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978-1-107-00633-1 - Introduction to Aberrations in Optical Imaging Systems

José Sasián

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## INTRODUCTION TO ABERRATIONS IN OPTICAL IMAGING SYSTEMS

The competent and intelligent optical design of today's state-of-the-art products requires an understanding of optical aberrations. This accessible book provides an excellent introduction to the wave theory of aberrations and will be valuable to graduate students in optical engineering, as well as to researchers and technicians in academia and industry interested in optical imaging systems.

Using a logical structure, uniform mathematical notation, and high-quality figures, the author helps readers to learn the theory of optical aberrations in a modern and efficient manner. In addition to essential topics such as the aberration function, wave aberrations, ray caustics, and aberration coefficients, this text covers pupil aberrations, the irradiance function, aberration fields, and polarization aberrations. It also provides a historical perspective by explaining the discovery of aberrations, and two chapters provide insight into classical image formation; these topics of discussion are often missing in comparable books.

JOSÉ SASIÁN is Professor of Optical Sciences at the College of Optical Sciences, University of Arizona. His research areas include aberration theory, optical design, light in gemstones, art in optics and optics in art, optical imaging, and light propagation in general.

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*University of Arizona*



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In appreciation and love to my family  
With love to Phitchanat

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In this sort of algebra one is to some extent dependent on luck (which no doubt favors the patient) in the reduction of apparently intractable expressions to something less resembling chaos.  
(Hans A. Buchdahl, *Optical Aberration Coefficients*)

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## Preface

This book provides an introduction to the theory of optical aberrations. Those interested will find a variety of topics that provide a solid foundation, and will appreciate the beautiful structure built in the theory of aberrations. Understanding the contents of this book will be useful for solving problems in optical design, optical imaging, and other related fields. The treatments in the book exploit symmetry properties to provide insight and derive useful results; highlighting symmetry properties is a recurring theme. The approach followed in the book is the wave aberration theory pioneered by H. H. Hopkins. However, the contents of this book take the wave theory of aberrations much further, and provide a comprehensive understanding of aberrations in optical imaging systems.

Chapter 1 provides an introduction and a historical overview. Chapter 2 provides basic concepts in geometrical optics. In order to appreciate the theory of aberrations it is necessary to have an understanding of optical image formation. To this end Chapter 3 provides a basic and insightful discussion on imaging with rays, and Chapter 4 provides a fresh and useful discussion on the fundamentals of imaging with light waves. Chapter 5 introduces and highlights the wave aberration function, which is central to the understanding of aberrations. Chapter 6 discusses second-order effects which determine the location and size of an image. Chapter 7 discusses the primary aberrations. Chapter 8 discusses aberrations using the concept of light rays. Chapter 9 provides a novel treatment of ray caustics. Chapter 10 derives aberration coefficients, and Chapter 11 presents a basic discussion of structural aberration coefficients. Chapter 12 provides a discussion of pupil aberrations and a useful interpretation. Pupil aberrations have received little attention in the past. Chapter 13 takes further the independent work of G. G. Slyusarev and M. Reiss on irradiance changes in optical systems, and develops the irradiance function. Chapter 14 provides a “well-rounded” sixth-order theory which further exhibits the beauty and structure of the wave theory of aberrations. Chapter 15 discusses two useful theories for understanding optical systems that lack an axis of rotational

symmetry. Chapter 15 discusses the aberration function, in wave form, for plane symmetric systems. This function turns out to be also useful for constructing polarization fields and in building the theory for multiple aperture systems. Chapter 16 discusses the topic of polarization aberrations. The treatment follows the notation of previous chapters and continues to exhibit the overall structure of aberration theory, this time by no longer treating the optical field as a scalar quantity, but writing the field amplitude in vector form; this is a new treatment of the subject.

