Evolution Inclusions and Variation Inequalities for Earth Data Processing III

Long-Time Behavior of Evolution Inclusions Solutions in Earth Data Analysis

Bearbeitet von
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Preface

Lately, due to the intense investigation processes in economics, ecology, geophysics, synergy, and geoinformatics there is the necessity for more detailed study of nonlinear effects, new classes of mathematical models with nonlinear, nonsmooth, discontinuous, and multivalued dependence between determinative parameters of problems, control problems for nonlinear processes and fields, and long-range forecast problems for state functions of solutions for such problems. The Institute for Applied System Analysis (NAS) of Ukraine carries out fundamental research on such problems in the following areas:

1. Nonlinear analysis and control for classes of nonlinear geophysics processes and fields.
2. Properties of solutions of differential-operator inclusions and multivariation inequalities for Earth Data Analysis.
3. Theory of global and trajectory attractors for infinite-dimensional dynamical systems.

New qualitative and constructive results concerning properties of state functions for problems of analysis and control with nonsmooth interaction functions have been obtained.

Note that there exist classes of real problems with regular interaction functions that allow non-classical effects (for example equations of hydrodynamic type). We have also developed an abstract theory of geophysical problems. We propose the system mathematical instrument including the theory of differential-operator inclusions and multivariation inequalities for Earth Data Processes, theory of global and trajectory attractors for infinite-dimensional m-semiflows, methods for investigation of existence of solutions for control problems. Applying stated results to real problems, we provide long-range forecasts in cases where only the abstract theory for solutions is known (processes of piezoelectricity, viscoelasticity, thermodynamics, hydrodynamics etc.) (Fig. 1).

Moreover, the hypotheses of our theorems are close to the corresponding necessary conditions and the results of the theorems are close to the maximal ones. Let us consider the following example: when interaction function loses its smoothness it can be easily seen that finite fractal dimension decreases as well. This fact causes lack of stronger estimates (for example, exponential ones) in particular cases and lack of exponential attractors in the general case as well, as the loss of fractal dimension is the governing factor here.

These results again make the following fact very clear: it is impossible to adequately describe processes and fields with nonlinear and non-smooth interactions using smoothing techniques and the linearization method. Also, the results stimulate the development of adequate analogues of such theorems for non-smooth models.

The first two volumes concentrate mostly on development of constructive methods according to which solutions of general classes of such problems can be studied. Here we answer the questions concerning the long-time behavior of such problems. This fact poses new mathematical problems concerning adequate choice and analysis of a mathematical model (especially problems related to the smoothness of interaction functions). On the other hand, speaking about control and optimization problems, there arises a question concerning the choice of such admissible control sets that would allow more appropriate solution behavior on corresponding attractors. Therefore the new theory has arisen—the theory of extremal solutions to differential-operator problems

$$y' + A(y, y) \ni f. \tag{1}$$

Here, according to the method of artificial control, the new parameter is introduced, and the optimization problem

$$\begin{cases} y' + A(y, u) \ni f, \\ F(u, y) = \|y - u\|_W \to \inf, \; u \in U \subset W \end{cases} \tag{2}$$

takes its place instead of the initial evolutionary inclusion (1). Here W is a functional class of solutions possessing the necessary physical properties, $U \subset W$ is a space of artificial controls.

When problem (1) admits a classical solution in the given class, then it coincides with the solution of problem (2). Otherwise we have an accurate approximation of the initial solution, and, in particular, we can study the asymptotical behavior of all such solutions. Moreover, the results of our investigations are not conditional, unlike other known similar results (in particular, we apply our results to the 3-dimensional Navier–Stokes equation).

Our theory supplements well-known results for mathematical models of processes with smooth interaction functions. Rejecting smoothness in general cases, we guarantee general topological properties for attractors, which exist for smooth models. At the same time, we may lose properties related to estimates of the attractor dimension, though these properties are not natural ones for smooth models in general cases.

This book arose from seminars and lecture courses on multi-valued and nonlinear analysis and their geophysical application. These courses were delivered for rather different categories of learners in the National Technical University of

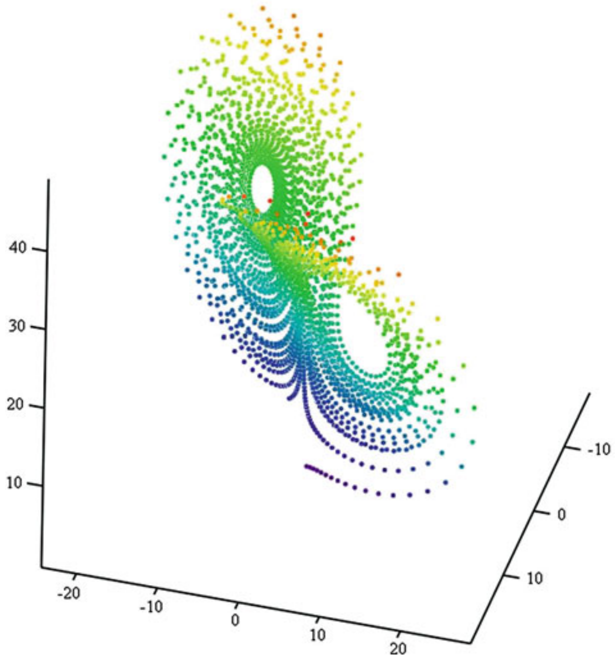

Fig. 1 Lorenz attractor (by M.Z. Zgurovsky, P.O. Kasyanov, O.V. Kapustyan, J. Valeró, N.V. Zadoianchuk, 2011)

Ukraine, “Kiev Polytechnic Institute”, Taras Shevchenko National University of Kyiv, Universidad Miguel Hernández de Elche, University of Salerno etc. over 10 years. It is meant for a wide circle of mathematical and engineering students.

It is unnecessary to state that the pioneering work of such authors as J.M. Ball, V.S. Mel’nik, J.-L. Lions, V.V. Obukhovskii, N. Panagiotopoulos, N.S. Papageoriou, and I.V. Sergienko who created and developed the theory of mentioned problems, exerted a powerful influence on this book.

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We apologize to people whose work has been missed inadvertently when making the references.

We will be grateful to readers for comments and corrections.

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