

## Introduction

“Chases and escapes” are activities that might be said to define human existence, no matter where or when, ranging over the entire spectrum of violence from the romantic pursuit of a future spouse to the military pursuit of a target to be destroyed. Young children are introduced to both chases and escapes when they learn the simple rules of the game of tag: played by at least two, one player is designated as *it*, who then chases after all the others, who, of course, attempt to escape. When the *it* player manages to catch one of the evaders he or she yells “Tag! You’re *it*!,” that evader and the pursuer change roles, and the game continues until all drop from exhaustion. Parents love tag! The related game of hide-and-seek, also of this genre, is equally attractive to young children (and to their parents). And surely all readers can remember the fun of watching the old Looney Tunes Road Runner/Wile E. Coyote chase-and-escape cartoons. Hollywood’s interest in chases dates, in fact, at least as far back as the 1914 serial *The Perils of Pauline*, with countless imaginative escapes by the heroine from her dastardly guardian’s attempts to do her in.

As we grow older we leave hide-and-seek, tag, and cartoons behind, but there remains, for most people, a deep interest in pursuit-and-evasion; the fascination simply becomes more sophisticated. The inherent conflict of chasing and escaping, of the battle between the hunter and the hunted, is the basis for at least half the fictional writing in the world. By their high school years, for example, most students

## 2 Introduction

have read the exciting *Collier's Magazine* story “The Most Dangerous Game” by Richard Connell. That 1924 tale of a shipwrecked man (Sangar Rainsford, a celebrated hunter) has the ironic twist of a famous hunter becoming the hunted. At first he finds what he thinks is salvation from drowning at sea as he stumbles ashore onto a nearby jungle-covered island, inhabited by one General Zaroff, who oddly finds himself regularly visited by shipwrecked sailors — probably because he has installed a false lighthouse to lure passing ships onto rocks. Rainsford soon changes his mind about salvation when he learns that the General provides his “visitors” with hunting clothes, food, and a knife, and then turns them loose back into the jungle. After a three-hour head start the General comes after them, armed only with a small caliber automatic pistol — if the General fails to make a kill within three days he promises to return his elusive prey to the mainland. As he explains to Rainsford, however, he has never failed to make the kill. The rest of the story, assigned to generations of high school English students by teachers who clearly admire Connell’s tale, is of the suspenseful chase.

The printed story won a prestigious O’Henry Short Story Award, and was made into an exciting movie of the same name in 1932 (Joel McCrea played Rainsford, with his first name changed for some reason from Sangar to *Bob!*, and Fay Wray played a female role not in the original story; this last point is particularly interesting to note since the film was shot between takes, on the same set, of the classic film *King Kong*, which was released the next year in 1933 — starring, of course, Fay Wray). *The Most Dangerous Game* was remade at least two more times: *A Game of Death* (1946) and *Run for the Sun* (1956). There were numerous obvious derivatives of an inferior nature, as well, for example, the 1961 *Bloodlust!*, in which a mad scientist hunts teenagers and displays his catches in glass cases (this movie is notable for having embarrassed one of its stars, Robert Reed, even more than did his later role as Mike Brady, the perfect father on television’s *The Brady Bunch*).

Jumping ahead a few decades, most readers of adventure fiction have enjoyed Frederick Forsyth’s masterful 1971 novel *The Day of the Jackal*, in which a mysterious assassin-for-hire attempts to kill Charles de Gaulle, the president of France. We all know that de Gaulle was

*not* assassinated in real life but, nonetheless, the writing of the killer's chase of his target — and of the intertwined police chase of the assassin — is so well done you think he's going to pull it off anyway. (The book was made into a successful film of the same title in 1973.) The novel was not the first of its type, being a clever reversal of roles from what I think the perfect pursuit-and-evasion fictional work — British writer Geoffrey Household's 1939 novel *Rogue Male* (made into the 1941 film *Man Hunt*). That work is a first-person, page-turning narrative of a world famous English hunter who makes the mistake of being caught sighting a high-powered telescopic rifle onto the distant figure of an unnamed European dictator. (Only a late-1930s reader as dense as a rock could have failed to recognize Hitler.) He is interrogated by his captors, that is, the Gestapo, and then escapes by a fantastic ruse back to England, where his enemies continue to pursue him ruthlessly. The book has a wonderful ending, telling the reader that matters have most definitely *not* ended and the chase is by no means over. This ending was literally a call-to-arms to the world of 1939 to challenge Hitler and defeat him.

