Lecture Notes in Physics 798

Introduction to the Functional Renormalization Group

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Preface

The renormalization group (RG) has nowadays achieved the status of a *meta-theory*, which is a theory about theories. The theory of the RG consists of a set of concepts and methods which can be used to understand phenomena in many different fields of physics, ranging from quantum field theory over classical statistical mechanics to nonequilibrium phenomena. RG methods are particularly useful to understand phenomena where fluctuations involving many different length or time scales lead to the emergence of new collective behavior in complex many-body systems. In view of the diversity of fields where RG methods have been successfully applied, it is not surprising that a variety of apparently different implementations of the RG idea have been proposed. Unfortunately, this makes it somewhat difficult for beginners to learn this technique. For example, the field-theoretical formulation of the RG idea looks at the first sight rather different from the RG approach pioneered by Wilson, the latter being based on the concept of the effective action which is iteratively calculated by successive elimination of the high-energy degrees of freedom. Moreover, the Wilsonian RG idea has been implemented in many different ways, depending on the particular problem at hand, and there seems to be no canonical way of setting up the RG procedure for a given problem. Fortunately, in the last decade the development of the so-called functional renormalization group (FRG) method has somewhat unified the field by providing a mathematically elegant and yet simple way of expressing Wilson's idea of successive mode elimination in terms of a formally exact functional differential equation for the suitably defined generating functionals of a given theory. While the basic ideas of the Wilsonian RG as well as the field-theoretical RG are explained in many excellent textbooks, a pedagogic introduction to the Wilsonian RG using its modern formulation in terms of the FRG seems not to exist in the literature. It is the purpose of this book to fill this gap.

The book is subdivided into three parts. In Part I, which consists of the first five chapters, we introduce the reader to the basic concepts of the RG. This part is elementary and requires only previous knowledge of some introductory equilibrium statistical mechanics. In the four chapters of Part II we then give a self-contained introduction to the FRG. Since we are aiming at applications of the FRG to nonrelativistic quantum many-body systems, we start in Chap. 6 with an introduction to functional methods, defining various types of generating functionals and vertex functions. With these preparations, we derive in the central Chap. 7 of this book formally exact FRG flow equations of general field theories involving fermions, bosons, or mixtures thereof. We also discuss in detail how to include the emergence of finite vacuum expectation values of some of the field components into the FRG. In the following two chapters we discuss the two most common truncation strategies of the FRG flow equations, namely the vertex expansion in Chap. 8, and the derivative expansion in Chap. 9. Finally, in Part III of this book we apply the FRG method to nonrelativistic fermions. This part consists of three chapters: we first discuss the purely fermionic FRG in Chap. 10, and then present in Chaps. 11 and 12 partially bosonized FRG flow equations for interacting Fermi systems, where certain types of interaction processes are represented by suitable bosonic fields. The selected topics of Part III reflect our own research. We would like to emphasize, however, that the formulation of the FRG method developed in Chap. 7 is rather general and should be useful beyond the limited scope of our own research interest.

This book is based on a special topics course taught by one of us (P.K.) at the Goethe-Universität Frankfurt during the summer semesters 2006 and 2008. The course consisted of two 90-min lectures and one two-hour tutorial each week. The complete material in Part I and the first two chapters of Part II can be covered in 13 weeks provided the audience is familiar with the functional integral formulation of quantum many-body theory as developed on the first 100 pages of the textbook by Negele and Orland. The exercises at the end of Chaps. 1–7 are sometimes nontrivial and should be solved by the students at home. A complete discussion of the solutions of all exercises requires 11 or 12 two-hour tutorials.

We would like to thank several people who, in one way or the other, helped us to complete this book. First of all, we are grateful to Andreas Kreisel, who skillfully used the open source vector graphics editor *inkscape* to create a large part of the figures presented in this book and helped us to optimize our presentation. We also thank our collaborators on topics related to the functional renormalization group: Alvaro Ferraz, Hermann Freire, Nils Hasselmann, Thomas Kloss, Sascha Ledowski, and Andreas Sinner. In particular, the long-term collaborations with Sascha Ledowski and Nils Hasselmann influenced some of the presentations in Part II and Part III of this book. We have also profited from many useful comments and suggestions from some of the students who attended the courses on the renormalization group taught by one of us at the Goethe-Universität Frankfurt; we especially thank Christopher Eichler, who made several useful suggestions. Finally, we would like to thank Nicolas Dupuis, Holger Gies, Carsten Honerkamp, Christoph Kopper, Brad Marston, Jan Martin Pawlowski, Oliver Rosten, and Manfred Salmhofer for illuminating discussions on the functional renormalization group. We are extremely grateful to Manfred Salmhofer for his comments on Chaps. 6 and 10.

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Last but not least, we thank the *Sonderforschungsbereich SFB/TRR49* and the *DAAD/CAPES PROBRAL* program for financial support, and Christine Dinges for patiently producing the first LaTeXversion of the manuscript from the original handwritten lecture notes. We will maintain a web page

http://itp.uni-frankfurt.de/∼rgbook/

where we will list all errors and points of confusion which will undoubtedly come to our attention.

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