Preface

This volume is dedicated to Themistocles M. Rassias, on the occasion of his 60th birthday. The articles published here present some recent developments and surveys in Nonlinear Analysis related to the mathematical theories of stability, approximation, and inequalities.

Themistocles M. Rassias was born in Pellana near Sparta in Greece in the year 1951. He is currently a Professor in the Department of Mathematics at the National Technical University of Athens. He received his Ph.D. in Mathematics in the year 1976 from the University of California at Berkeley with Stephen Smale as his thesis advisor.

Rassias' work extends over several fields of Mathematical Analysis. It includes Global Analysis, Calculus of Variations, Nonlinear Functional Analysis, Approximation Theory, Functional Equations, and Mathematical Inequalities and their Applications. Rassias' work has been embraced by several mathematicians internationally and some of his research has been established with the scientific terminology "Hyers–Ulam–Rassias stability", "Cauchy–Rassias stability", "Aleksandrov–Rassias problem for isometric mappings".

The stability theory of functional equations has its roots primarily in the investigations by S.M. Ulam, who posed the fundamental problem for approximate homomorphisms in the year 1940, the stability theorem for the additive mapping due to D.H. Hyers (1941), and the stability theorem for the linear mapping of Th.M. Rassias (1978). Much of the modern stability of functional equations has been influenced by the seminal paper of Th.M. Rassias, entitled "On the stability of the linear mapping in Banach spaces" [*Proceedings of the American Mathematical Society* **72**, 297–300 (1978)], which has provided a theoretical breakthrough. For an extensive discussion of various advances in stability theory of functional equations, the reader is referred to the recently published book of S.-M. Jung, "*Hyers–Ulam–Rassias Stability of Functional Equations in Nonlinear Analysis*", ©Springer, New York, 2011.

In the formulation as well as in the solution of stability problems of functional equations, one frequently encounters the interplay of Mathematical Analysis, Geometry, Algebra, and Topology. Rassias has contributed also to other subjects such as Minimal Surfaces (Plateau problem), Isometric Mappings (Aleksandrov problem), Complex Analysis (Poincaré inequality and Möbius transformations), and Approximation theory (Extremal problems). He has published more than 230 scientific research papers, 6 research books and monographs, and 30 edited volumes on current research topics in Mathematics. He has also published 4 textbooks in Mathematics for Greek university students.

Some of the honors and positions that he has received include “Membership” at the School of Mathematics of the Institute for Advanced Study at Princeton for the academic years 1977–1978 and 1978–1979 (which he did not accept for family reasons); “Research Associate” at the Department of Mathematics of Harvard University (1980) invited by Raoul Bott, “Visiting Research Professor” at the Department of Mathematics of the Massachusetts Institute of Technology (1980) invited by F.P. Peterson; “Accademico Ordinario” of the Accedemia Tiberina Roma (since 1987); “Fellow” of the Royal Astronomical Society of London (since 1991); “Teacher of the year” (1985–1986 and 1986–1987) and “Outstanding Faculty Member” (1989–1990, 1990–1991, and 1991–1992) of the University of La Verne, California (Athens Campus); “Ulam Prize in Mathematics” (2010). In addition to the above, during the last few years, Th.M. Rassias had been bestowed with honorary degrees “Doctor Honoris Causa” from the University of Alba Iulia in Romania (2008) and an “Honorary Doctorate” from the University of Niš in Serbia (2010). In 2003, a volume entitled “*Stability of Functional Equations of Ulam–Hyers–Rassias Type*” was dedicated to the 25 years since the publication of Th. M. Rassias’ stability theorem (edited by S. Czerwik, Florida, USA). In 2009, a special issue of the *Journal of Nonlinear Functional Analysis and Applications* (Vol. 14, No. 5) was dedicated to the 30th Anniversary of Th.M. Rassias’ stability theorem. In 2007, a special volume of the *Banach Journal of Mathematical Analysis* (Vol. 1, Issues 1 & 2) was dedicated to the 30th Anniversary of Th.M. Rassias’ stability theorem. He is an “editor” or “advisory editor” of several international mathematical journals published in the USA, Europe, and Asia. He has delivered lectures at several universities in North America and Europe, including Harvard University, MIT, Yale University, Princeton University, Stanford University, University of Michigan, University of Montréal, Imperial College London, Technion—Israel Institute of Technology (Haifa), Technische Universität Berlin, and the Universität Göttingen.

The contributed papers in the present volume highlight some of the most recent achievements that have been made in Mathematical Analysis.

