STOCHASTIC CALCULUS AND DIFFERENTIAL EQUATIONS FOR PHYSICS AND FINANCE

Stochastic calculus provides a powerful description of a specific class of stochastic processes in physics and finance. However, many econophysicists struggle to understand it. This book presents the subject simply and systematically, giving graduate students and practitioners a better understanding and enabling them to apply the methods in practice.

The book develops Ito calculus and Fokker–Planck equations as parallel approaches to stochastic processes, using those methods in a unified way. The focus is on nonstationary processes, and statistical ensembles are emphasized in time series analysis. Stochastic calculus is developed using general martingales. Scaling and fat tails are presented via diffusive models. Fractional Brownian motion is thoroughly analyzed and contrasted with Ito processes. The Chapman–Kolmogorov and Fokker–Planck equations are shown in theory and by example to be more general than a Markov process. The book also presents new ideas in financial economics and a critical survey of econometrics.

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For our youngest ones,
Will, Justin, Joshua, Kayleigh, and Charlie

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Abbreviations

$B(t)$, Wiener process

$x(t)$ or $X(t)$, random variable at time t in a stochastic process

$f_n(x_n, t_n; \ldots; x_1, t_1)$, n-point density of a continuous random variable x at n different times $t_1 \leq t_2 \leq \ldots \leq t_n$.

$p_2(x, t|y, s)$, conditional density to get x at time t, given that y was observed at time $s < t$.

$\langle x(t)\rangle_c = \int dx x p_2(x, t|y, s)$, avg. of x at time t conditioned on having observed y at time s. Using a bracket to denote an average is standard in physics since the time of Dirac.

$A(x, t)$, dynamical variable, meaning a function of a random variable x and also the time t.

$\langle A(t)\rangle = \int dx A(x, t) f_1(x, t)$, absolute average of a dynamical variable A.

$\langle x(t)y(s)\rangle = \int dx dy x y f_2(x, t; y, s)$, pair correlation function

$\langle x(t)\rangle = \int dx dy x p_2(x, t|y, s) f_1(y, s)$, absolute average of x at time t; $\langle x(t)\rangle = \int dx A(x) f_1(x, t)$ since $\int dy p_2(x, t|y, s) = 1$.

$\langle x(t)\rangle_c = \int dx x p_2(x, t|y, s) = y$, martingale process

$x(t, T) = x(t + T) - x(t)$, an increment/displacement/difference

$\langle x^2(t, T)\rangle$, mean square fluctuation about an arbitrary point x observed at time t.

$dX = R(X, t)dt + b(X, s)dB(t)$, Ito process;

$b^2(x, t) = D(x, t)$ is the diffusion coefficient

$M(t)$, a martingale in Ito calculus, $dM(t) = \pm\sqrt{D(M, t)}dB(t)$

$\{X\} = \int d(X)^2$ where $(dX)^2 = D(X, t)dt$[1]

$\{X, Y\} = \frac{1}{4}(\{X + Y\} - \{X - Y\})$

fBm, fractional Brownian motion, a mathematical model with stationary increments and long-time correlations

ratex, rational expectations, a mathematized ideology

[1] This is a special notation used in Chapter 10 where stochastic calculus is extended to martingales $dX = b(X, t)dB(t)$. It differs from Durrett's notation because we use his bracket symbol $\langle\,\rangle$ to denote averages.