Preface

Stochastic orders and inequalities have been used during the last 40 years, at an accelerated rate, in many diverse areas of probability and statistics. Such areas include reliability theory, queuing theory, survival analysis, biology, economics, insurance, actuarial science, operations research, and management science. The purpose of this book is to collect in one place essentially all that is known about these orders up to the present. In addition, the book illustrates some of the usefulness and applicability of these stochastic orders.

This book is a major extension of the first six chapters in Shaked and Shanthikumar [515]. The idea that led us to write those six chapters arose as follows. In our own research in reliability theory and operations research we have been using, for years, several notions of stochastic orders. Often we would encounter a result that we could easily (or not so easily) prove, but we could not tell whether it was known or new. Even when we were sure that a result was known, we would not know right away where it could be found. Also, sometimes we would prove a result for the purpose of an application, only to realize later that a stronger result (stronger than what we needed) had already been derived elsewhere. We also often have had difficulties giving a reference for *one* source that contained everything about stochastic orders that we needed in a particular paper. In order to avoid such difficulties we wrote the first six chapters in Shaked and Shanthikumar [515].

Since 1994 the theory of stochastic orders has grown significantly. We think that now is the time to put in one place essentially all that is known about these orders. This book is the result of this effort.

The simplest way of comparing two distribution functions is by the comparison of the associated means. However, such a comparison is based on only two single numbers (the means), and therefore it is often not very informative. In addition to this, the means sometimes do not exist. In many instances in applications one has more detailed information, for the purpose of comparison of two distribution functions, than just the two means. Several orders of distribution functions, that take into account various forms of possible knowledge about the two underlying distribution functions, are studied in Chapters 1 and 2.

When one wishes to compare two distribution functions that have the same mean (or that are centered about the same value), one is usually interested in the comparison of the dispersion of these distributions. The simplest way of doing it is by the comparison of the associated standard deviations. However, such a comparison, again, is based on only two single numbers, and therefore it is often not very informative. In addition to this, again, the standard deviations sometimes do not exist. Several orders of distribution functions, which take into account various forms of possible knowledge about the two underlying distribution functions (in addition to the fact that they are centered about the same value), are studied in Chapter 3. Orders that can be used for the joint comparison of both the location and the dispersion of distribution functions are studied in Chapters 4 and 5. The analogous orders for multivariate distribution functions are studied in Chapters 6 and 7.

When one is interested in the comparison of a sequence of distribution functions, associated with the random variables X_i, $i = 1, 2, \ldots$, then one can use, of course, any of the orders described in Chapters 1–7 for the purpose of comparing any two of these distributions. However, the parameter i may now introduce some patterns that connect all the underlying distributions. For example, suppose not only that the random variables X_i, $i = 1, 2, \ldots$, increase stochastically in i, but also that the increase is sharper for larger i's. Then the sequence X_i, $i = 1, 2, \ldots$, is stochastically increasing in a convex sense. Such notions of stochastic convexity and concavity are studied in Chapter 8.

Notions of positive dependence of two random variables X_1 and X_2 have been introduced in the literature in an effort to mathematically describe the property that "large (respectively, small) values of X_1 go together with large (respectively, small) values of X_2." Many of these notions of positive dependence are defined by means of some comparison of the joint distribution of X_1 and X_2 with their distribution under the theoretical assumption that X_1 and X_2 are independent. Often such a comparison can be extended to general pairs of bivariate distributions with given marginals. This fact led researchers to introduce various notions of positive dependence orders. These orders are designed to compare the strength of the positive dependence of the two underlying bivariate distributions. Many of these orders can be further extended to comparisons of general multivariate distributions that have the same marginals. In Chapter 9 we describe these orders.

We have in mind a wide spectrum of readers and users of this book. On one hand, the text can be useful for those who are already familiar with many aspects of stochastic orders, but who are not aware of all the developments in this area. On the other hand, people who are not very familiar with stochastic orders, but who know something about them, can use this book for the purpose of studying or widening their knowledge and understanding of this important area.

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