Rassias’ curiosity, enthusiasm as well as his passion for doing research as well as teaching are unlimited. He has served as a mentor in Mathematics to several students at universities where he has taught.

His research work has received up-to-date more than 7,000 citations (see, e.g., the Google Scholar). That is an impressive number of citations for a mathematician. Thus, Rassias has achieved international distinction in the broadest sense.

The reader is referred to the article of Per Enflo and M. Sal Moslehian, An interview with Themistocles M. Rassias, *Banach Journal of Mathematical Analysis* **1**, 252–260 (2007) [see also www.math.ntua.gr/~trassias/].

In what follows, we present a brief outline of the contributed papers in this volume, which are collected in an alphabetical order of the contributors.

In Chap. 1, S. Abramovich deals with Jensen's type inequality, its bounds and refinements, and with eigenvalues of the Sturm–Liouville system.

In Chap. 2, M. Adam and S. Czerwik consider some quadratic difference operators (e.g., Lobaczewski difference operators) and quadratic-linear difference operators (e.g., d'Alembert difference operators and quadratic difference operators) in some special function spaces. They prove a stability result in the sense of Ulam–Hyers–Rassias for the quadratic functional equation in a special class of differentiable functions.

In Chap. 3, C. Affane-Aji and N.K. Govil present a study concerning the location of the zeros of a polynomial starting from the results of Gauss and Cauchy to some of the most recent investigation on the topic.

Chapter 4 by D. Andrica and V. Bulgarean is devoted to isometry groups $Iso_{d_p}(\mathbb{R}^n)$ for $p \geqq 1$, $p \neq 2$ and $p = \infty$, where the metric d_p is appropriately defined.

In Chap. 5, I. Biswas, M. Logares, and V. Muñoz prove that the moduli spaces $\mathscr{M}_\tau(r, \Lambda)$ are, in many cases, rational. Here the moduli spaces are defined by using a concept of τ-stable pairs of rank r and fixed determinant Λ.

In Chap. 6, D. Breaz, Y. Polatoğlu, and N. Breaz investigate a subclass of generalized p-valent Janowski type convex functions and its application to harmonic mappings.

In Chap. 7, J. Brzdęk, D. Popa, and B. Xu present some observations concerning stability of the following linear functional equation:

$$\varphi\big(f^m(x)\big) = \sum_{i=1}^{m} a_i(x)\varphi\big(f^{m-i}(x)\big) + F(x)$$

in the class of functions φ mapping a nonempty set S into a Banach space X over a field $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$, where m is a fixed positive integer and the functions $f : S \to S$, $F : S \to X$ and $a_i : S \to \mathbb{K}$ $(i = 1, \ldots, m)$ are given.

In Chap. 8, M.J. Cantero and A. Iserles examine the limiting behavior of solutions to an infinite set of recursions involving q-factorial terms as $q \to 1$.

In Chap. 9, E.A. Chávez and P.K. Sahoo determine the general solutions of the following functional equations:

$$f_1(x+y) + f_2(x+\sigma y) = f_3(x) \quad \text{and}$$
$$f_1(x+y) + f_2(x+\sigma y) = f_3(x) + f_4(y), \quad \big(x, y \in S^n\big),$$

where $f_1, f_2, f_3, f_4 : S^n \to G$ are unknown functions, S is a commutative semigroup, $\sigma : S \to S$ is an endomorphism of order 2, G is a 2-cancellative abelian group, and n is a positive integer.

In Chap. 10, W.-S. Cheung, G. Leng, J. Pečarić, and D. Zhao present recent developments of Bohr-type inequalities.

In Chap. 11, S. Ding and Y. Xing establish some basic norm inequalities, including the Poincaré inequality, weak reverse Hölder inequality, and the Caccioppoli inequality, for conjugate harmonic forms. They also prove the Caccioppoli inequality with the Orlicz norm for conjugate harmonic forms.

In Chap. 12, S.S. Dragomir presents a survey about some recent inequalities related to the celebrated Jensen's result for positive linear or sublinear functionals and convex functions.

In Chap. 13, A. Ebadian and N. Ghobadipour prove the generalized Hyers–Ulam–Rassias stability of bi-quadratic bi-homomorphisms in C^*-ternary algebras and quasi-Banach algebras.

In Chap. 14, E. Elhoucien and M. Youssef apply a fixed point theorem to prove the Hyers–Ulam–Rassias stability of the following quadratic functional equation:

$$f(kx+y)+f\big(kx+\sigma(y)\big)=2k^2f(x)+2f(y).$$

In Chap. 15, M. Fujii, M. Sal Moslehian, and J. Mićić survey several significant results on the Bohr inequality and present its generalizations involving some new approaches.

