Cambridge University Press
978-1-107-01469-5 - Stochastic Geometry for Wireless Networks
Martin Haenggi
Frontmatter
More information

Stochastic Geometry for Wireless Networks

Covering point process theory, random geometric graphs, and coverage processes, this rigorous introduction to stochastic geometry will enable you to obtain powerful, general estimates and bounds of wireless network performance, and make good design choices for future wireless architectures and protocols that efficiently manage interference effects.

Practical engineering applications are integrated with mathematical theory, with an understanding of probability the only prerequisite. At the same time, stochastic geometry is connected to percolation theory and the theory of random geometric graphs, and is accompanied by a brief introduction to the R statistical computing language.

Combining theory and hands-on analytical techniques, this is a comprehensive guide to the spatial stochastic models essential for modeling and analysis of wireless network performance.

Martin Haenggi is a Professor of Electrical Engineering, and a Concurrent Professor of Applied and Computational Mathematics and Statistics, at the University of Notre Dame, Indiana. He has conducted extensive research on the use of stochastic geometry for the analysis and design of modern wireless systems, publishing over 100 articles on the subject. He is the recipient of a US NSF CAREER award and the 2010 IEEE Communications Society Best Tutorial Paper Award, and is a Senior Member of the IEEE and ACM.

Cambridge University Press
978-1-107-01469-5 - Stochastic Geometry for Wireless Networks
Martin Haenggi
Frontmatter
More information

"Stochastic geometry is a rigorous mathematical basis for a number of applications. It has recently been applied to wireless networking concepts and design, and it is fair to say that it forms a valuable anchor of scientific support for the somewhat chaotic field of ad hoc networking. This monograph does a superior job in explaining the theory and demonstrating its use. It is the most complete, readable, and useful document to date that illuminates the intricate web of wireless networks and transforms it from a 'dark art' to a solid engineering discipline with a scientific foundation."

Anthony Ephremides, University of Maryland

Cambridge University Press
978-1-107-01469-5 - Stochastic Geometry for Wireless Networks
Martin Haenggi
Frontmatter
More information

Stochastic Geometry for Wireless Networks

MARTIN HAENGGI
University of Notre Dame, Indiana

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore,
São Paulo, Delhi, Mexico City

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9781107014695

First published 2013

Printed and bound in the United Kingdom by the MPG Books Group

A catalog record for this publication is available from the British Library

Library of Congress Cataloging in Publication data

Haenggi, Martin, author.
Stochastic geometry for wireless networks / Martin Haenggi, University of Notre Dame, Indiana.
pages cm
Includes bibliographical references and index.
ISBN 978-1-107-01469-5 (Hardback)
1. Wireless communication systems–Mathematics. 2. Stochastic models. I. Title.
TK5102.83.H33 2013
621.39′80151922–dc23
2012034887

ISBN 978-1-107-01469-5 Hardback

For Roxana and my parents

Cambridge University Press
978-1-107-01469-5 - Stochastic Geometry for Wireless Networks
Martin Haenggi
Frontmatter
More information

Contents

Preface — *page* xi
Notation — xiii

Part I Point process theory — 1

1 Introduction — 3
1.1 What is stochastic geometry? — 3
1.2 Point processes as spatial models for wireless networks — 3
1.3 Asymptotic notation — 5
1.4 Sets and measurability — 6
Problems — 7

2 Description of point processes — 9
2.1 Description of one-dimensional point processes — 9
2.2 Point process duality — 10
2.3 Description of general point processes — 11
2.4 Basic point processes — 15
2.5 Distributional characterization — 24
2.6 Properties of point processes — 31
2.7 Point process transformations — 35
2.8 Distances — 38
2.9 Applications — 40
Bibliographical notes — 43
Problems — 44

3 Point process models — 47
3.1 Introduction — 47
3.2 General finite point processes — 47
3.3 Cox processes — 50
3.4 Cluster processes — 53
3.5 Hard-core processes — 57
3.6 Gibbs processes — 60
3.7 Shot-noise random fields — 72