Overall, those who read and follow the material in this book will obtain a strong perspective in aberrations and appreciate the beauty and structure of wave aberration theory. The whole matter revolves around an understanding of how the optical field changes and propagates in an optical system. This understanding is essential for the intelligent design, fabrication, and test of optical systems.

José Sasián  
College of Optical Sciences  
University of Arizona  
Tucson, Arizona, 2012

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Simon Capelin, Editorial Director at Cambridge University Press, prompted me to embark on the task. I would like to thank him, Claire L. Poole, Antoaneta Ouzounova, Abigail Jones, and Cambridge University Press, for kindly editing and publishing this book. I also thank Mairi Sutherland for her detailed editing of the manuscript. Takeshi Nakazawa and Chia-Ling Li helped me with proofing the draft and in producing the figures. I thank them as their work considerably helped me to finish this book.

I would like to acknowledge my colleagues Lakshmi Narayan Hazra at the University of Calcutta, and Yongtian Wang at the Beijing Institute of Technology, for valuable discussions on aberration theory. I thank Andrew Rakish from the European Southern Observatory for stimulating discussions about historical aspects of aberrations.

I would like to thank Christine Hopkins for kind permission to use the photograph of Harold H. Hopkins.

I would like to thank Tina E. Kidger for her help in obtaining the photograph of Harold H. Hopkins.

I would like to thank Pamela Shack for kind permission to include a photograph of Roland V. Shack.

I am grateful to Margy Green for providing a photo of the painting by artist Don Cowen used on the front cover of this book. The painting is located at the College

of Optical Sciences at the University of Arizona. I also thank Kristin M. Waller for kindly obtaining permission to publish the photograph.

I would like to thank the Royal Society of England for kind permission to use Figure 1.4 in this book, which appeared as Fig. 28 in the Bakerian Lecture: Thomas Young, “On the Mechanism of the Eye,” *Phil. Trans. R. Soc. Lond.* **91**(1801), 23–88.

I would like to thank Roland V. Shack for permission to use material from his class notes for the course OPTI 518 “Introduction to Aberrations,” at the College of Optical Sciences at the University of Arizona. The treatment of the transverse ray aberrations, the definition of caustic, the treatment of the structural aberration coefficients, and the use of grid surfaces to illustrate the wavefront deformation, presented in this book, are due to him. The presentation of these treatments does not necessarily reflect his opinion on the subjects.



## Harold H. Hopkins

The wave theory of aberrations was pioneered by H. H. Hopkins.<sup>1</sup> Of the numerous contributions to optics of H. H. Hopkins an important one is the equation that describes the process of physical imaging formation, namely,

$$I(u', v') = \int \int \int_{\infty} \Gamma(u_1 - u_2, v_1 - v_2) E(u_1, v_1) F(u' - u_1, v' - v_1) E^*(u_2, v_2) \\ \times F(u' - u_2, v' - v_2) du_1 du_2 dv_1 dv_2.$$



Figure P.1 Harold H. Hopkins. With kind permission of Mrs. Christine Hopkins.

This equation, which relates the irradiance variations of an image, was published in *Proc. R. Soc. Lond. A* **217** (1952), 408–431 in a paper entitled “On the diffraction theory of optical images.” It considers the properties of the illumination, the object

<sup>1</sup> A biography of H. H. Hopkins can be found in C. W. McCombie and J. C. Smith, “Harold Horace Hopkins,” in *Biographical Memoirs of Fellows of the Royal Society*, Vol. 44, the Royal Society, 1998, 238–252.

properties, and the imaging system. H. H. Hopkins also provided the equivalent form (here with different notation and derived in Chapter 4),

$$I(x, y) = \left(\frac{1}{f\lambda}\right)^2 \iint_{\infty} \sigma(x_0, y_0) |s(x, y) t(x, y) ** p s f(x, y)|^2 dx_0 dy_0.$$

H. H. Hopkins left for us the insights and line of thought that led him to the discovery of these fundamental equations in the paper “The concept of partial coherence in optics,” *Proc. R. Soc. Lond. A* **208** (1951), 263–277.