In Chap. 16, P. Găvruţa and L. Găvruţa provide an introduction to the Hyers–Ulam–Rassias stability of orthogonally additive mappings.

In Chap. 17, M. Eshaghi Gordji, N. Ghobadipour, A. Ebadian, M. Bavand Savadkouhi, and C. Park investigate ternary Jordan homomorphisms on Banach ternary algebras associated with the following functional equation:

$$f\left(\frac{x_1}{2}+x_2+x_3\right)=\frac{1}{2}f(x_1)+f(x_2)+f(x_3).$$

In Chap. 18, F. Habibian, R. Bolghanabadi, and M. Eshaghi Gordji investigate the Hyers–Ulam–Rassias stability of cubic n-derivations from non-archimedean Banach algebras into non-archimedean Banach modules.

In Chap. 19, S.-S. Jin and Y.-H. Lee investigate a fuzzy version of stability for the following functional equation:

$$2f(x+y)+f(x-y)+f(y-x)-f(2x)-f(2y)=0$$

in the sense of M. Mirmostafaee and M.S. Moslehian.

In Chap. 20, K.-W. Jun, H.-M. Kim, and E.-Y. Son prove the generalized Hyers–Ulam stability of the following Cauchy–Jensen functional equation:

$$f(x)+f(y)+nf(z)=nf\left(\frac{x+y}{n}+z\right),$$

in an n-divisible abelian group G for any fixed positive integer $n\geqq 2$.

In Chap. 21, S.-M. Jung applies the fixed point method for proving the Hyers–Ulam–Rassias stability of the gamma functional equation.

In Chap. 22, H.A. Kenary proves the generalized Hyers–Ulam stability in random normed spaces of the following additive-quadratic-cubic-quartic functional equation:

$$f(x+2y)+f(x-2y)=4f(x+y)+4f(x-y)-6f(x) \\ +f(2y)+f(-2y)-4f(y)-4f(-y).$$

In Chap. 23, S.V. Konyagin and Yu.V. Malykhin prove the existence of an infinite-dimensional separable Banach space with a basis set such that no arrangement of it forms a Schauder basis.

In Chap. 24, S. Koumandos presents a survey of recent results on positive trigonometric sums.

In Chap. 25, P. Mihăilescu presents a proof of a slightly more general result than the one of Vandiver and Sitaraman, concerning the first case of Fermat's Last Theorem, with consequences for a larger family of Diophantine equations.

In Chap. 26, G.V. Milovanović and M.P. Stanić present a survey of multiple orthogonal polynomials defined by using orthogonality conditions spread out over r different measures. A method for the numerical construction of such polynomials by using the discretized Stieltjes–Gautschi procedure is given.

In Chap. 27, F. Moradlou and G.Z. Eskandani prove the Hyers–Ulam–Rassias stability of C^*-algebra homomorphisms and of generalized derivations on C^*-algebras for the following Cauchy–Jensen functional equation:

$$f\left(\left(\sum_{i=1}^{n} z_i\right)-\left(\sum_{i=1}^{n} x_i\right)\right)+f\left(\left(\sum_{i=1}^{n} z_i\right)-\left(\sum_{i=1}^{n} y_i\right)\right) \\ =2f\left(\left(\sum_{i=1}^{n} z_i\right)-\frac{(\sum_{i=1}^{n} x_i)+(\sum_{i=1}^{n} y_i)}{2}\right).$$

In Chap. 28, D. Motreanu and P. Winkert present a survey on the Fučík spectrum of the negative p-Laplacian with different boundary conditions such as the Dirichlet, Neumann, Steklov, and Robin boundary conditions.

In Chap. 29, M. Mursaleen and S.A. Mohiuddine use the notion of almost convergence and statistical convergence in order to prove the Korovkin type approximation theorem by means of the test functions $1, e^{-x}, e^{-2x}$.

In Chap. 30, A. Najati proves the Hyers–Ulam stability of the following functional equation:

$$f(x+y+xy)=f(x+y)+f(xy).$$

In Chap. 31, M.A. Noor, K.I. Noor, and E. Al-Said make use of the projection technique in order to study a new class of quasi-variational inequalities, which they call the extended general nonconvex quasi-variational inequalities, and establish their equivalence with the fixed point problem. They also apply this equivalence to the existence of a solution of the above-named inequalities under some suitable conditions.