A little thought will convince you that Hollywood continues to love a good chase and (perhaps) escape — who, for example, can think of Steve McQueen's *Bullitt* (1968) without instantly visualizing that incredible car chase through the hilly streets of San Francisco? Other successful "chase" films include *Predator* (1987), *The Hunt for Red October* (1990), *The Fugitive* (1993), *Alien* (1979), *Catch Me If You Can* (2002), and the very best such film of all (in my opinion), Cornel Wilde's powerful portrayal of a naked man on the run from headhunters in his brilliant *The Naked Prey* (1961).

A relatively new (since the early 1980s) form of entertainment, computer video games, has embraced the chase-and-escape concept as the basis for some of the most successful of all such games (e.g., *Thief*, *Call of Duty*, *Half-Life*, *Max Payne*, *CounterStrike*, *Medal of Honor*, and *Hitman*, all of which have had sequels to their original appearances). These games, called first-person shooters (so named because the screen image is the "world" as seen through the eyes of the protagonist) can be quite violent — typically, that image includes "your" hand holding a weapon for instance, a knife, gun, or club. The sales pitch for one of the *Hitman* games, for example, is "See the world, meet

## 4 Introduction

interesting people, kill them” — in *Hitman* you play as an assassin-for-hire who is assigned targets, mostly international terrorists and crime lords, to hunt for “termination with extreme prejudice.” These games can be subtle, too, however — the emphasis in the *Thief* games is not on killing at all (although you *do* carry a sword or dagger, and a bow with arrows), but rather on being able as a medieval thief to skillfully skulk about in the shadows and, if at all possible, to *avoid* confrontation while attempting to complete the mission goals. Whether violent or (relatively) benign, the successful first-person-shooter games *are* fun — many are even educational — to experience: playing as the conscript Alexei Ivanovich (a Russian sniper at the Second World War Battle of Stalingrad) in *Call of Duty* can only be described as both ear-shattering and eye-opening, as well as masterfully capturing the horror of what happened there on the rooftops, in the sewers, and on the bombed-out Stalingrad streets in a way that I think equal to the best historical writing (e.g., Antony Beevor’s 1998 *Stalingrad, The Fateful Siege: 1942–1943*).

The mathematical puzzles of chases and escapes were recognized quite early in the history of mathematics, for example, with the paradox given by the fifth-century B.C. Greek mathematician Zeno of Elea, of the race between Achilles (Homer’s hero in the *Iliad*) and the tortoise. As Smith (1917) says about the origin of all pursuit problems, “It would be difficult to conceive of a problem that would seem more real, since we commonly overtake a friend in walking, or are in turn overtaken. It would therefore seem very certain that this problem is among the ancient ones in what was once looked upon as higher analysis. We have a striking proof that this must be the case in the famous paradox of Achilles and the Tortoise . . .” (Zeno may well have been inspired to think upon the problem of *linear* pursuit — a chase along a straight line — by the even more ancient Aesop fable of the race between the hare and the tortoise. That tale predates Zeno by at least a century; its intent, unlike Zeno’s paradox, was not mathematical at all, but rather to serve as an illustration of the *moral lesson* that “perseverance wins the race.”) Zeno’s paradox is easy to explain.

