Preface

In the fall semester of 1979 I gave a course on deformation theory at Berkeley. My goal was to understand completely Grothendieck's local study of the Hilbert scheme using the cohomology of the normal bundle to characterize the Zariski tangent space and the obstructions to deformations. At the same time I started writing lecture notes for the course. However, the writing project soon foundered as the subject became more intricate, and the result was no more than five of a projected thirteen sections, corresponding roughly to sections 1, 2, 3, 5, 6 of the present book.

These handwritten notes circulated quietly for many years until David Eisenbud urged me to complete them and at the same time (without consulting me) mentioned to an editor at Springer, "You know Robin has these notes on deformation theory, which could easily become a book." When asked by Springer if I would write such a book, I immediately refused, since I was then planning another book on space curves. But on second thought, I decided this was, after all, a worthy project, and that by writing I might finally understand the subject myself.

So during 2004 I expanded the old notes into a rough draft, which I used to teach a course during the spring semester of 2005. Those notes, rewritten once more, with the addition of exercises, form the book you are now reading.

My goal in this book is to introduce the main ideas of deformation theory in algebraic geometry and to illustrate their use in a number of typical situations. I have made no effort to state results in the most general form, since I preferred to let the basic ideas shine forth unencumbered by technical details. Nor have I attempted to phrase results in the current "state of the art" language of stacks, since that requires a formidable apparatus of category theory. I hope that my elementary approach will be useful as a preparation for the new language in the same way that a thorough study of varieties is a good basis for understanding schemes and cohomology.

The prerequisite for reading this book is a basic familiarity with algebraic geometry as developed for example in [57].