Astrophysics Through Computation

With Mathematica® Support

This new text surveys a series of fundamental problems in astrophysics, both analytically and computationally, for advanced students in physics and astrophysics. The contents are supported by more than 110 class-tested *Mathematica*® notebooks, allowing rigorous solutions to be explored in a visually engaging way. Topics covered include many classical and historically interesting problems, enabling students to appreciate the mathematical and scientific challenges that have been overcome in the subject's development. The text also shows the advantages and disadvantages of using analytical and computational methods. It will serve students, professionals, and capable amateurs to master the quantitative details of modern astrophysics and the computational aspects of their research projects.

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Brian Koberlein is Senior Lecturer of physics and astronomy at the Rochester Institute of Technology.

David Meisel is Distinguished Professor of physics and astronomy at the State University of New York, Geneseo.

DDM thanks Carolyn for 56 years of shared entropy that made his contribution possible.
BDK thanks Julia and Douglas for bringing love and joy to his universe.

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BRIAN KOBERLEIN
Rochester Institute of Technology, New York

DAVID MEISEL
State University of New York, Geneseo

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Preface

Why another book on astrophysics? Undergraduates and first-year graduate students are deluged with complex information that they are expected to "know" at least qualitatively, and there are plenty of texts that present just such a broad comprehensive survey of astronomy and astrophysics (either observationally or theoretically) and do that just fine. In fact, there are many things being taught in introductory astronomy classes today that 50 years ago appeared only in doctoral theses. But in those same times there were those (Chandrasekhar, Einstein, Hubble, and Spitzer, for example) who, having no access to the powerful computers of today, developed the elegant analytical and observational theories upon which our modern ideas are based. Not only did these early people work with incomplete data sets and poorly understood physical concepts, but they also had to invent their own mathematical and computational methodologies to make their concepts quantitative. The scientific progress of those times was largely the product of sheer intellect from beginning to end.

Astronomy and astrophysics now make such immense strides almost continuously that undergraduate and beginning graduate students are rarely aware of the extraordinary quantitative foundations given to these disciplines during the nineteenth and twentieth centuries, and this knowledge gap widens with each passing day. There are two factors at work here. First, the analytical mathematics used by these early masters was quite above that usually considered suitable for undergraduate instruction, and second the actual computational programming required to produce realistic modern models these days is considered too sophisticated to be meaningfully approached by all but the most advanced undergraduates and beginning graduate students. Yet NASA and other space agencies have often honored the intellectual giants of this era by naming spacecraft after them without explaining to modern students those lines of mathematical/quantitative reasoning that made possible the revolutions in thought of those scientists.

The purpose of this text is to use a modern computer mathematics system to give undergraduate and first-year graduate students a quantitative bridge between the old and the new. We do not intend for this volume to be a comprehensive survey of astronomy and astrophysics, either observationally or theoretically. Instead we cover a series of topics where it is evident (at least to us) that the mathematical (analytical or numerical) development in the hands of a skilled "practitioner" was critical to the understanding of available observations and/or proposed models. Our selected quantitative tasks had to fit the ready availability of the mathematical tools in the chosen computer math system, and a number of topics were rejected as being unable to be solved conveniently because of an unreasonable amount of processing time, because the data were too big for a desktop computer, or because the complexity of the problem obscured the concept we were trying to illustrate. As a result, the level of presentation varies greatly with the difficulty of the problem. We do not avoid a discussion just because the concept is considered too difficult if the math can be used to produce intelligible results. In general, we do not expect students to write their own software from scratch, but we do hope that they can use the text or notebooks as templates for their own applications.

If we do not include a favorite "moment in history" for every reader of the text, please consider those chosen as steppingstones to an extraordinary wealth of material from which you can formulate your own examples. In every case we consider, we extend (and encourage the student to do likewise) the exploration of a cited early work by means of modern computer technology. This graphically illustrates the quantitative directions that the work might have taken had such technological marvels been available in those early times. These computational activities also reveal various shortcomings in the early work that might have been avoided had advanced computation been available. Students will learn to recognize these for themselves as they work through the text and notebooks. We do this so that students will develop a sense of connection between those days when most ideas started in the human brain and today's modern world of supercomputers where visualization is so complicated that the brain needs computer "filtering" to understand what the numerical results are saying. But just because we have tended to develop the ideas of astrophysics of the past (and some of the present) does not mean we neglect to show some of the connections with the present and future.

