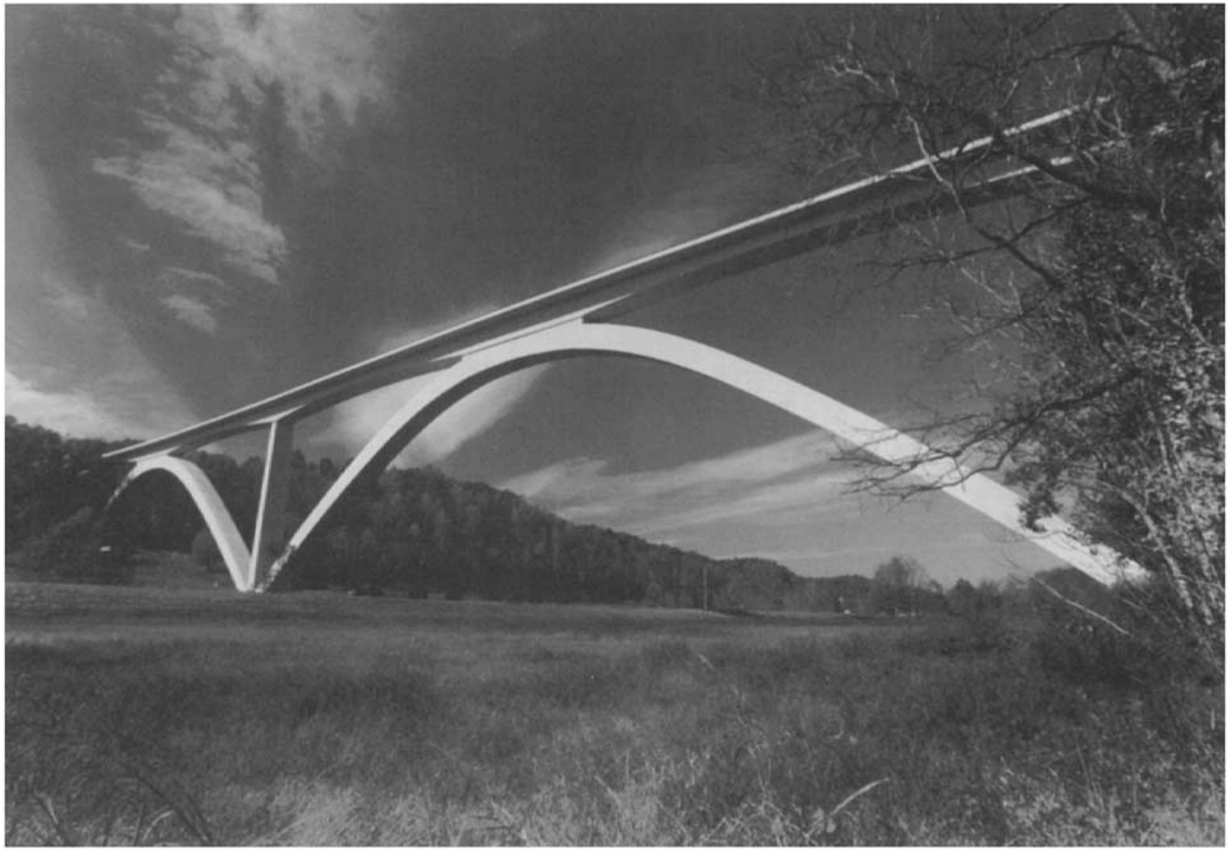
The study of structural analysis and design is a central subject in civil, aeronautical, and mechanical engineering. This book presents a modern and unified introduction to structural analysis, with a strong focus on how structures actually behave.

The unifying theme is the application of energy methods, developed without the formal mathematics of the calculus of variations. The energy approach makes it possible to articulate the logical relationship between equilibrium and compatibility; emphasize the unity of structural analysis, particularly for indeterminate structures; and identify the roles of idealization and discretization in structural modeling. Thus, energy methods also serve as a prelude to the main ideas behind modern computational approaches to structural analysis and design.

Overall, the author intends to convey a style of thinking about and modeling structures and their behavior, and to introduce the intellectual roots from which most computer tools derive. As an aid to upper-level undergraduate students in mastering this material, the text includes numerous worked examples, as well as homework problems.

Structural Modeling and Analysis



The Natchez Trace Parkway Bridge, located near Franklin Tennessee, was completed in 1994. It earned the 1995 Outstanding Civil Engineering Achievement Award of Merit from the American Society of Civil Engineers. It was the first arch bridge built in the United States from precast concrete segments, as shown in the figure immediately preceding Chapter 1. As such, this bridge combines original technical design with breathtaking aesthetic vision. (Photo courtesy of Figg Engineering Group.)

