Preface

This is the second of three books devoted to classical mechanics. The first book, entitled *Classical Mechanics: Statics and Kinematics*; the third, being coauthored by Z. Koruba, is entitled *Classical Mechanics: Applied Mechanics and Mechatronics*. All three books reference each other, and hence they are highly recommended to the reader. In this book dynamical and advanced mechanics problems are stated, illustrated, and discussed, and a few novel concepts, in comparison to standard textbooks and monographs. Apart from being addressed to a wide spectrum of graduate and postgraduate students, researchers, and instructors from the fields of mechanical and civil engineering, this volume is also intended to be used as a self-contained body of material for applied mathematicians and physical scientists and researchers.

In Chap. 1 the dynamics of a particle and system of particles, as well as rigid-body motion about a point, are studied. Section 1.1 is focused on particle dynamics. First, Newton's second law of motion is revisited. Classification of forces is carried out, and Newton's second law is formulated in cylindrical, spherical, and polar coordinates. Forward and inverse dynamics problems are defined and analyzed. The dynamics of a particle subjected to the action of a particular excitation from the previously classified forces is studied. Governing second-order ordinary differential equations (ODEs) are derived and then solved. An illustrative example of particle motion along an ellipse is provided. The laws of conservation of momentum, angular momentum, kinetic energy, and total (mechanical) energy are introduced, illustrated, and examined. In Sect. 1.2, the fundamental laws of the collection of particles (discrete or continuous) are introduced and studied. In the case of momentum conservation of a continuous mechanical system, two important theorems are formulated. Then the conservation of the center of gravity of either a discrete or continuous mechanical system is described. Essential corollaries and principles are formulated, and an illustrative example is provided. Next, the conservation of the angular (kinetic) momentum of a discrete mechanical system is considered. Five important definitions are introduced including Köning's systems, and Köning's theorem is formulated. A kinetic-energy formula is derived. Next, the conservation of angular momentum of a discrete (lumped) mechanical system is studied.

A theorem regarding the necessary and sufficient condition for the existence of the first integral of angular momentum is formulated and proved. In addition, two examples are provided. In what follows the formulation of the law of conservation of kinetic energy is given. Body motion about a point is analyzed in Sect. 1.3.

Mathematical and physical pendulums are studied in Chap. 2. In Sect. 2.1 second-order ODE governing the dynamics of a mathematical pendulum is derived and then explicitly solved for two different sets of initial conditions. In addition, a mathematical pendulum oscillating in a plane rotating with constant angular velocity is analyzed. A physical pendulum is studied in Sect. 2.2. Again a governing dynamics equation is formulated. In the case of a conservative system mechanical energy of the physical pendulum is also derived. Section 2.3 concerns the plane dynamics of a triple physical pendulum. Initially, three second-order ODEs governing the dynamics of a triple pendulum are derived, and then they are presented in matrix notation. Since the obtained differential equations are strongly non-linear, they are then solved numerically. In particular, periodic, quasiperiodic, and chaotic motions are illustrated and discussed. Furthermore, the dynamic reactions in pendulum joints are determined and monitored.

In Chap. 3 static and dynamic problems of discrete mechanical systems are discussed. In Sect. 3.1 the constraints and generalized coordinates are defined. That is, geometric, kinematic (differential), and rheonomic (time-dependent), as well as holonomic and non-holonomic, constraints are illustrated and analyzed through several examples. Furthermore, unilateral and bilateral constraints are introduced and explained using two illustrative examples. Possible and ideal virtual displacements are also introduced and further examples are provided. Variational principles of Jourdain and Gauss are introduced in Sect. 3.2, and their direct application to static problems is illustrated through two examples. The general equations of statics, as well as the stability of equilibrium configurations of mechanical systems embedded in a potential force field, are considered in Sect. 3.3. Important theorems as well as four principles are formulated, and two examples illustrating theoretical considerations are provided. In Sect. 3.4 the Lagrange equations of the second and first kind are rigorously derived. Both discrete and continuous mechanical systems are considered, and some particular cases of the introduced various constraints are analyzed separately. Five illustrative examples are given. In Sect. 3.5 properties of Lagrange's equation, i.e., covariance, calibration invariance, kinetic-energy form, non-singularity, and the least action principle, are briefly described. The first integrals of the Lagrange systems are derived and discussed in Sect. 3.6. Cyclic coordinates are introduced, and two theorems are formulated. Routhian mechanics is briefly introduced in Sect. 3.7. Using the Legendre transformation, we derive Routh's equations. Next, in Sect. 3.8 the cyclic coordinates are discussed, and their validity is exhibited through examples. A three-degree-of-freedom manipulator serving as an example of rigid-body kinetics is studied in Sect. 3.9. First, physical and mathematical models are introduced, then the Denavit–Hartenberg notation is applied, and the obtained differential equations are solved numerically. Furthermore, the results of numerical simulations are discussed on the basis of an analysis of three different cases, and some conclusions are formulated.

