PREFACE

Through its rapid progress in the last decade, H^∞control became an established control technology to achieve desirable performances of control systems. Several highly developed software packages are now available to easily compute an H^∞controller for anybody who wishes to use H^∞control.

It is questionable, however, that theoretical implications of H^∞control are well understood by the majority of its users. It is true that H^∞control theory is harder to learn due to its intrinsic mathematical nature, and it may not be necessary for those who simply want to apply it to understand the whole body of the theory. In general, however, the more we understand the theory, the better we can use it. It is at least helpful for selecting the design options in reasonable ways to know the theoretical core of H^∞control.

The question arises: *What is the theoretical core of H^∞control?* I wonder whether the majority of control theorists can answer this question with confidence. Some theorists may say that the interpolation theory is the true essence of H^∞control, whereas others may assert that unitary dilation is the fundamental underlying idea of H^∞control. The J-spectral factorization is also well known as a framework of H^∞control. A substantial number of researchers may take differential game as the most salient feature of H^∞control, and others may assert that the Bounded Real Lemma is the most fundamental building block. It may be argued that, since the Bounded Real Lemma is just a special case of linear matrix inequality (LMI), the LMI is more fundamental as the theoretical foundation of H^∞control and is a panacea that eliminates all the difficulties of H^∞control.

All these opinions contain some truth. It is remarkable that H^∞control allows such a multitude of approaches. It looks entirely different from different viewpoints. This fact certainly implies that H^∞control theory is quite rich in logical structure and is versatile as an engineering tool. However, the original question of what is the theoretical core of H^∞control remains unanswered. Indeed, every fundamental notion mentioned has a method of solving the H^∞control problem associated with it. Unfortunately, however, lengthy chains of reasoning and highly technical manipulations are their common characteristic features. For instance, the method of unitary dilation, which first gave the complete solution to the H^∞control problem, requires several steps of problem reductions starting with the *Nehari problem* [28]. The game-theoretic approach seems to be the most comprehensible because it reduces the problem to a simple completion of squares. This approach, however, introduces unnecessary complications concerning the initial condition. The issue of internal stability as well as the optimality is not well addressed by this approach. It is indeed remarkable that we have not yet found a proper framework to describe H^∞control theory in a clear and self-contained way so that the intrinsic features are explicitly exposed.

This book is a compilation of the author's recent work as an attempt to give a unified, systematic, and self-contained exposition of H^∞control theory based on the three key notions: *chain-scattering representation, conjugation*, and *J-lossless factorization.* In this new framework, the H^∞control problem is reduced to a J-lossless factorization of chain-scattering representation of the plant. This is comparable with the similar fact that the LQG (Linear Quadratic Gaussian) problem is reduced to the Wiener-Hopf factorization of a transfer function associated with the plant and the performance index. It is the author's belief that the approach proposed in this book is the simplest way to expose the fundamental structure of H^∞control embodying all its essential features of H^∞control that are relevant to the design of control systems.

Our scenario of solving the problem is roughly described as follows.

(1) The cascade structure of feedback systems is exploited based on the chain-scattering representation, and H^∞control is embedded in the more general framework of cascade synthesis.

(2) The H^∞control problem is reduced to a new type of factorization problem called a J-lossless factorization, within the cascade framework of control system design.

(3) The factorization problem is solved based on J-lossless conjugation.

Our approach is not entirely new. Among the three key notions, the first two are already well known. The chain-scattering representation is used extensively in various fields of engineering to represent the scattering properties of the physical system, although the term *chain-scattering* is not widely used except in circuit theory and signal processing where this notion plays a fundamental role. It was introduced in the control literature by the author. J-lossless factorization is an alternative expression of the well-known factorization in the literature of H^∞control which is usually referred to as *J-spectral factorization* in order to facilitate the cascade structure of synthesis. The conjugation is entirely new and was formulated by the author in 1989 as a state-space representation of the classical interpolation theory. It turned out to be a powerful tool for carrying out the J-lossless factorization.

Finally, the approach in this book throws a new light on the meaning of duality in H^∞control. In the chain-scattering formalism, *the dual is equivalent to the inverse.* The duality between the well-known two Riccati equations is explained as a natural consequence of the chain-scattering formalism itself.

In writing the book, we considered the following points to be of paramount importance.
(1) The exposition must be self-contained.
(2) The technicalities should be reduced to a minimum.
(3) The theory must be accessible to engineers.

Concerning point (1), the reader is not required to have a background of linear system theory beyond the very elementary level which is actually supplied in this book in Chapters 2 and 3. All the technical complications are due to the augmentations of the plant in forming the chain-scattering representation. These augmentations have their own relevance to the synthesis problem beyond the technicalities. In Chapters 2 and 3, some basic preliminaries of linear system theory and associated mathematical results are briefly reviewed. Chapter 4 is devoted to the introduction of chain-scattering representations of the plant. Various properties of the chain-scattering representations are discussed. The J-lossless matrix is introduced in this chapter as a chain-scattering representation of lossless systems. The notion of J-lossless conjugation is introduced in Chapter 5 with emphasis on its relation to classical interpolation problems. Chapter 6 deals with J-lossless factorization based on J-lossless conjugation. The well-known Riccati equations are introduced characterizing the existence of J-lossless factorization. The H^∞control problem is formulated in Chapter 7 in the frequency domain. The problem is reduced to the J-lossless factorization of the chain-scattering representation of the plant. The state-space solution to the H^∞control problem is obtained in Chapter 8. Chapter 9 discusses the closed-loop structure of H^∞control systems.

This book was projected five years ago, but the writing didn't go smoothly due to the author's lack of time to concentrate on it. The first drive to write the book was obtained when the author stayed at Delft University of Technology for three months in the summer of 1994. Professor Dewilde taught me a lot about the deep theory of chain-scattering representation through stimulating discussions. I would like to express my sincere gratitude to him for giving me this unusual opportunity. The second drive to complete the book was gained when the author stayed at UC Berkeley as a Springer Professor in the Spring of 1995. I was given the good fortune to deliver a series of lectures on H^∞ control. This book took its final shape during these lectures. I owe a great deal to Professor Tomizuka for giving me this valuable experience and to Professor Packard for his patience in attending all of my lectures. My gratitude also goes to my colleagues, Dr. Kawatani, Professor Fujii, Dr. Hashimoto, Dr. Ohta, and Dr. Yamamoto, and many students who helped me in my research on H^∞control; Mr. Okunishi, Dr. Xin, Dr. Zhou, Dr. Baramov, Miss Kongprawechnon, Mr. Ushida, and many others. I am greatly indebted to a research group on control theory in Japan which gave me continual encouragement during the last decade through the private workshops, colloqia, correspondences, and so on. The discussions with Professors Hosoe, Mita, and Hara gave me many ideas and suggestions. I am also grateful to Messrs. Monden, Oishi, Suh, and Oku, and Mrs. Ushida who typed my manuscript and made extensive corrections through the careful reading of the draft. Finally, I am thankful to Ms. Yoshiko Kitagami who created nice artwork for the cover of this book.