Structural Modeling and Analysis

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Contents

List of Photographs*

*Photographs indicated by an unadorned chapter number appear on the page immediately preceding the chapter.

Preface

1 Still Another Introduction to Structural Analysis?

I studied civil engineering as an undergraduate at the Cooper Union, earning my B. C. E. in 1962. I took three courses in structures and two on the strength of materials, as well as two design courses that heavily featured structural design. The curriculum totaled 143.5 credits. Recently, engineering schools have begun to reduce their total credit hours in an emerging trend toward a total of 120 credits. Further, civil engineering curricula have been reoriented away from a traditional emphasis on structural and geotechnical engineering toward more general programs that include environmental engineering, transportation engineering, systems analysis, and computer-aided engineering. Consequently, the number of required courses in structural analysis taken by civil engineering undergraduates has declined steadily, reaching a point where, typically, civil engineering majors take only one *required* course in structures, usually after (only) one course in the strength of materials.

In the face of such changes, you might expect that the way in which we teach structures has also shifted markedly. However, beyond extending the classical approaches to this subject to include elementary structures programs on attached floppy disks, the basic textbooks on structural analysis remain largely unchanged. That is, those books that present the first course in structures are not terribly different than those that were available when I was an undergraduate. To be sure, there are many books on computational techniques, especially on finite element methods, but they are intended for advanced undergraduate and graduate courses. There are one or two elementary books that purport to support computer-based modeling, but in my view they won't provide the reader with a sound basis – about both theory and behavior – for learning much about structures. I think we can do better, and this book is my contribution to the continuing dialogue on how we should transmit the beginnings of an extraordinary body of knowledge.

We clearly need a shift away from traditional approaches to the subject. Structural analysis is often presented as a collection of tools – often seemingly unrelated – for handling a set of fairly specific problem types. A major dividing line on the problem-type axis is the distinction between statically determinate and statically indeterminate structures. Further, the tools are often presented in an order reflecting their respective degree of difficulty of application, rather than in an order reflecting a coherent view of the discipline. In fact, structural analysis is taught in two or three different disciplines, including civil engineering, aeronautical engineering, and mechanical engineering. (It is also taught in architecture programs, but that really is a horse of another color.) Perhaps reflecting long-standing differences in the different engineering disciplines, traditional approaches to structural analysis are often presented as distinct from their logical underpinnings in mechanics, especially engineering or applied mechanics. There are, I feel, better ways to do this.

I haven't said much about computers, yet there is no doubt that the ways in which structures are analyzed and designed are dramatically different than what was done in my professional youth. Computers are ubiquitous both in the classroom and in the design office. However, in terms of introducing structural analysis in a first course, the major response to this change in the working environment has been the inclusion of elementary computer programs within a shrinking structural curriculum. One common result of this is more time spent on generating numbers and less time spent on understanding what meaning – if any – to attach to the numbers that are generated with these programs. In short, I believe that as computers become ever more powerful, it is even more important to teach *basic structural modeling*, with a heavy emphasis on *understanding behavior*, as well as on interpreting results in terms of the limitations of the models being applied. In fact, I have heard it argued that the generation of numerical analyses for particular cases is, in the real world, a task increasingly performed by technicians – and not by professional engineers. As numerical analysis becomes both more common and significantly easier, the premiums will be earned by those who know *which* calculations to perform and *what meaning* to attach to the subsequent results.

As I've already indicated, there have been many, many books written about structures. I have included a bibliography of recent and classical textbooks and tradebooks on structural mechanics, structural analysis, and structural engineering, organized into five broad categories: classical civil engineering structures, energy methods, finite element and matrix methods, special topics, and general structural behavior.

Any teacher of structures could, of course, choose a selection of books from several categories to generate material for a single course. However, it seems pretty clear that there is no single book that, in a one semester course, could: present a unified approach to analysis tools; successfully integrate structural modeling and analysis with elements of structural behavior; provide a solid basis for further learning for those going on to further work in structural engineering; and, finally, make structural analysis a useful and more interesting subject for those students whose primary focus is not structural engineering. This book represents my attempt, classroom tested here at Harvey Mudd, to write a book that achieves these ends.

2 What I am Trying to Do in This Book

Meeting the four goals just outlined is a formidable challenge, especially within the constraint of a one-semester introduction to a complex subject. The approach I take includes (1) choosing a unifying theme and (2) limiting the scope of the implementation to a manageable feast.

