Preface

A group is polycyclic if it can be built from a finite number of cyclic groups in a natural and simple way that we specify below. Polycyclic groups also crop up in a number of different contexts and can be defined in perhaps a surprising number of different ways. The most striking of these are, I think, that they are exactly the soluble groups with faithful matrix representations over the integers and are exactly the soluble groups satisfying the maximal condition on subgroups. As a result polycyclic groups appear in a large number of works that are not specifically about polycyclic groups, including works outside of group theory in areas such as ring theory, topology and even to a small extent geometry and number theory, the latter usually via arithmetic groups.

The theory of polycyclic groups started as a purely group theoretic exercise by Kurt Hirsch between the late 1930's to the early 1950's. I think he thought of it as a sort of analogue, a very superficial analogue, of Emmy Noether's theory of what are now called commutative Noetherian rings. It was Hirsch who first sparked my interest in polycyclic groups when I attended a year-long course he gave on polycyclic groups in 1964/5. This basic theory we cover in Chap. 2. The theory came of age with the introduction in the 1950's, by A.I. Mal'cev particularly, of the application of matrix group theory. This phase continued for some fifteen to twenty years. We describe how matrix theory is used to study polycyclic groups in Chaps. 4 and 5. Then came a halcyon period for the theory of polycyclic groups. In the 1970's and early 1980's two quite different and very powerful collections of techniques were developed by a number of people in a great rush of significant papers. The first can be briefly summarized as ring theoretic, especially non-commutative ring theoretic, with J.E. Roseblade being a major contributor. Explaining this is the prime object of my book and Chaps. 3, 6, 7, 8 and 9 are devoted to it. The theorems on polycyclic groups in these chapters seem to be of use as much outside the area of polycyclic groups as in it.

The second of these powerful collections of techniques can be loosely described as number, especially arithmetic, theoretic including the use of arithmetic groups. These were generally aimed more inside the theory of polycyclic groups rather than at those areas outside but involving at some level polycyclic groups. This is very well described and developed, along with very significant uses and theorems by Dan

Segal in the second half his book ‘Polycyclic Groups’; Segal was a major player in this area. Thus I make no attempt to delve into this. Rather than rework Segal’s splendid book, my aim is to complement it by concentrating on the ring theoretic area, an area that Segal does not try to cover. Since the second halves of our books have little if anything in common, this naturally affects our choice and order of development in the first halves. Also I have the opportunity to refer the reader to much later material even in the earlier part of the book.

Apart from advanced students of algebra, I want this book to be accessible to research workers in areas other than group theory who find themselves involved with polycyclic groups. I definitely intended to keep the book short and readable from start to finish. Towards the end of most sections and chapters I describe, often in some detail, the subsequent development of the topic, particularly with regard to more specialized results. Probably only a minority of readers will want to delve into the full proofs of any one of these discussions. My intention is that all readers should be able to read these discussions reasonably easily without getting too bogged down in the details and to get some idea of what sort of results are available in these areas, certainly enough of an idea for them to decide whether they need the full details, and then to inform them where to find these details.

The first half of this book, that is Chaps. 2 to 5, I covered in a one-semester University of London M.Sc. course. The second half I covered in a one-semester University of London course aimed at M.Phil. and Ph.D. students. Actually in both cases my book is a somewhat expanded version of these two courses with additional proofs and information. For the first half I assume that the reader has covered the equivalent of a one-semester course in group theory, but not necessarily recently. For readers whose group theory is inadequate or rusty I start the book with a Chap. 1, revising the general group theory that I need, with one or two less common results towards its end. I advise readers to start at Chap. 2 and only read Chap. 1 if they find the group theory really tough going. Other readers may well want to read up the odd result from Chap. 1, but I recommend they do so when they find it being quoted elsewhere in the book.

Particularly when it comes to the latter parts of the book I use quite a lot from ring and module theory. I give full references for theorems that I use. Unlike the group theory the ring theory that I quote tends to be just bits here and there so I have decided to leave it to the reader to read up whatever they find they need. I assume the reader has seen notions like ring, ideal, homomorphism, module, submodule and module homomorphism before. If not the reader would be well advised to read the first sections on rings and modules in some general algebra text such as the first volume of either P.M. Cohn’s ‘Algebra’ or N. Jacobson’s ‘Basic Algebra’ (or in one of the numerous other algebra texts). At various points I do need more advanced theorems. I hope I have stated what is being used sufficiently clearly that the reader can continue reading my proof with understanding without needing to refer immediately to another text. I stress that for a first reading it should not be necessary to actually have these standard texts to hand and in any case the reader may well have there own favorite texts containing the relevant material.

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