Chapter 4 is devoted to classical equations of mechanics. Section 4.1 concerns Hamiltonian mechanics, Sect. 4.2 describes methods of solution of the Euler–Lagrange equations, Sect. 4.3 deals with Whittaker equations, Sect. 4.4 concerns Voronets and Chaplygin equations, and, finally, Sect. 4.5 includes both a derivation and discussion of Appell's equations. In Sect. 4.1.1 the canonically conjugate variables are introduced, and Hamilton's canonical equations are derived. An example is added for clarification. In Sect. 4.1.2 the Jacobi–Poisson theorem is formulated and proved, followed by an introduction to the Poisson bracket. Canonical transformations are discussed in Sect. 4.1.3, where six theorems are also given. Non-singular canonical transformations and guiding functions are introduced in Sect. 4.1.4, whereas the Jacobi method and the Hamilton–Jacobi equations are presented in Sect. 4.1.5. Then two particular cases of Hamilton–Jacobi equations are considered in Sect. 4.1.6, i.e., the Hamilton–Jacobi equations for cyclic coordinates and conservative systems. In Sects. 4.2.1–4.2.4 solutions of the Euler–Lagrange equations are presented. Section 4.2.2 includes definitions of weak and strong minima and Euler's theorem. The Bogomol'nyi equation and decomposition are briefly stated in Sect. 4.2.3, whereas the Bäcklund transformation is described in Sect. 4.2.4 supplemented with two examples. Whittaker's equations are derived in Sect. 4.3, whereas the Voronets and Chaplygin equations are formulated in Sect. 4.4. Applications of Chaplygin's equations are presented as an example of a homogeneous disk rolling on a horizontal plane. The Appell equations, followed by an example, are derived in Sect. 4.5.

Classical impact theory is introduced and illustrated in Chap. 5. Basic concepts of phenomena associated with impact are presented in Sect. 5.1. The fundamental laws of an impact theory such as conservation of momentum and angular momentum are given in Sect. 5.2, where two theorems are also formulated. A particle's impact against an obstacle is studied in Sect. 5.3, and the physical interpretation of impact is given in Sect. 5.4. Next (Sect. 5.5) the collision of two balls in translational motion is analyzed. In particular, kinetic-energy loss during collision is estimated through the introduced restitution coefficient. In Sect. 5.6 a collision of two rigid bodies moving freely is studied, and in Sect. 5.7 a center of percussion is defined using as an example the impact of bullet against a compound pendulum.

Chapter 6 deals with vibrations of mechanical systems. After a short introduction multi-degree-of-freedom mechanical systems are studied. In Sect. 6.2 linear and non-linear sets of second-order ODEs are derived from Lagrange's equations. Classification and properties of mechanical forces are presented for linear systems in Sect. 6.3. Subsequently, the dissipative, gyroscopic, conservative, and circulatory forces are illustrated and discussed in general and using an example. In Sect. 6.4 small vibrations of linear one-degree-of-freedom mechanical systems are presented. Both autonomous and non-autonomous cases are considered, and (contrary to standard approaches) solutions are determined for homogenous/non-homogenous equations using the notion of complex variables. Amplitude and phase responses are plotted in two graphs, allowing for direct observation of the influence of damping magnitude on the amplitude-frequency and phase-frequency plots. In particular, resonance and non-resonance cases are discussed. In addition, transverse vibrations of a disk mounted on a flexible steel shaft modeled by two second-order linear ODEs are analyzed, and the critical speeds of the shaft are defined. Both cases, i.e., with and without damping, are studied. The phenomenon of shaft self-centering is illustrated. Finally, a one-degree-of-freedom system driven by an arbitrary time-dependent force is studied using Laplace transformations. An illustrative example is given. A one-degree-of-freedom non-linear autonomous and conservative system is studied in Sect. 6.5. It is shown step by step how to obtain its period of vibration. In addition, the so-called non-dimensional Duffing equation is derived. One-degree-of-freedom systems excited in a piecewise linear or impulsive fashion are studied in Sect. 6.6. It is shown how to find the corresponding solutions.