In Chap. 32, M.A. Noor, K.I. Noor, and E. Al-Said study a system of general nonconvex variational inequalities involving four different operators. Their results can be viewed as a refinement and improvement of previously known results for variational inequalities.

In Chap. 33, B. Paneah presents a survey on results about a general linear functional operator, which includes Cauchy type functional operators, Jensen type functional operators, and quasiquadratic functional operators.

In Chap. 34, C. Park proves the generalized Hyers–Ulam stability of the following functional equation:

$$2f(x+y)+f(x-y)+f(y-x)=3f(x)+f(-x)+3f(y)+f(-y)$$

in Banach spaces.

In Chap. 35, C. Park, M.E. Gordji, and R. Saadati classify and prove the generalized Hyers–Ulam stability of linear, quadratic, cubic, quartic, and quintic functional equations in complex Banach spaces.

In Chap. 36, A. Prástaro presents results about local and global existence and stability theorems for exotic n-d'Alembert PDEs, previously introduced by the author.

In Chap. 37, V.Yu. Protasov studies the precision of approximation of a function in linear spaces by affine functionals in case their restrictions to every straight line can be approximated by affine functions on that line with a given precision (in the uniform metric).

Chapter 38 is a survey-cum-expository article by H.M. Srivastava who presents a systematic account of some recent developments on univalent and bi-univalent analytic functions, thereby encouraging future researches on these topics in *Geometric Function Theory of Complex Analysis*.

In Chap. 39, Á. Száz presents a detailed survey on the famous Hyers–Ulam stability theorems, Hahn–Banach extension theorems, and their set-valued generalizations. He also reviews the most basic additivity and homogeneity properties of relations and investigates, in greater detail, some elementary operations on relations. These operations and the intersection convolutions of relations allow a new view of relational generalizations of the Hyers–Ulam and the Hahn–Banach theorems.

In Chap. 40, L. Székelyhidi presents a survey on spectral analysis and spectral synthesis over locally compact abelian groups.

In Chap. 41, A. Ungar presents a theory which extracts the Möbius addition in the ball of the Euclidean n-space, from the Möbius transformation of the complex open unit disc, and demonstrates the hyperbolic geometric isomorphism between the resulting Möbius addition and the famous Einstein velocity addition of special relativity theory.

In Chap. 42, B. Yang defines a general Hilbert-type integral operator and studies six particular kinds of this operator with different measurable kernels in several normed spaces.

In Chap. 43, X. Zhao and X. Yang study the stability of the following Pexider type sine functional equation:

$$h(x)k(y)=f^2\left(\frac{x+y}{2}\right)-g^2\left(\frac{x+\sigma y}{2}\right)$$

and extend the results to Banach algebras.

In Chap. 44, Z. Wang and W. Zhang establish some stability results concerning the following additive-quadratic functional equation:

$$f(2x + y) + f(2x - y) = f(x + y) + f(x - y) + 2f(2x) - 2f(x)$$

in intuitionistic fuzzy normed spaces (IFNS).

We wish to express our deepest appreciation to the above-named mathematicians from the international mathematical community who contributed their papers for publication in this volume on the occasion of the 60th birthday anniversary of Themistocles M. Rassias. In addition, we are also very thankful to Springer for its generous support to this publication.

Gainesville, USA — Panos M. Pardalos
Victoria, Canada — Hari M. Srivastava
Gainesville, USA — Pando G. Georgiev

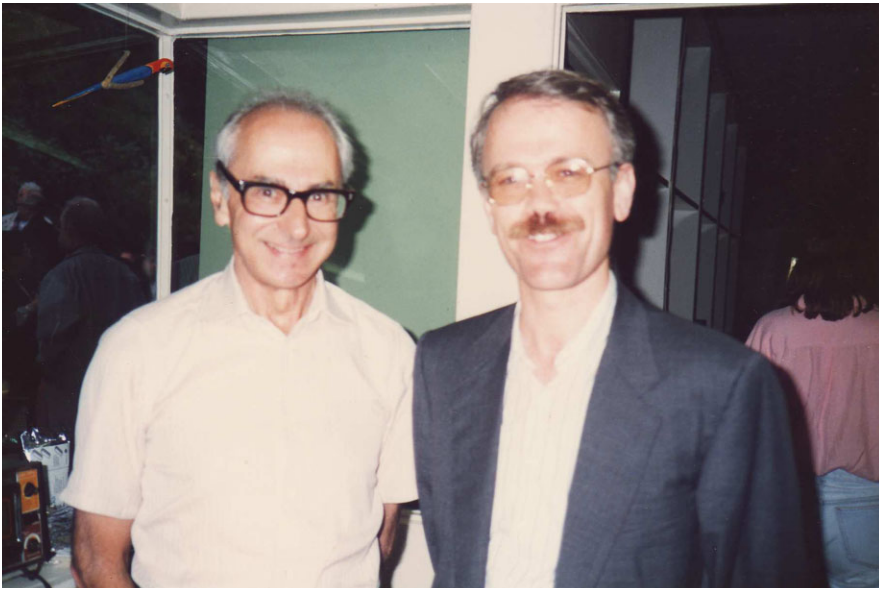
Themistocles M. Rassias and Stephen Smale at Berkeley, 1990

