

# Preface

*Utinam exitus acta probet*

Global optimization plays an outstanding role in applied mathematics, because a huge number of problems arising in natural sciences, engineering, and economics can be formulated as global optimization problems. It is impossible to overlook the literature dealing with the subject. This is due to the fact that-unlike local optimization-only small and very special classes of global optimization problems have been investigated and solved using a variety of mathematical tools and numerical approximations. In summary, global optimization seems to be a very inhomogeneous discipline of applied mathematics (comparable to the theory of partial differential equations). Furthermore, the more comprehensive the considered class of global optimization problems, the smaller the tractable scale of problems.

In this book, we try to overcome these drawbacks by the development of a homogeneous class of numerical methods for a very large class of global optimization problems. The main idea goes back to 1953, when Metropolis et al. proposed their algorithm for the efficient simulation of the evolution of a solid to thermal equilibrium (see [Met.etal53]).

In [Pin70], the analogy between statistical mechanics and optimization is already noticed for the first time. Since 1985, one tries to use this analogy for solving unconstrained global optimization problems with twice continuously differentiable objective functions (see [Al-Pe.etal85], [GemHwa86], and [Chi.etal87]). These new algorithms are known as *simulated annealing*. Unfortunately, simulated annealing algorithms use so-called cooling strategies inspired by statistical mechanics in order to solve global optimization problems but neglect the existence of efficient local optimization procedures. Hence, these cooling strategies lead to unsatisfactory practical results in general.

The analogy between statistical mechanics and global optimization with constant temperature is analyzed for the first time in [Schä93] using Brownian Motion and using the stability of random dynamical systems. This analysis forms the basis of all methods developed in this book. As a result, the application of the equilibrium theory of statistical mechanics with fixed temperature in combination with the stability theory of random dynamical systems leads to the algorithmic generation of pseudorandom vectors, which are located in the region of attraction of a global optimum point of a given objective function.

Here is an outline of the book. In Chap. 1, stochastic methods in global optimization are summarized. Surveys of deterministic approaches can be found in [Flo00], [HorTui96], and [StrSer00] for instance. In Chap. 2, we develop unconstrained local minimization problems and their numerical analysis using a special type of dynamical systems given by the curve of steepest descent. This approach allows the interpretation of several numerical methods like the Newton method or the trust region method from a unified point of view.

The treatment of global optimization begins with Chap. 3, in which we consider unconstrained global minimization problems. A suitable randomization of the curve of steepest descent by a Brownian Motion yields a class of new non-deterministic algorithms for unconstrained global minimization problems. These algorithms are applicable to a large class of objective functions, and their efficiency does not substantially depend on the dimension of the given optimization problem, which is confirmed by numerical examples. In Chap. 4, we propose a very important application of the results of Chap. 3, namely, the optimal decoding of high-dimensional block codes in digital communications. Chapter 5 is concerned with constrained global minimization problems. Beginning with equality constraints, the projected curve of steepest descent and its randomized counterpart are introduced for local and global optimization, respectively. Furthermore, the penalty approach is analyzed in this context. Besides the application of slack variables, an active set strategy for inequality constraints is developed. The main ideas for global minimization of real-valued objective functions can be generalized to vector-valued objective functions. This is done in the final chapter by the introduction of randomized curves of dominated points.

Appendix A offers a short course in probability theory from a measure-theoretic point of view and Appendix B deals with the algorithmical generation of pseudorandom numbers, which represents the fundament of all numerical investigations in this book. Since we have chosen a stochastic approach for the analysis and numerical solution of global optimization problems, we evidently have to ask whether this approach is adequate when dealing with stochastic global optimization problems. This question is answered in Appendix C by the investigation of gradient information additively disturbed by a white noise process.

The reader of this book should be familiar with

- Initial value problems
- Theory and practice of local optimization
- Topics in probability theory summarized in Appendix A

I gratefully acknowledge the critical comments and suggestions of Prof. K. Pilzweger, Prof. M. Richter, Prof. S. M. Robinson, and Dr. R. von Chossy. Obviously, all errors are my sole responsibility.

I am deeply indebted to the publisher, in particular, to the publisher's representative, Ms. Vaishali Damle, for her patience and understanding.

*This book is dedicated to my teachers and promoters Prof. Klaus Ritter and Prof. Stephen M. Robinson.*