	Bibliographical notes	73
	Problems	74
4	**Sums and products over point processes**	77
	4.1 Introduction	77
	4.2 The mean of a sum	78
	4.3 The probability generating functional	79
	4.4 The Laplace functional	82
	4.5 The moment-generating function of sums over Poisson processes	83
	4.6 The probability generating and Laplace functionals for the Poisson point process	86
	4.7 Summary of relationships	87
	4.8 Functionals of other point processes	88
	Bibliographical notes	92
	Problems	92
5	**Interference and outage in wireless networks**	93
	5.1 Interference characterization	93
	5.2 Outage probability in Poisson networks	104
	5.3 Spatial throughput in Poisson bipolar networks	105
	5.4 Transmission capacity	107
	5.5 Temporal correlation of the interference	111
	5.6 Temporal correlation of outage probabilities	115
	Bibliographical notes	116
	Problems	117
6	**Moment measures of point processes**	119
	6.1 Introduction	119
	6.2 The first-order moment measure	119
	6.3 Second moment measures	120
	6.4 Second moment density	124
	6.5 Second moments for stationary processes	129
	Bibliographical notes	136
	Problems	136
7	**Marked point processes**	138
	7.1 Introduction and definition	138
	7.2 Theory of marked point processes	143
	7.3 Applications	148
	Bibliographical notes	150
	Problems	150
8	**Conditioning and Palm theory**	152
	8.1 Introduction	152
	8.2 The Palm distribution for stationary processes	157

8.3 The Palm distribution for general point processes 160
8.4 The reduced Palm distribution 164
8.5 Palm distribution for Poisson processes and Slivnyak's theorem 168
8.6 Second moments and Palm distributions for stationary processes 170
8.7 Palm distributions for Neyman–Scott cluster processes 171
8.8 Palm distribution for marked point processes 172
8.9 Applications 173
Bibliographical notes 175
Problems 175

Part II Percolation, connectivity, and coverage 177

9 Introduction 179
9.1 Motivation 179
9.2 What is percolation? 179

10 Bond and site percolation 181
10.1 Random trees and branching processes 181
10.2 Preliminaries for bond percolation on the lattice 190
10.3 General behavior of the percolation probability 191
10.4 Basic techniques 198
10.5 Critical threshold for bond percolation on the square lattice 205
10.6 Further results in bond percolation 213
10.7 Site percolation 215
Bibliographical notes 218
Problems 218

11 Random geometric graphs and continuum percolation 221
11.1 Introduction 221
11.2 Percolation on Gilbert's disk graph 223
11.3 Other percolation models 229
11.4 Applications 233
Bibliographical notes 244
Problems 244

12 Connectivity 246
12.1 Introduction 246
12.2 Connectivity of the random lattice 246
12.3 Connectivity of the disk graph 249
12.4 Connectivity of basic random geometric graphs 251
12.5 Other graphs 251
Bibliographical notes 253
Problems 253

Cambridge University Press
978-1-107-01469-5 - Stochastic Geometry for Wireless Networks
Martin Haenggi
Frontmatter
More information

13	**Coverage**	255
	13.1 Introduction	255
	13.2 Germ–grain and Boolean models	255
	13.3 Boolean model with fixed disks	259
	13.4 Applications	263
	Bibliographical notes	270
	Problems	270
	Appendix Introduction to R	272
	References	275
	Index	281

Cambridge University Press
978-1-107-01469-5 - Stochastic Geometry for Wireless Networks
Martin Haenggi
Frontmatter
More information

Preface

The performance of wireless systems depends strongly on the locations of the users or nodes. In modern networks, these locations are subject to considerable uncertainty and thus need to be modeled as a stochastic process of points in the two- or three-dimensional space.

The area of mathematics providing such models and methods to analyze their properties is *stochastic geometry*, in particular point process theory. Hence wireless network modeling and analysis is a very natural application of stochastic geometry, and, indeed, the last decade has witnessed a significant growth in this area. The goal of this book is to make the mathematical theory accessible to graduate students, researchers, and practitioners who are working in the field of wireless networks. This not only includes a coherent presentation of the theory as it applies to wireless networks, but also enables the reader to understand the related research articles and to define and solve new problems. The field is young enough to leave many opportunities for exciting and relevant new results. Indeed, not all the theoretical concepts covered in this book have found applications to wireless networks yet.