The dynamics of planets is briefly studied in Chap. 7. In the introduction, Galileo's principle of relativity is discussed. Second-order vector ODEs are presented and the homogeneity of space and time are defined. Potential force fields are introduced in Sect. 7.2, whereas Sect. 7.3 is devoted to the analysis of two-particle dynamics. Total system energy, momentum, and angular momentum are derived. In addition, a surface integral and the Laplace vector are defined explicitly, and their geometrical interpretations are given. First and second cosmic velocities are defined, among others. Kepler's three laws are revisited.

The dynamics of variable mass systems is studied in Chap. 8. After a short introduction, the change in the quantity of motion and angular momentum is described in Sect. 8.2. Then an equation of motion of a particle of variable mass (the Meshcherskiy equation) is derived. Two Tsiolkovsky problems are studied in Sect. 8.4. Finally, in Sect. 8.4.1 an equation of motion of a body with variable mass is derived and studied. Two illustrative examples are given.

Body and multibody dynamics are studied in Chap. 9. First, in Sect. 9.1, the rotational motion of a rigid body about a fixed axis is introduced. In Sect. 9.2 Euler's dynamic equations are derived, and the so-called Euler case is analyzed. Poinsot's geometric interpretation of rigid-body motion with one fixed point is illustrated. The roles of a polhode and a herpolhode are discussed. In Sect. 9.3 the dynamics of a rigid body about a fixed point in the gravitational field is studied. The Euler, Lagrange, and Kovalevskaya cases, where first integrals have been found, are also briefly described. The general free motion of a rigid body is analyzed in Sect. 9.4. In Sect. 9.5, the motion of a homogenous ball on a horizontal plane in the gravitational field with Coulomb friction is modeled and analyzed. Equations of motion are derived and then solved. The roles of angular velocities of spinning and rolling and the associated roles of the rolling and spinning torques are illustrated and discussed. Section 9.6 deals with the motion of a rigid body of convex surface on a horizontal plane. Equations of motion are supplemented by the Poinsot equation, and the dynamic reaction is derived. Dynamics of a multibody system coupled by universal joints is studied in Sect. 9.7. Equations of motion are derived using Euler's angles and Lagrange equations of the second kind. Conservative vibrations of a rigid body supported elastically in the gravitational field are analyzed in Sect. 9.8, and one illustrative example is provided. Wobblestone dynamics is studied in Sect. 9.9. The Coulomb–Contensou friction model is first revisited, and the importance of the problem is exhibited emphasizing a lack of a correct and complete solution of the stated task in Sect. 9.9.1. Three vectorial equations of motion are derived, followed by a tenth scalar equation governing the perpendicularity condition. Several numerical simulation results are presented. Next, a hyperbolic tangent approximation of friction spatial models are introduced and discussed in Sect. 9.9.2. The advantages and disadvantages of the introduced approximation versus the Padé approximations are outlined. A few numerical simulation results are given.

Stationary motions of a rigid body and their stability is studied in Chap. 10. It includes problems related to stationary conservative dynamics (Sect. 10.1) and invariant sets of conservative systems (Sect. 10.2).

A geometrical approach to dynamical problems is the theme of Chap. 11. In Sect. 11.1, the correspondence between dynamics and a purely geometrical approach through the Riemannian space concept is derived. It is shown how dynamical problems, supported by a configuration space and the Jacobi metric, are reduced to an equation of geodesic deviation known also as the Jacobi–Levi–Civita (JLC) equation (Sect. 11.3). Next, in Sects. 11.4 and 11.5, the Jacobi metric on a configuration space is rigorously defined, the JLC equation is derived, and then it is rewritten in geodesic coordinates. Finally, a two-degree-of-freedom mechanical system is used to illustrate the theoretical background introduced earlier in Sect. 11.6.