To achieve our goals we have taken the broadest definition of computation possible to include data analysis as well as theoretical modeling, as this is the way modern astrophysics has evolved. Scientific papers in all fields have become a synthesis of both theory and data analysis, and today's students need to be able to navigate equally well in either capacity. Throughout the book we leave "nuggets" for future computational exploration with only brief comments about them, almost in passing. Only in the suggested computational projects do we make any reference to some of these. Instructors and students alike are expected to think about these on their own.

It may be wondered why we picked Wolfram *Mathematica*® for our programming system when others are readily available. One of us (DM) has used *Mathematica* as a programming system for the last decade and has seen it evolve from a somewhat hard-to-learn and abstract mathematical programming language into a comprehensive mathematical analysis system that suits both theoretical development and extensive data analysis. Its coherent structure allows use for all research tasks without having to change software for different subtasks: first, it is practically the only system available that is transparently and completely cross platform; second, whereas other packages keep subdividing as new features are added, *Mathematica* gets more and more unified as befits modern research that is cross-disciplinary and interdisciplinary. The amount of scientific and computational capability in its current version is nothing short of spectacular, as illustrated in the more than 115 notebooks we have used as the foundation of this book. *Mathematica* is now available internationally with versions that can be purchased directly by students and interested laypersons even if their home institutions have no licensed versions. There is also a free reader that allows anyone to at least read each notebook and in some instances some of the multimedia included. If one has professional colleagues who do not own a copy, they can apply for a special *Mathematica* version that allows editing and reworking of selected documents. If you want a taste of *Mathematica* at no cost, then there is always *WolframAlpha* online as a massive public demonstration of *Mathematica*'s capabilities.

Although only snippets of the needed *Mathematica* expressions at critical junctures are given in the textbook, the full notebooks with comments are available online. Because each of these is a self-contained "program" or collection of related algorithms, there are various levels of complexity involved in examining the contents there.

1. **Beginner's level**: At this level, the best situation is to have a working copy of the regular, student or home *Mathematica* because in Version 9 there are several features that make learning the system much easier than in the past, including online direct connections to the Wolfram site and its new video tutorials for beginners or the availability of natural language commands. If full *Mathematica* capability is not available to the reader, we have provided .pdf copies of all the notebooks that can be

studied independently. An alternative to this is to get a free copy of the *Wolfram CDF Reader* from their website as it will the read .nb files directly and allow native plot display including rotating the 3D plots and printing them plus rapid scrolling, none of which is available in the .pdf copies.

2. **Intermediate level**: Running the notebooks or portions of notebooks with different data in "what-if" mode using some original *Mathematica* utilities provided in their own short programs or projects. If original routines or Wolfram-owned routines are used in publicly available notebooks, be sure to copy over the appropriate copyright notice into your own notebook. *Mathematica* or *Reader Pro* is required for this.
3. **Mastery level**: Use these notebooks as templates for generating new versions or extensions of the full notebooks for projects. Flow charts are recommended before starting drastic changes, and always keep double copies of all modifications separate from the original notebooks as downloaded. A full copy of *Mathematica* is required here.

Finally, we have resisted doing "fancy" programming in our notebooks, preferring to stay within products that instructors are likely to see within their own classroom. These notebooks are not necessarily efficient or compact, but they do work well enough to get the point across. Our experience is that program authors are never satisfied with any of the versions they produce, but in this case we are. We are sure that talented student programmers can review these notebooks and produce better versions, and they are welcome to try their hands at it. For them, these notebooks should provide templates upon which more refined versions can be developed to their own personal tastes while they sharpen their programming abilities. *Mathematica* can be self-documenting within limits, but because of its built-in LaTeX word processing properties, it has allowed us to provide much more documentation than a usual program listing contains. We have even provided some documentation to get students started in *Mathematica* programming and usage. But this book is not a programming manual, as for that there are plenty of texts available. We have concentrated instead on computational matters of direct concern in astronomy and astrophysics.