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## Roland V. Shack

R. V. Shack had many contributions to optics and to aberration theory.<sup>1</sup> His writing of the aberration function using the field and aperture vectors,

$$W(\vec{H}, \vec{\rho}) = \sum_{j,m,n}^{\infty} W_{k,l,m}(\vec{H} \cdot \vec{H})^j (\vec{H} \cdot \vec{\rho})^m (\vec{\rho} \cdot \vec{\rho})^n,$$

an apparently trivial substitution, led to the discovery of binodal astigmatism and more generally to the concept of aberration fields and nodes.

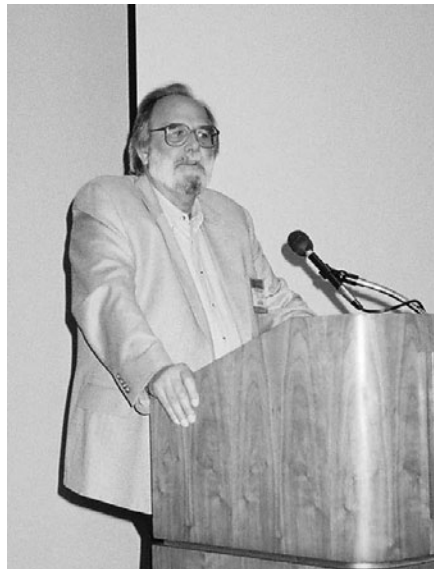


Figure P.2 Roland V. Shack. With kind permission of Mrs. Pamela Shack. Photo by José Sasián.

<sup>1</sup> See, for example, J. E. Harvey and R. B. Hooker (eds.), *Robert Shannon and Roland Shack: Legends in Applied Optics*, SPIE Press 2005.

Several of the advancements in aberration theory presented in this book have been made possible by using Shack's form of the aberration function.

R. V. Shack was a student of Hopkins at Imperial College London in England. While R. V. Shack was professor at the College of Optical Sciences at the University of Arizona, he had an unusual gift for motivating and inspiring students and colleagues.

Roland V. Shack taught a variety of topics in aberration theory and he freely shared his knowledge of the subject. Meeting him in his office was a great pleasure as his conversation was highly motivational and inspiring. In explaining optics he used insightful and appealing figures and models. They were also artistic, which added an element of pleasure. A favorite model was for the ray caustic of astigmatism: two separated and perforated plates supporting a tight string, going back and forth many times between the plates, showed the ray paths and astigmatic line segments.

## Symbols

Symbol	Description
$\lambda$	Wavelength of light
$\vec{H}$	Field vector
$\vec{\rho}$	Aperture vector
$\phi$	Angle between the field and aperture vectors
$\mathcal{K}$	Lagrange invariant
$\Phi, \phi$	Optical power
$n$	Index of refraction
$r$	Surface radius of curvature
$u$	Marginal ray slope
$\bar{u}$	Chief ray slope
$s$	Object conjugate distance
$s'$	Image conjugate distance
$y$	Marginal ray height
$\bar{y}$	Chief ray height
$i$	Marginal ray slope of incidence
$\bar{i}$	Chief ray slope of incidence
$A = ni$	Marginal ray refraction invariant
$\bar{A} = n\bar{i}$	Chief ray refraction invariant
$v = \frac{n_d - 1}{n_F - n_C}$	Glass V-number
$W$	Wavefront deformation
$W_{k,l,m}$	Image wave aberration coefficient
$\bar{W}_{k,l,m}$	Pupil wave aberration coefficient
$I_{k,l,m}$	Irradiance coefficient
$\bar{S}$	Stop shifting parameter
$S$	Object shifting parameter
$\sigma$	Structural coefficient
$\bar{S}_\sigma$	Structural stop shifting parameter
$FT \{ \}$	Fourier transform
**	Convolution operation

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