It is assumed that the reader has a solid background in basic probability and perhaps has had some exposure to point processes in one dimension, most likely in the form of traffic models for queueing theory.

While being rigorous, the book is not pedantic and does not dwell on measure-theoretic details. The interested reader can always study these intricacies from the mathematical literature; others may simply take measure theory for granted. For example, while the Radon–Nikodým theorem is mentioned on several occasions, it is not essential to follow the exposition. The many examples should illustrate the theoretical concepts and help develop a good intuition. To assist in this process, problems are included at the end of each chapter, some of which are based on the R statistical software for simulation and numerical studies. Also, many chapters have a dedicated applications section at the end, where the theory in that chapter is used to solve pertinent problems in wireless networking.

To the extent that it exists, standard terminology is used. Unfortunately, the notation is hardly consistent in the literature, which is a consequence of the fact that researchers from many different areas have made important contributions to the theory. Consequently, to help the reader understand books and articles with their specific terminology and definitions, different names and conventions are mentioned as appropriate.

The first part of the book gives an introduction to stochastic geometry, in particular point process theory. In order to be consistent with the application, the focus is on simple point processes on the Euclidean space $\mathbb{R}^d$. Particular care is given to the functionals and their relationships, to higher-order statistics, and Palm distributions, covered in Chapters 4, 6, and 8. These are the topics that are perhaps the most difficult to learn from the mathematical literature. Chapter 5 is entirely devoted to important applications in wireless networks, in particular those modeled using Poisson point processes.

The second part of the book discusses percolation theory, connectivity, and coverage. It uses the material from Part I to model the locations of the points or nodes, but then focuses on how or whether they are connected or whether a certain region is covered by a set of nodes if each node can cover a certain small area around itself. Connectivity is closely tied to percolation, which is the question of the existence of giant components in a network. Many results for the continuous case (for which points sit in $\mathbb{R}^d$) are based on arguments from discrete percolation, i.e., percolation on trees or lattices, which is discussed in Chapter 10. The next chapter introduces random geometric graphs and continuum percolation, while the last two chapters provide an introduction to connectivity and coverage problems.

The book is suitable both for self-learners and as a textbook for a graduate course, e.g., for a one-semester course on point process theory (Part I only), or for a course that covers both parts. In the latter case, some chapters in Part I will probably have to be discussed in less detail. For example, it is possible to leave out the sections on Gibbs processes, Janossy measures, and the Papangelou conditional intensity without losing the context. In a quarter-based system, each part could serve as a basis for a course.

Anticipating the use of the book as a textbook for a graduate course, I have refrained from using fonts that are not easily reproduced on a black- or white-board. As a consequence, x may denote a generic (deterministic) location in $\mathbb{R}^d$ and also an element of a point process (and thus a random variable). It should always be clear what is meant from the context and the use of the term location on the one hand and point on the other.

I would like to thank my friends and collaborators Jeff Andrews, Radha Krishna Ganti, Nihar Jindal, Amites Sarkar, and Steven Weber. This book benefited greatly from our discussions. Also, I am grateful to Phil Meyler from Cambridge University Press for encouraging me to undertake this endeavor. Special thanks go to Jeff, Radha, and Amites for providing detailed comments on parts of a draft. Of course, I am fully responsible for all mistakes that may still be present.

M.H.

Cambridge University Press
978-1-107-01469-5 - Stochastic Geometry for Wireless Networks
Martin Haenggi
Frontmatter
More information

