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Michael D. Lee and Eric-Jan Wagenmakers
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Bayesian Cognitive Modeling

A Practical Course

Bayesian inference has become a standard method of analysis in many fields of science. Students and researchers in experimental psychology and cognitive science, however, have failed to take full advantage of the new and exciting possibilities that the Bayesian approach affords. Ideal for teaching and self study, this book demonstrates how to do Bayesian modeling. Short, to-the-point chapters offer examples, exercises, and computer code (using WinBUGS or JAGS, and supported by Matlab and R), with additional support available online. No advance knowledge of statistics is required and, from the very start, readers are encouraged to apply and adjust Bayesian analyses by themselves. The book contains a series of chapters on parameter estimation and model selection, followed by detailed case studies from cognitive science. After working through this book, readers should be able to build their own Bayesian models, apply the models to their own data, and draw their own conclusions.

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ERIC-JAN WAGENMAKERS



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CAMBRIDGE
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University Printing House, Cambridge CB2 8BS, United Kingdom

Published in the United States of America by Cambridge University Press, New York

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning, and research at the highest international levels of excellence.

www.cambridge.org

Information on this title: www.cambridge.org/9781107603578

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First published 2013

Printed in the United Kingdom by Clays, St Ives plc

A catalogue record for this publication is available from the British Library

ISBN 978-1-107-01845-7 Hardback

ISBN 978-1-107-60357-8 Paperback

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For Colleen and David, and Helen and Mitchell — Michael

Preface

This book, together with the code, answers to questions, and other material at www.bayesmodels.com, teaches you how to do Bayesian modeling. Using modern computer software—and, in particular, the WinBUGS program—this turns out to be surprisingly straightforward. After working through the examples provided in this book, you should be able to build your own models, apply them to your own data, and draw your own conclusions.

This book is based on three principles. The first is that of *accessibility*: the book’s only prerequisite is that you know how to operate a computer; you do not need any advanced knowledge of statistics or mathematics. The second principle is that of *applicability*: the examples in this book are meant to illustrate how Bayesian modeling can be useful for problems that people in cognitive science care about. The third principle is that of *practicality*: this book offers a hands-on, “just do it” approach that we feel keeps students interested and motivated.

In line with these three principles, this book has little content that is purely theoretical. Hence, you will not learn from this book why the Bayesian philosophy to inference is as compelling as it is; neither will you learn much about the intricate details of modern sampling algorithms such as Markov chain Monte Carlo, even though this book could not exist without them.

The goal of this book is to facilitate and promote the use of Bayesian modeling in cognitive science. As shown by means of examples throughout this book, Bayesian modeling is ideally suited for applications in cognitive science. It is easy to construct a basic model, and then add individual differences, add substantive prior information, add covariates, add a contaminant process, and so on. Bayesian modeling is flexible and respects the complexities that are inherent in the modeling of cognitive phenomena.

We hope that after completing this book, you will have gained not only a new understanding of statistics (yes, it can make sense), but also the technical skills to implement statistical models that professional but non-Bayesian cognitive scientists dare only dream about.

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Acknowledgements

The plan to produce this book was hatched in 2006. Since then, the core material has undergone a steady stream of additions and revisions. The revisions were inspired in part by students and colleagues who relentlessly suggested improvements, pointed out mistakes, and attended us to inconsistencies and inefficiencies. We would especially like to thank Ryan Bennett, Adrian Brasoveanu, Eddy Davelaar, Joram van Driel, Wouter Kruijne, Alexander Ly, John Miyamoto, James Negen, Thomas Palmeri, James Pooley, Don van Ravenzwaaij, Hedderik van Rijn, J. P. de Ruiter, Anja Sommvilla, Helen Steingroever, Wolf Vanpaemel, and Ruud Wetzels for their constructive comments and contributions. We are particularly grateful to Dora Matzke for her help in programming and plotting. Any remaining mistakes are the sole responsibility of the authors. A list of corrections and typographical errors will be available on www.bayesmodels.com. When you spot a mistake or omission that is not on the list please do not hesitate to email us at BayesModels@gmail.com.

The material in this book is not independent of our publications in the cognitive science literature. Sometimes, an article was turned into a book chapter; at other times, a book chapter spawned an article. Here we would like to acknowledge our published articles that contain text and figures resembling, to varying degrees, those used in this book. These articles often may be consulted for a more extensive and formal exposition of the material at hand.

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Chapter 15: The SIMPLE model of memory

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Chapter 16: The BART model of risk taking

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