

Preface

The aim of these notes is to include in a unitary presentation some topics related to the theory of degenerate nonlinear diffusion equations, treated in the mathematical framework of evolution equations with multivalued maximal monotone operators in Hilbert spaces. The problems concern nonlinear parabolic equations involving two cases of degeneracy. More exactly, one case is due to the vanishing of the time derivative coefficient and the other is provided by the vanishing of the diffusion coefficient on subsets of positive measure of the domain.

From the mathematical point of view, the results presented in these notes can be considered as general results in the theory of degenerate nonlinear diffusion equations. However, this work does not seek to present an exhaustive study of degenerate diffusion equations, but rather to emphasize some rigorous and efficient functional methods for approaching these problems.

The main objective is to present various techniques in which a degenerate boundary value problem with initial data can be approached and to introduce relevant solving methods different for each case apart. The work focuses on the theoretical part, but some attention is paid to the link between the abstract formulation and examples concerning applications to boundary value problems which describe real phenomena. Numerical simulations by which the theoretical results are applied to some concrete real-world problems are included with a double scope: for verifying the theory and for illustrating the response given by the theoretical results to the problems arisen in applied sciences.

The material is organized in four chapters, each divided into several sections. The Definitions, results (Theorems, Propositions, Lemmas), and figures are continuously numbered inside a chapter.

The readers are assumed to be familiar with functional analysis, partial differential equations, and some concepts and basic results from the theory of monotone operators. However, the book is self-contained as possible, some specific definitions and results being either introduced at the first place where they are evoked, or indicated by citations. The work addresses to advanced

graduate students in mathematics and engineering sciences, researchers in partial differential equations, applied mathematics and control theory. It can serve as a basis for an advanced course and seminars on applied mathematics for students during the Ph.D. level, and in this respect it is aimed to open to the readers the way toward applications.

The writing of these notes has been developed during the visits of the second author to the Department of Mathematics at the University of Bologna, especially in the periods April–May 2010 when she was a visiting professor, thanks for the financial support of Istituto Nazionale di Alta Matematica “F. Severi”—Gruppo Nazionale per l’Analisi Matematica, la Probabilità e le loro Applicazioni, Italy, May and November 2011. The work is mainly based on the common results obtained with the first author, and some parts of it completed in 2011 are new and not published in other works.

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