

PREFACE

Over the years that I have been teaching logic, I have become convinced that to teach it effectively, one needs to convey two things: the how and the why of logic. For (to adapt a phrase from Kant) the why without the how is empty, and the how without the why is blind.

Conveying the how and the why of logic is the aim of this book. The book explains how to do logic: it presents the tools and techniques of modern logic in a clear and accessible way. It also explains why things are done the way they are in logic: the purpose of the tools and the rationales behind the techniques. In a nutshell, the why comes down to this: the aim of logic is to discern the laws of truth.

Coverage

The book is a thorough introduction to classical logic (also known as first-order logic with identity). Part I covers propositional logic, and Part II covers predicate logic and identity. Part III covers three topics: basic metatheory for the system of tree proofs used in Parts I and II (soundness and completeness results are proven; decidability and undecidability results are discussed but not proven), the major alternative systems of proof (axiomatic proofs, natural deduction, and sequent calculus), and basic notions from set theory. Some of these notions from set theory are employed earlier in the book, so the final chapter, in which these notions are explained, is more in the nature of an appendix: it can be read piecemeal, as and when necessary. (When an earlier section presupposes something explained in the final chapter, there is a reference forward at that point to the relevant section of the final chapter.)

Readership

When writing the book, I had two primary target audiences in mind. First, the book is designed for use as a textbook in a standard comprehensive introductory logic course—that is, a course that has no prerequisites and is open to

students from all faculties and majors. Such courses are often taught by philosophy departments but attract students from across the humanities, natural sciences, social sciences, mathematics, engineering, computer science, law and health sciences.

Second, the book is intended to be suitable for independent study. In particular, students at the end of their undergraduate work or the beginning of their graduate studies—in philosophy, linguistics, and other subjects—often find that they need to know logic, but the opportunity has passed for them to take an undergraduate course in the subject. Others have learned some of the techniques of formal logic but want a better sense of how they relate to one another, or a deeper understanding of what these techniques amount to. This book should be well suited to such readers.

Parts of the book also constitute a contribution to the philosophy of logic and should be of interest even to specialists. Chapter 11 and §13.6.3 are the most obvious—but not the only—examples.

Choice of Proof Systems

The first question when introducing logic is: what system of proof should be used? The most popular choices are trees, or one or another flavor of natural deduction. In this book, logic is introduced via trees, in Parts I and II. Then in Part III—once the reader already has a good understanding of logic via trees—Chapter 15 presents all major forms of all other major proof systems: axiomatic, natural deduction, and sequent calculus. This broad exposure is, I believe, important, because after a proper introduction to logic, one should not find oneself in the position of picking up a different logic book and thinking “what on Earth is this?!”—a position students all too commonly find themselves in when they learn, say, trees and then encounter natural deduction (or vice versa), or even when they learn, say, Fitch-style natural deduction and then pick up a book that uses Gentzen-style natural deduction.

Selecting Material

The basic rationale behind the choice of material to cover was as follows. Studies in introductory logic lead naturally to further studies in many different areas, for example,

1. nonclassical logics (extensions of and alternatives to classical logic, e.g., modal, tense, intuitionist, relevance, many-valued, fuzzy, and free logics; as in Priest [2008], Burgess [2009]),
2. mathematical logic (more advanced metatheory, e.g., Löwenheim-Skolem and compactness theorems, undecidability of first-order logic,

Gödel's incompleteness theorems; as in Enderton [2001], Boolos et al. [2007]),

3. theory of computation (e.g., models of computation—automata of various kinds, Turing machines, register machines; computable and uncomputable functions; computational complexity; algorithmic complexity; as in Davis et al. [1994], Sipser [2006]),
4. philosophical logic (e.g., monism versus pluralism, normativity of logic, logic and reasoning, logic and ordinary language, theories of truth, analysis of logical consequence; as in Haack [1978], Hughes [1993]),
5. set theory (e.g., axiomatic set theories, consistency and independence, foundations of mathematics; as in Devlin [1993], Hrbacek and Jech [1999]), and
6. formal semantics (e.g., generalized quantifiers, theory of types, categorial grammar, intensional semantics, Montague grammar; as in Gamut [1991b], Heim and Kratzer [1998]).

The idea behind the present book was not to cover these further areas, but to cover introductory logic in sufficient detail to enable readers interested in any of these areas to see how they connect to introductory logic. At the same time, I wanted to cover the core of introductory logic in a way that will be useful and accessible to readers not intending to pursue logic any further.

The upshot of this approach is that there is more in the book than can be covered in a typical introductory logic course. The core material is as follows (excluding endnotes):

- Propositional logic: Chapter 1 (excluding §1.2.2), Chapter 2 (excluding §2.2.1, §2.5.4 after the first three paragraphs, §2.5.5, and §2.5.6), Chapter 3 (excluding §3.5), Chapter 4 (excluding §4.3.2 after the first paragraph) and Chapter 7.
- Predicate logic: Chapter 8 (excluding §8.3.3 after the first two paragraphs), Chapter 9, Chapter 10 (excluding §§10.3.3–10.3.7), and Chapter 12 (excluding §12.4 and §12.5).
- Identity: Chapter 13 (excluding §§13.5–13.7).

In these core parts of the book, the pace is gentle and the presentation maximally accessible. At the same time, the material is presented in proper detail (in contrast to texts that are gentle and reader friendly at the cost of presenting a simplified version of the material, e.g., a watered-down version of the model theory for first-order logic, or no model theory at all). When moving outside the core sections, readers may at times notice a slight increase in pace.

Those parts of the book not included in the above list of core material can be used in various ways. Independent readers and teachers with the requisite time available can cover one or more of them, according to time and interests. They can be used as material for extension classes. Students wishing to prepare for subsequent logic courses during the summer after introductory logic can read one or more of these parts. They can also be used as bridging material at the beginning of subsequent logic courses. To help readers work out which of the non-core parts they might like to cover, here is a rough indication of how these parts relate to the six areas of further study mentioned above. The numbers in boldface refer to the earlier list of six areas of study; **G** indicates general relevance: §1.2.2: **4, 6**. §2.2.1: **4, 6**. §2.5.4 after the first three paragraphs: **G**. §2.5.5: **G**. §2.5.6: **2, 3**. §3.5: **G**. §4.3.2 after the first paragraph: **G**. Chapter 5: **G**. Chapter 6 up to §6.6: **1, 4, 6**; §6.6: **G**. §8.3.3 after the first two paragraphs: **4, 6**. §§10.3.3–10.3.7: **2, 3**. Chapter 11 as a whole: **1, 4, 6**; §11.3: **2**. §12.4: **4, 6**. §12.5: **2**. §13.5: **G**. §13.6: **1, 4, 6**. §13.7: **2, 6**. Chapter 14 as a whole: **2**; §14.2, §14.3: **3**; §14.4: **1, 4, 6**. Chapter 15 as a whole: **1, 2, 4**; §15.1: **5**. Chapter 16: **G**.

Exercises and Solutions

Working out problems is crucial to learning logic, so the book contains numerous exercises. Being able to see whether one has done the exercises correctly is also crucial; hence, answers are available on the accompanying website (<http://www.press.princeton.edu/titles/9727.html>). Because the answers are online, the exercises cannot be used as take-home assessment tasks, but they could not be used for this purpose anyway: as soon as exercises are published, answers to them begin to circulate, either online or through personal networks within colleges and universities.

The exercise questions are also reproduced on the website. Additional exercises—and perhaps other resources—may be added in time. Any typographical errors or other mistakes in the exercises or answers will be corrected in the online version as I become aware of them.