Notation

General notation

$\emptyset$	the empty set
$\mathbb{N}$	natural numbers $\{1, 2, \ldots\}$
$\mathbb{N}_0$	$\mathbb{N} \cup \{0\}$
$[n]$	the set $\{1, 2, \ldots, n\}$
$\mathbb{R}^d$	d-dimensional Euclidean space
$\mathbb{R}^+$	non-negative real numbers
$\circ$	concatenation of functions
$\otimes$	product of measures
o	origin of $\mathbb{R}^d$
$\lvert \cdot \rvert \equiv \nu_d$	Lebesgue measure (of appropriate dimension)
$\lVert x \rVert$	Euclidean metric of $x \in \mathbb{R}^d$
$\lfloor x \rfloor$	largest integer smaller than or equal to $x \in \mathbb{R}$
$\#\{\cdot\}$	number of elements in set
Φ	point process on $\mathbb{R}^d$ as random countable set or counting measure
φ	locally finite countable subset of $\mathbb{R}^d$ or counting measure
$\mathcal{N}$	space of counting measures
N	point process as random counting measure
$\mathfrak{N}$	σ-algebra of counting measures
$\mathbb{M}$	mark space
$\hat{\Phi}$	marked point process; point process on $\mathbb{R}^d \times \mathbb{M}$
$\hat{\mathcal{N}}$	space of counting measures on $\mathbb{R}^d \times \mathbb{M}$
$\hat{\mathfrak{N}}$	σ-algebra of $\mathbb{R}^d \times \mathbb{M}$
$\mathcal{B}^d$	Borel σ-algebra in $\mathbb{R}^d$
$\mathcal{B}$	$= \mathcal{B}^1$
$\mathbb{P}, \mathsf{P}, P$	probability measures
P_o	Palm distribution
$\mathsf{P}_o^!$	reduced Palm distribution
Ψ	non-negative random measure (possibly a point process)
$\mathcal{M}$	space of non-negative random measures
$\mathfrak{M}$	σ-algebra of random measures

$\mathbf{1}(\cdot)$	indicator function
$\mathbf{1}_A(x)$	$\equiv \mathbf{1}(x \in A)$, indicator function of condition $x \in A$
$\delta(x)$	Dirac delta function
$\delta_x(A)$	$\equiv \mathbf{1}_A(x) \equiv \int_A \delta(y-x)\mathrm{d}y$, Dirac measure at point x
$b(x,r)$	d-dimensional (closed) ball of radius r centered at x
c_d	$\triangleq \lvert b(o,1)\rvert$, volume of d-dimensional unit ball
Λ	intensity measure (or first-order moment measure)
$\hat{\Lambda}$	intensity measure of marked point process
$\lambda(x)$	intensity function
$\lambda(x,\Phi)$	Papangelou conditional intensity
$\mathcal{V}$	$[0,1]$-valued functions v with $1-v$ of bounded support
$\mathcal{U}$	non-negative-valued functions of bounded support
$\mathcal{L}_X(s)$	Laplace transform $\mathbb{E}(\mathrm{e}^{-sX})$ of random variable X
$\mu^{(2)}$	second-order moment measure
$\alpha^{(2)}$	second-order factorial moment measure
$\varrho^{(2)}$	second moment density (or second-order product density)
g	pair correlation function
ℓ, $\tilde{\ell}$	path loss function, radial path loss function
$\mathcal{K}$	reduced second moment measure
K	Ripley's K function
$S[f]$	sum of $f(x)$, where $x \in \Phi$
$G[v]$	probability generating functional for $v \in \mathcal{V}$
$L_\Psi[u]$	Laplace functional for $u \in \mathcal{U}$ of random measure Ψ
$V(B)$	vacancy indicator of $B \in \mathcal{B}^d$
$V(x)$	Voronoi cell of point x
$\sum^{\neq}_{x,y\in A}$	double sum $\sum_{x\in A}\sum_{y\in A;x\neq y}$
A_x	translation of set A by x: $A_x \triangleq \{y \in A\colon y+x\}$
$A \oplus B$	Minkowski addition $\{x \in A, y \in B\colon x+y\}$
$A \star B$	Cartesian product of sets including only distinct points
$\lVert x - A\rVert$	minimum distance $\min_{y\in A}\{\lVert x-y\rVert\}$
p_c	critical probability in percolation models
$\triangleq$	definition
$\stackrel{\mathrm{d}}{=}$	equality in distribution

Abbreviations

PPP	Poisson point process
BPP	binomial point process
a.s.	almost surely (with probability 1)
a.a.s.	asymptotically almost surely
iid	independent and identically distributed
fidi	finite-dimensional
pdf	probability density function
cdf	cumulative distribution function
pgfl	probability generating functional
FKG (inequality)	inequality named after Fortuin, Kasteleyn, and Ginibre
BK (inequality)	inequality named after van den Berg and Kesten
LR (crossing)	left–right (crossing) in percolation models
TB (crossing)	top–bottom (crossing) in percolation models
RGG	random geometric graph
MAC	medium access control (for channel access)