

# Preface

This book was conceived as an introduction to  $C_p$ -theory with the intent that it could also be used as a reference guide for specialists. The author's idea was to make this introduction accessible to any reader having a minimum of mathematical culture and, possibly, with no topological background at all. This work is a unique attempt to offer any undergraduate student an opportunity to plunge directly into the theory of function spaces endowed with the topology of pointwise convergence. The main difficulty in this quest is to achieve the level of understanding in general topology needed to be able to work in  $C_p$ -theory. It often happens that a student who wants to study function spaces is told that he (or she) has first to learn some general topology. Many students are lost along this path because they feel that they have to put a considerable effort into an area they have not chosen. Some students eventually do become topologists and discover that they need not look for another field. General topology could be accused of many things but lack of interesting unsolved problems is not one of them.

The author became a specialist in function spaces after publishing three papers which had nothing to do with spaces  $C_p(X)$ . At that time there were no books on this topic at all, so all that was accessible were several (very good!) surveys of Arhangel'skii. However, at the Department of Mechanics and Mathematics at the Moscow State University, we had a very strong topology group in which progress could be discussed and excellent tutors were available for advice and for instruction. For this reason our department produced several hundred international-level topologists over a 30-year period with few texts, but many research papers with which to consult.

In 1974 A.V. Arhangel'skii and V.I. Ponomarev published (in Russian) a book entitled "Fundamentals of General Topology. Problems and Exercises". This work was translated into English ten years later. The author considers that this book contributed dramatically to his development as a topologist. Of course, outside the USSR there was the topologists' vademecum written by Engelking [1977] which is a truly excellent book. Unfortunately, it first came into the author's hands after he had written two papers on general topology; so he used it only as a reference guide.

Even now, after almost twenty years of heavy duty use of both books, when it comes to choosing an introduction to general topology, the author prefers the book of Arhangel'skii and Ponomarev. While it is true that it presents less material than Engelking's, the book of Engelking gives complete proofs only for the textbook results. Engelking's book gives the reader an opportunity to arrive to the limit of modern knowledge in quite a few areas of topology, but even a very strong student won't always be able to assimilate professionally its outlined proofs and constructions.

Even though the book of Arhangel'skii and Ponomarev has fewer topics than Engelking's, the range of their material is impressive. Additionally, each area is presented in great detail and the proofs and constructions are complete. Another advantage to their text is that all proofs are concentrated in the final parts of each chapter and the reader mostly sees results (presented like problems) and definitions. This makes it easier to grasp the general view of the topic and the methods that are used.

The author accepts that he might be biased about the book of Arhangel'skii and Ponomarev because he grew up having been nourished by it. All this praise is more an explanation of why he wanted to write a book on  $C_p$ -theory resembling the same format. Finally, this outline summarizes the points showing the potential usefulness of the present work:

- *The only background needed is some knowledge of set theory and real numbers. Any reasonable course in calculus covers everything needed to understand this book.*
- *The student can learn all of the general topology required without referring to any textbook or papers. The amount of general topology in the text is strictly minimal and is presented in such a way that the student works with the spaces  $C_p(X)$  from the very beginning.*
- *This text can also be used as a reference for mathematicians working in, or working outside of topology (in functional analysis, for example) wanting to use results or methods of  $C_p$ -theory. He (or she) will find them easily in a concentrated form or with full proofs if there is such a need.*
- *The material presented here is up to date and brings the reader to the frontier of a reasonable number of important areas of  $C_p$ -theory.*
- *This book appears to be the first self-contained introduction to  $C_p$ -theory. Although there is an excellent textbook written by Arhangel'skii [1992a], it heavily depends on the reader's good knowledge of general topology.*

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