Achilles, fleet of foot, runs ten times as fast as the tortoise and so, to make their race more interesting, he gives the tortoise a ten-mile head start. Zeno’s paradox is the result of a seductive argument

that seems to prove that Achilles can never even catch the tortoise, much less pass him to win the race. This is a conclusion that is seen to be incorrect if one simply watches a real race! And that's the paradox, of course — where does Zeno's argument (which *is* pretty convincing to most young students when they first encounter it) go wrong? Zeno's argument is as follows: by the time Achilles runs through the initial ten mile head start, the tortoise will have run through one mile, and by the time Achilles runs through that one mile lead the tortoise will have run through one-tenth of a mile, and by the time Achilles runs through that one-tenth of a mile lead the tortoise will have run through one-hundreth of a mile, etc., etc., etc. This process continues through an *infinity* of ever decreasing leads that the tortoise has over Achilles and so, concludes Zeno, Achilles can never catch the tortoise.

The flaw in Zeno's argument escaped understanding for a very long time. Intellects as profound as those of Plato and Aristotle were befuddled by the problem, with Aristotle (in his *Physics*) correctly calling it a fallacy without being able to explain *why*. Today, twenty-five centuries later, high school students learn that the answer is that an infinity of numbers (the infinity of times it takes Achilles to run through the infinity of leads the tortoise has) does *not* necessarily have to add up to infinity. To see this, let  $v$  be the speed of the tortoise (and so  $10v$  is the speed of Achilles). Thus, to run through the ten-mile head start takes Achilles a time of  $10/10v$ . Then, to run through the now one-mile lead of the tortoise takes Achilles  $1/10v$ . To run through the now one-tenth-of-a-mile lead takes Achilles  $1/10/10v$ . To run through the now one-hundreth-of-a-mile lead takes Achilles  $1/100/10v$ . And so on. So, the total time required for Achilles to catch the tortoise is

$$\frac{10}{10v} + \frac{1}{10v} + \frac{1/10}{10v} + \frac{1/100}{10v} + \cdots = \frac{1}{v} \left( 1 + \frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \cdots \right).$$

The expression in the parentheses is a geometric series, easily summed to  $10/9$ , that is, the total time it takes Achilles to catch the tortoise is the finite time  $10/9v$ , which of course we see is simply the time to run through the initial head start of ten miles at Achilles' *closing speed* of  $10v - v = 9v$ .

## 6 Introduction

A different sort of pursuit-and-evasion problem, of great interest today among computer scientists because it requires skill in handling not a differential equation but rather what is called a *data structure*, also has an ancient origin: the Greek mythological story of Theseus and the Minotaur, which appears in the written word as early as in Ovid's *Metamorphoses* (dating from the time of Christ), although the myth itself is even more ancient, by *centuries*. The Minotaur, the result of an unnatural union between the wife of King Minos of Crete and a bull, was a flesh-eating monster with the head of a bull and the body of a man — but Edith Hamilton's classic book *Mythology* contains an illustration of the Minotaur with the head of a man and the body of a bull. This monster was kept confined by Minos to a complicated maze of connected passages called the Labyrinth, and once a year the creature was given a sacrifice offering of seven youths and seven maidens. These unfortunates were simply tossed into the Labyrinth, which was so complicated that they soon became lost and unable to find their way free. Eventually the Minotaur would come upon them in their confused wanderings and they were quickly devoured. The Minotaur's reign of terror was finally ended by Theseus, who, before entering the Labyrinth, tied one end of a ball of string to the entry. Then, after wandering through the maze until he found the Minotaur asleep — which he then killed — Theseus was able to escape by simply following the string back to the entry point. A popular undergraduate computer science programming project assignment is the writing of the code to simulate Theseus' hunt for the Minotaur through a Labyrinth of any desired structure, that is, one with arbitrarily complex connectivity.

All of the above discussion was to make the case that the subject of this book resonates with a fundamental psychological aspect of human nature. But let's now leave mere fictional games of chases and escapes, and turn our attention to the really good stuff; let's take a long look at the *mathematics* of pursuit-and-evasion. The intellectual demands will be significant; as Davis (1962) wrote, pursuit problems with "their curious difficulties have intrigued the fancy and strained the ingenuity of modern mathematicians." You'll agree by the time you finish, but I also think you'll see it will have been well worth the effort.