN 978-1-107-01765-8 (hardback)
1. Geometry, Algebraic. I. Title.
QA564.D638 2012
516.3′5–dc23
2012018486

ISBN 978-1-107-01765-8 Hardback

Dedicated to
Natasha, Denis and Andrey

Contents

Preface

The main purpose of the present treatise is to give an account of some of the topics in algebraic geometry which, while having occupied the minds of many mathematicians in previous generations, have fallen out of fashion in modern times. In the history of mathematics new ideas and techniques often make the work of previous generations of researchers obsolete; this applies especially to the foundations of the subject and the fundamental general theoretical facts used heavily in research. Even the greatest achievements of past generations, which can be found for example in the work of F. Severi on algebraic cycles or in O. Zariski's work in the theory of algebraic surfaces, have been greatly generalized and clarified so that they now remain only of historical interest. In contrast, the fact that a nonsingular cubic surface has 27 lines or that a plane quartic has 28 bitangents is something that cannot be improved upon and continues to fascinate modern geometers. One of the goals of this present work is to save from oblivion the work of many mathematicians who discovered these classic tenets and many other beautiful results.

In writing this book the greatest challenge the author has faced was distilling the material down to what should be covered. The number of concrete facts, examples of special varieties and beautiful geometric constructions that have accumulated during the classical period of development of algebraic geometry is enormous, and what the reader is going to find in the book is really only the tip of the iceberg; a work that is like a taste sampler of classical algebraic geometry. It avoids most of the material found in other modern books on the subject, such as [10], where one can find many of the classical results on algebraic curves. Instead, it tries to assemble or, in other words, to create a compendium of material that either cannot be found, is too dispersed to be found easily, or is simply not treated adequately by contemporary research papers. On the other hand, while most of the material treated in the book exists in classical treatises in algebraic geometry, their somewhat archaic terminology and what is by now completely forgotten background knowledge makes these books useful to only a handful of experts in the classical literature. Lastly, one must admit that the personal taste of the author also has much sway in the choice of material.

The reader should be warned that the book is by no means an introduction to algebraic geometry. Although some of the exposition can be followed with only a minimum background in algebraic geometry, for example, based on Shafarevich's book [523], it often relies on current cohomological techniques, such as those found in Hartshorne's book [279]. The idea was to reconstruct a result by using modern techniques but not necessarily its original proof. For one, the ingenious geometric constructions in those proofs were often beyond the author's abilities to follow completely. Understandably, the price of this was often to replace a beautiful geometric argument with a dull cohomological one. For those looking for a less demanding sample of some of the topics covered in the book, the recent beautiful book [39] may be of great use.

No attempt has been made to give a complete bibliography. To give an idea of such an enormous task one could mention that the report on the status of topics in algebraic geometry submitted to the National Research Council in Washington in 1928 [528] contains more than 500 items of bibliography by 130 different authors only in the subject of planar Cremona transformations (covered in one of the chapters of the present book.) Another example is the bibliography on cubic surfaces compiled by J. E. Hill [292] in 1896 which alone contains 205 titles. Meyer's article [381] cites around 130 papers published 1896–1928. The title search in MathSciNet reveals more than 200 papers refereed since 1940, many of them published only in the past 20 years. How sad it is when one considers the impossibility of saving from oblivion so many names of researchers of the past who have contributed so much to our subject.

A word about exercises: some of them are easy and follow from the definitions, some of them are hard and are meant to provide additional facts not covered in the main text. In this case we indicate the sources for the statements and solutions.

I am very grateful to many people for their comments and corrections to many previous versions of the manuscript. I am especially thankful to Sergey Tikhomirov whose help in the mathematical editing of the book was essential for getting rid of many mistakes in the previous versions. For all the errors still found in the book the author bears sole responsibility.

Igor Dolgachev
Ann Arbor, May 2012