Themistocles M. Rassias with Henri Cartan in Paris, 1993

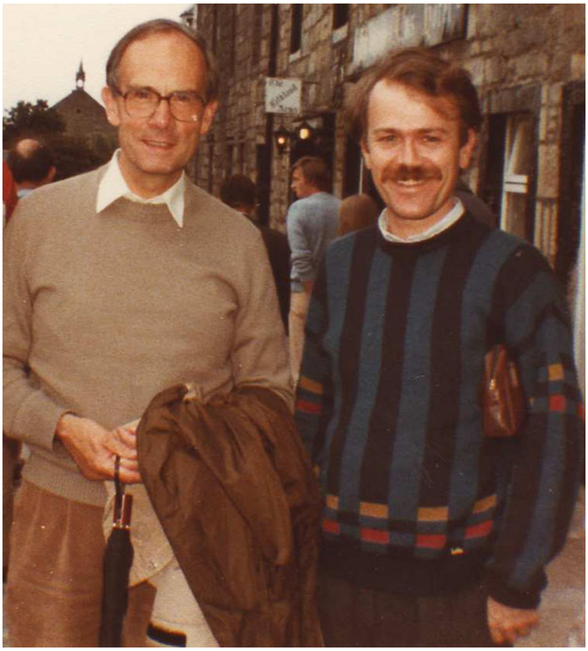
Themistocles M. Rassias with Lars V. Ahlfors at Harvard, 1995

Themistocles M. Rassias with Paul Erdős in Zurich, 1994

Themistocles M. Rassias with Vladimir I. Arnold in Paris, 1993



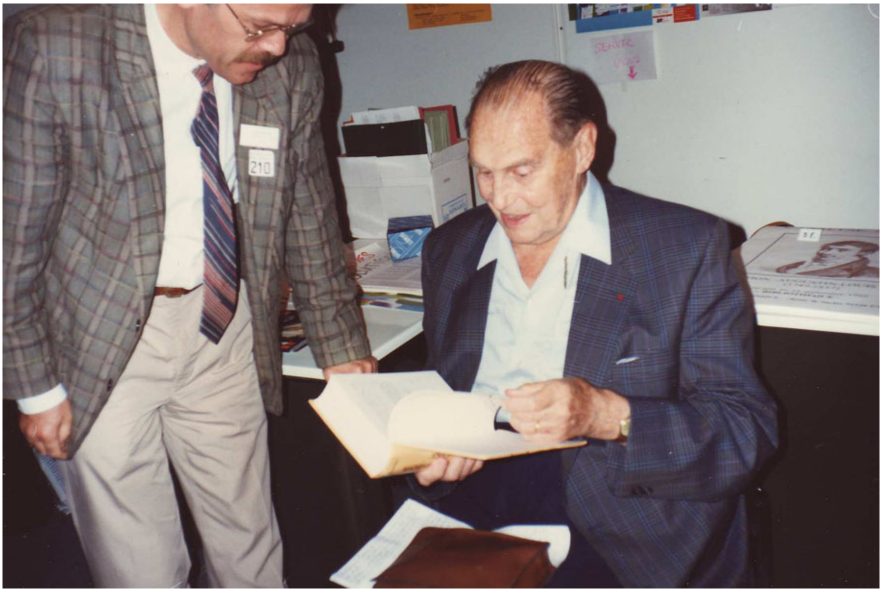
Themistocles M. Rassias with Serge Lang at Berkeley, 1990



Themistocles M. Rassias with Friedrich E.P. Hirzebruch in Bonn, 1987

Themistocles M. Rassias with Israel M. Gelfand in London, 1994

Themistocles M. Rassias with Shizuo Kakutani at Yale, 1990



Themistocles M. Rassias with Jean Dieudonné in Paris, 1989