Preface

In the past half century, dynamical system theory, also more grandly called “nonlinear science”, has developed rapidly. In particular, the discovery of chaos has revolutionized the field. The study of chaos is closely related to research in the area of bifurcation and stability, and, in particular, the theory of limit cycles has had a significant impact since chaos may be considered to be a summation of motions with an infinite number of frequencies (such as the scenario of “period doubling”, which leads to chaos). Although limit cycles involve a comparatively simple motion, they appear in almost all disciplines of science and engineering including mechanics, aeronautics, electrical circuits, control systems, population problems, economics, financial systems, stock markets, ecological systems, etc. In fact, most of the early work in the theory of limit cycles was stimulated by practical problems displaying periodic behavior. Therefore, the study of the bifurcation of limit cycles is not only significant for its theoretical development, but also plays an important role in solving practical problems.

The bifurcation of limit cycles may be generally classified into three categories: (i) Hopf bifurcation from a center or a focus; (ii) Poincaré bifurcation from closed orbits; and (iii) separatrix cycle bifurcations from a homoclinic or a heteroclinic loop. Although the study of Hopf bifurcation has been ongoing for more than half century, and one of the main tools for the study of bifurcation, normal form theory, dates back 100 years, efficient computation of the normal form is still a major challenge in this field. Computation becomes extremely difficult for higher-order normal forms or focus values, which is particularly important in studying the well-known Hilbert 16th problem. The second and third categories of limit cycle bifurcation, on the other hand, are comparatively more difficult; not only are there few techniques available, but there is also no general computational method developed for the Melnikov function, which is the main tool. It is the case that researchers new to the field often have difficulty in obtaining systematic material for study. Frontier research results can be difficult to access and even an experienced researcher in the field can be unaware of them. It appears necessary and significant that monographs on the topic of the bifurcation of limit cycles be available. The present monograph is intended to introduce the most recent new developments, and to provide major advances in the fundamental theory of limit cycles.

This monograph comes mainly from our recent research results and experience of teaching the topics of the stability and bifurcation of limit cycles to graduate students. It is divided into two parts: the first part (Chaps. 2–5) is focused on limit cycles bifurcating from a Hopf singularity and the use of normal form theory, while the second part (Chaps. 6–10) considers near-Hamiltonian systems and the main mathematical tool used is the Melnikov function.

Chapter 1 is an introduction, presenting the background for nonlinear dynamics, bifurcation and stability, the normal form method, the Melnikov function and Hilbert's 16th problem.

In Chap. 2, the computation of normal forms is discussed. First, a general approach which combines center manifold theory with computation of the normal form is presented. Then, a perturbation method which has proved computationally efficient is discussed in detail.

Chapter 3 is devoted to studying the computational efficiency of existing methods for computing focus values. Three typical methods: the Poincaré method or Takens method, a perturbation technique, and the singular point value method are particularly discussed. It is shown that these three methods have the same order of computational complexity, and no method has so far been developed for computing the "minimal singular point values".

In Chap. 4, Hopf bifurcation and the computation of normal forms are applied to consider planar vector fields and focused on the well-known Hilbert 16th problem. General cubic- and higher-order systems are considered to find the maximum number of limit cycles that such systems can have, i.e., to find a lower bound for the Hilbert number for certain vector fields. The Liénard system is investigated, and also, the critical periods of bifurcating periodic solutions from two special types of planar system are studied.

Chapter 5 is focused on the application of Hopf bifurcation theory and normal form computation to practical problems, including those from engineering and biological systems, as well as problems arising from the area of Hopf bifurcation control.

Chapter 6 introduces the fundamental theory of the Melnikov function method. Basic definitions and fundamental lemmas are presented and the main theory on the number of limit cycles is given.

In Chap. 7, particular attention is given to the bifurcation of limit cycles near a center. After normalizing the Hamiltonian function, detailed steps for computing the Melnikov function are described, and formulas are explicitly given. Maple programs for computing the coefficients of the Melnikov function are developed and illustrative examples are presented.

Chapter 8 considers the bifurcation of limit cycles near a homoclinic or heteroclinic loop. The method for computing the Melnikov function near a homoclinic or heteroclinic loop is developed and explicit formulas for the coefficients in the expansion of the Melnikov function are derived. Double homoclinic loops are also studied in this chapter.

In Chap. 9, an idea for finding more limit cycles is introduced, which combines the bifurcation of limit cycles from centers, homoclinic and heteroclinic loops.

A generalized theorem is presented. In particular, two polynomial systems are studied. By using the theorems and results obtained in Chaps. 6–9 it is shown that one system can have seven limit cycles while the other can have five.

Chapter 10 investigates the bifurcation of limit cycles in equivariant systems, including S-equivariant vector fields, Z_q-equivariant vector fields and S_q-reversible vector fields. In particular, an S-equivariant quadratic system, a Z_3-equivariant system and a cubic $(2\pi/3)$-equivariant system are studied.

The Maple programs for the semisimple case (see Sect. 2.2) and Hopf bifurcation (see Sect. 2.3) can be found on "Springer Extras" as *Electronic Supplementary Material*. These can be accessed and downloaded online anytime, anywhere. To use the content on Springer Extras, please visit extras.springer.com and search using the book's ISBN. You will then be asked to enter a password, which is given on the copyright page of this print book. More Maple programs for other singularities can be found from http://pyu1.apmaths.uwo.ca/~pyu/pub/index/software.html.

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