The unifying theme is the application of energy methods. I believe these methods can be developed and applied in a relatively straightforward way, without the heavy machinery of the formal mathematics of the calculus of variations. An energy approach allows us to develop a logical relationship between equilibrium concepts and compatibility concepts. Further, energy approaches also provide a sound base for developing approximate solutions for structural analysis (e.g., Rayleigh–Ritz methods) as a prelude to introducing the central ideas behind numerical approaches (e.g., finite element methods (FEM)). In fact, it is possible to go further in this direction and identify *indirect* energy methods with the *idealization* part of the modeling process and *direct* energy methods with *idealization and discretization* in structural modeling.

I also use the energy-based approach to emphasize the unity of structural analysis, particularly in the consideration of indeterminate structures. While there are important differences between determinate and indeterminate structures that emerge as a consequence of design considerations (e.g., stress and deflection limitations, redundancy, stability, etc.), their analysis should reflect behavioral and computational differences, not that they are different branches of structural analysis (as is often the case with traditional approaches).

Some will feel that energy methods themselves require enough background and sophistication that they are not readily accessible to undergraduates. However, I argue from my own classroom experience that energy methods can be readily introduced by focusing first on elementary structural models such as simple springs and discrete structures (e.g., the bent beam as a pair of rigid links joined by a rotational spring). Further, as I've said before, energy approaches can be introduced without the heavy machinery of the calculus of variations by introducing small systematic variations in an informal and intuitive manner, and then focusing on the pragmatics of applying the *delta* or "δ operator." The subject can thus be introduced correctly and without too much jargon.

Trying to do all of this in one-semester is tough, very tough. This severe constraint is reflected by (generally) limiting considerations to linearly elastic trusses, linearly elastic beams, and frames with symmetrical crosssections, responding in bending about a principal axis. This class of structures is sufficiently complex and practically interesting to illustrate the major points. It should also provide an adequate base for studying more complex structures in further courses. In addition, not all of the material presented need be covered in a one-semester, first course on structures. For example, the general statements of energy methods (Chapter 6) can be summarized or even deleted, as can some of the material in Chapters 4 and 7 where classical approximate solutions for the extensions of bars and the bending deflections of beams are demonstrated.

I will also emphasize two other points in this book. First, I will normally do examples and solve problems in terms of variables, with numerical values inserted irregularly and then only in the very last steps. I do this to emphasize the validation and interpretation of results in terms of (1) dimensional consistency and (2) the range of values of proper dimensionless ratios (e.g., the thickness-to-length ratio of a long, slender beam). This approach also lays the foundation for doing "back of the envelope calculations" in support of both analysis and design, particularly as an aid in the evaluation of computer results.

Second, I will try to emphasize structural behavior in terms of reasonableness of deflected shapes and of force and stress distributions (e.g., shear and moment diagrams). Again, I want to reinforce the notion that an intuitive feel for how a structure will respond can be developed, and that this intuition can often be expressed in sketches or simple formulas that help one interpret more complex analyses (or experiments).

There is an important point here that is intended particularly for student readers; it will use language that you will see again in what follows. The models of various structural elements that I will present are done with the intention of conveying a *style of modeling and thinking about structures and their behaviors*. Thus, specific numerical results are less important here than *knowing what sorts of things to look for*, including the right dimensions, the right dimensional ratios, and the presence and absence of terms embodying specific kinds of behavior. Certainly getting the magnitudes right is important, which is where the numerical results come in. All of the results I have given can be done for specific cases on a computer with one program or another. And there are certainly countless problems you can find where you can do it all numerically. However, the kinds of approaches emphasized here are intended to convey a flavor of what we always need to look for whenever we are doing any numerical work, but most especially when we are using computer-based tools, sometimes called "black boxes." Remember, a computer can't tell you whether the axial force *should* be greater than or smaller than the moment, it can only give you numbers. *You* have to apply some engineering judgement to see if you want to accept those numbers.

As with many other engineering courses, mastery of the material is developed and reflected in the ability to solve problems. There are many examples worked out in the main text, forming an integral part of the narrative, and they are also highlighted in a special format. The homework problems given include examples of the physical systems being modeled, often showing results that there was no time to get to in the main text. Therefore, it is very important for you, as reader, to *do* as many of the given problems as you can, and still more that you can find elsewhere. Remember, you can never do too many problems!

