Frontiers in Mathematics

Stability of Vector Differential Delay Equations

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1. Auflage 2013. Taschenbuch. x, 259 S. Paperback
ISBN 978 3 0348 0576 6
Format (B x L): 16,8 x 24 cm
Gewicht: 466 g

Weitere Fachgebiete > Mathematik > Mathematische Analysis > Differentialrechnungen und -gleichungen

Preface

1. The core of this book is an investigation of linear and nonlinear vector differential delay equations, extended to coverage of the topic of causal mappings. Explicit conditions for exponential, absolute and input-to-state stabilities are suggested. Moreover, solution estimates for these classes of equations are established. They provide the bounds for regions of attraction of steady states. We are also interested in the existence of periodic solutions. In addition, the Hill method for ordinary differential equations with periodic coefficients is developed for these equations.

The main methodology presented in the book is based on a combined usage of recent norm estimates for matrix-valued functions with the following methods and results:

a) the generalized Bohl–Perron principle and the integral version of the generalized Bohl–Perron principle;
b) the freezing method;
c) the positivity of fundamental solutions.

A significant part of the book is devoted to a solution of the Aizerman–Myshkis problem and integrally small perturbations of linear equations.

2. Functional differential equations naturally arise in various applications, such as control systems, viscoelasticity, mechanics, nuclear reactors, distributed networks, heat flow, neural networks, combustion, interaction of species, microbiology, learning models, epidemiology, physiology, and many others. The theory of functional differential equations has been developed in the works of V. Volterra, A.D. Myshkis, N.N. Krasovskii, B. Razumikhin, N. Minorsky, R. Bellman, A. Halanay, J. Hale and other mathematicians.

The problem of stability analysis of various equations continues to attract the attention of many specialists despite its long history. It is still one of the most burning problems because of the absence of its complete solution. For many years the basic method for stability analysis has been the use of Lyapunov functionals, from which many strong results have been obtained. We do not discuss this method here because it has been well covered in several excellent books. It should be noted that finding Lyapunov type functionals for vector equations is often connected with serious mathematical difficulties, especially in regard to nonautonomous equations. To the contrary, the stability conditions presented in this book are mainly formulated in terms of the determinants and eigenvalues of auxiliary matrices dependent on a parameter. This fact allows us to apply well-known results of the theory of matrices to stability analysis.

One of the methods considered in the book is the freezing method. That method was introduced by V.M. Alekseev in 1960 forstability analysis of ordinary differential equations and extended to functional differential equations by the author.

We also consider some classes of equations with causal mappings. These equations include differential, differential-delay, integro-differential and other traditional equations. The stability theory of nonlinear equations with causal mappings is in an early stage of development.

Furthermore, in 1949 M.A. Aizerman conjectured that a single input-single output system is absolutely stable in the Hurwitz angle. That hypothesis created great interest among specialists. Counter-examples were set up that demonstrated it was not, in general, true. Therefore, the following problem arose: to find the class of systems that satisfy Aizerman's hypothesis. The author has shown that any system satisfies the Aizerman hypothesis if its impulse function is non-negative. A similar result was proved for multivariable systems.

On the other hand, in 1977 A.D. Myshkis pointed out the importance of consideration of the generalized Aizerman problem for retarded systems. In 2000 it was proved by the author, that a retarded system satisfies the generalized Aizerman hypothesis if its Green function is non-negative.

3. The aim of the book is to provide new tools for specialists in the stability theory of functional differential equations, control system theory and mechanics.

This is the first book that:

i) gives a systematic exposition of an approach to stability analysis of vector differential delay equations based on estimates for matrix-valued functions allowing us to investigate various classes of equations from a unified viewpoint;
ii) contains a solution of the Aizerman–Myshkis problem;
iii) develops the Hill method for functional differential equations with periodic coefficients;
iv) presents an integral version of the generalized Bohl–Perron principle.

It also includes the freezing method for systems with delay and investigates integrally small perturbations of differential delay equations with matrix coefficients.

The book is intended not only for specialists in stability theory, but for anyone interested in various applications who has had at least a first year graduate level course in analysis.

I was very fortunate to have fruitful discussions with the late Professors M.A. Aizerman, M.A. Krasnosel'skii, A.D. Myshkis, A. Pokrovskii, and A.A. Voronov, to whom I am very grateful for their interest in my investigations.