Finally, it is rather impossible nowadays to write a comprehensive book on classical mechanics and include an exhaustive bibliography related to classical mechanics. Therefore, this volume and the two related books mentioned earlier, are rather located in standard classical mechanics putting emphasis on some important topics being rarely mentioned in published literature on mechanics.

Furthermore, in the particular case of dynamics/dynamical systems, there is a vast number of books that are either devoted directly to classical dynamics or that include novel branches of dynamics like stability problems, bifurcational behavior, or deterministic chaos. Although the latter material is beyond the book contents, the reader may be acquainted with my authored co-authored books/monographs devoted to the mentioned subjects, i.e., *Bifurcation and Chaos in Simple Dynamical Systems*, J. Awrejcewicz (World Scientific, Singapore, 1989); *Bifurcation and Chaos in Coupled Oscillators*, J. Awrejcewicz (World Scientific, Singapore, 1991); *Bifurcation and Chaos: Theory and Application*, J. Awrejcewicz (Ed.) (Springer, New York, 1995); *Nonlinear Dynamics: New Theoretical and Applied Results*, J. Awrejcewicz (Ed.) (Akademie Verlag, Berlin, 1995); *Asymptotic Approach in Nonlinear Dynamics: New Trends and Applications*, J. Awrejcewicz, I.V. Andrianov, and L.I. Manevitch (Springer, Berlin, 1998); *Bifurcation and Chaos in Nonsmooth Mechanical Systems*, J. Awrejcewicz and C.-H. Lamarque (World Scientific, Singapore, 2003); *Nonlinear Dynamics of a Wheeled Vehicle*, J. Awrejcewicz and R. Andrzejewski (Springer, Berlin, 2005); *Smooth and Nonsmooth High Dimensional Chaos and the Melnikov-Type Methods*, J. Awrejcewicz and M.M. Holicke (World Scientific, Singapore, 2007); *Modeling, Simulation and Control of Nonlinear Engineering Dynamical Systems: State of the Art, Perspectives and Applications*, J. Awrejcewicz (Ed.) (Springer, Berlin, 2009).

The dynamics of continuous mechanical systems governed by PDEs and completely omitted in this book is widely described in the following books/monographs authored/co-authored by the present author: *Nonclassical Thermoelastic Problems in Nonlinear Dynamics of Shells*, J. Awrejcewicz and V.A. Krysko (Springer, Berlin, 2003); *Asymptotical Mechanics of Thin Walled Structures: A Handbook*, J. Awrejcewicz, I.V. Andrianov, and L.I. Manevitch (Springer, Berlin 2004); *Nonlinear Dynamics of Continuous Elastic Systems*, J. Awrejcewicz, V.A. Krysko and A.F. Vakakis (Springer, Berlin, 2004); *Thermodynamics of Plates and Shells*, J. Awrejcewicz, V.A. Krysko, and A.V. Krysko (Springer, Berlin, 2007); *Chaos in Structural Mechanics*, J. Awrejcewicz and V.A. Krysko (Springer, Berlin, 2008); *Nonsmooth Dynamics of Contacting Thermoelastic Bodies*, J. Awrejcewicz and Yu. Pyryev (Springer, New York, 2009).

The author wishes to express his thanks to P. Dąbek and M. Kamierczak for their help in the book preparation.

Many helpful comments and suggestions given by the author's coworkers from the Łódź University of Technology, Poland, and in particular the contributions of K. Januszkiewicz, J. Mrozowski, and G. Kudra are gratefully acknowledged.

It should also be emphasized that a part of Chaps. 3 and 9 was prepared together with G. Kudra, and Chap. 11 was written in cooperation with D. Sendkowski.

Finally, I acknowledge the Humboldt Foundation Award and the hospitality of Professor P. Hagedorn during my stay at Fraunhofer Institute in Darmstadt, Germany, as well as the support of the Master Award of the Polish Foundation of Science (FNP) during the final stage of the book's preparation.

Łódź and Darmstadt Jan Awrejcewicz