

Preface

Inverse limits with set-valued functions were introduced in 2004 by Bill Mahavier as inverse limits with closed subsets of the unit square. In the short time since then, the subject has rapidly developed into a rich topic of research, particularly among continuum theorists. This new form of inverse limit can also show up in applications to economics and in dynamical systems. For instance, certain models in economics, notably in backward economics, can involve two mappings, and the flexibility to study the effects of using either function at each stage in the model is a valuable feature of inverse limits with set-valued functions. Recent work of Benjamin Marlin on the Lorenz attractor has provided evidence of the value of using set-valued functions in studying attractors in dynamical systems. This short book is not specifically concerned with these applications of set-valued functions. Instead, it is intended to provide a quick introduction to the subject of inverse limits with set-valued functions. The recently published Springer book *Inverse Limits: From Continua to Chaos* contains most of the background a researcher might need to make use of these inverse limits in his or her work. However, a shorter volume such as this one dedicated to inverse limits with set-valued functions could be helpful to someone wanting a quick introduction to this specific subject. This short book is intended to provide just such an introduction. The emphasis here is on inverse limits on the interval $[0, 1]$ much as the first chapter of *Inverse Limits: From Continua to Chaos* serves as an introduction to inverse limits on $[0, 1]$ with mappings. We believe that a deeper understanding of inverse limits can be obtained by studying examples. A major feature of this book is the inclusion of numerous examples and, in many instances, models of the inverse limits. Several of the examples in this volume have not appeared elsewhere in print.

Major differences between the theory of inverse limits with mappings and the theory with set-valued functions arise early in this study. These differences are featured prominently in this book. However, instead of viewing these differences as a negative development, we consider them as an opportunity for further study and research. Indeed, much of the research in the subject is devoted to resolving questions arising for these very reasons. We have included an extensive, but by no means exhaustive, list of currently unsolved problems in the final chapter of the book.

A senior-level course in analysis and, perhaps, an additional one in topology should provide a sufficient introduction to the topology of metric spaces and the topology of product spaces to make the material in this book accessible to advanced undergraduates and, certainly, to graduate students in mathematics. An alternate source of background material is the appendix in the Springer book *Inverse Limits: From Continua to Chaos*. In some of the examples, we assume some familiarity with inverse limits with mappings, but no deep understanding of ordinary inverse limits is necessary to read this book.

This book is based on a series of lectures given by the author at a workshop in the summer of 2011 at the Instituto de Matemáticas, Universidad Nacional Autónoma de México, in Mexico City. Many thanks go to all who participated in that short course. We are particularly indebted to Verónica Martínez de la Vega and Alejandro Illanes who organized the workshop and helped make our stay in Mexico City such a pleasant experience.

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