Finally, a word on notation, always a difficult issue. Whereas many aspects of structural engineering notation are fairly standard (e.g., E for the modulus of elasticity), many others vary – and oft times they simply conflict. This is particularly true for deflection and displacement quantities, and the concern is made even more difficult because I use the traditional δ for the variational operator, although it is often used to denote a deflection. So, the policy will be as follows. In Chapter 3, where I discuss discrete elements (i.e., springs), I denote extensions by ξ, whereas in Chapter 4, where I discuss one-dimensional structural elements, I use Δ for displacements and deflections. Later on, in an attempt to be consistent also with other works on structural mechanics, I use lower case "deltas" with subscripts, (e.g., δ_{Ch} or δ_T) to denote particular structural deflections – with one exception: δ_D is introduced in Eq. (4.9) as the Dirac delta function.

3 How This Book Is Organized

This book is organized as follows. In Chapter 1 I outline some important aspects of structural behavior, focusing on what purposes structures and structural elements are designed to achieve. I also discuss in general terms considerations of design for both strength and stiffness, as well as notions of load paths, redundancy, and safety. In Chapter 2 I review some fundamental structural models, focusing here mostly on *idealization*, although I complete the chapter by bringing *discretization* under the modeling umbrella as well. I also review the meaning of determinacy. I devote Chapter 3 to introducing the minimum potential energy principle for the elastic spring and for discrete models of structures. This will constitute the first introduction to the δ operator and to energy principles. These ideas are reinforced and extended in Chapter 4, which is devoted to a discussion of axially-loaded members or bars. These structural types provide a simple but robust platform for introducing potential and complementary energy principles, including the Castigliano theorems, the second of which is used to derive the standard unit-load calculation for the displacements of truss joints. Bars also offer the opportunity to introduce both indirect and direct energy approaches, the latter of which allows me to reinforce the idea of discretization and to introduce both matrix notation and the notion of interpolation. In Chapter 5 I describe how Castigliano's theorems are used to analyze those assemblages of bars called trusses, and I give a brief review of the classical methods of sections and of joints.

In Chapter 6, building on the examples presented in Chapters 3, 4, and 5, I provide formal statements of the energy principles in three dimensions (in Cartesian coordinates) and in matrix form. (And, as noted earlier, this material can be summarized or even deleted in a first course.) In Chapter 7 I use energy-based tools to derive the basic models for elastic beams and I introduce and apply some ideas about discretization and the direct displacement approach. In Chapter 8 I use energy-based tools, including Castigliano's second theorem, to calculate (and use) beam deflections, including examples of beams supported by elastic restraints. In Chapter 9 I introduce frames as assemblages of beams, again making frequent use of Castigliano's second theorem. Last, in Chapter 10 I present an overview of the force (flexibility) and displacement (stiffness) methods of analyzing structures, describe how they are represented in matrix format, and outline how these methods and formats are used in computational-based approaches to structural analysis and design.

A comment on the topic of modern computational-based approaches to structural engineering. While I summarize that in Chapter 10, you will find that it is a topic I highlight and discuss at several points in the book (e.g., Sections 4.6, 6.5, and 7.3). However, I am *not* going to tell you how to write programs or format input, nor do I provide programs, disks, or user's manuals. This is because, as I've already said, a goodly chunk of what this book is about is providing a basis for understanding the intellectual roots from which most of these computer tools derive, what these tools can do, why they do it in the ways they do, and most important, *how their results can be assessed for good use*. That is, the emphasis is on knowing the *why* of whatever calculations or computations are being done.

4 A Note on Style

Writing style is an important issue, even for a writer of technical textbooks or monographs. For example, my older brother, Harry, is a mathematician who writes with an economy and elegance of style that I can only admire. I try to choose words as carefully as topics and equations, and I wanted this book to be informal, even conversational, hoping that this would be more effective than a more formal pedagogic style. Thus, I intend the book to "sound" as I think I do in a relaxed, informal classroom setting, and I hope it works for you!

תושלב״ע

Acknowledgments

I have long been interested in structures and structural engineering, starting even before I was introduced to their analysis and design by Professor Anthony E. Armenàkas while studying civil engineering at Cooper Union. My interest in the variational approaches was inspired and encouraged by Professors Joseph Kempner of Polytechnic University (nee "Brooklyn Poly") and Nicholas J. Hoff and Jean Mayers of Stanford University. (Nicholas was my doctoral advisor; he earned his own doctorate under Stephen P. Timoshenko at Stanford in 1942, so I am part of a very distinguished tradition in applied mechanics. This lineage also shows a Russian immigrant to America mentoring an Hungarian immigrant, who in turn mentored an immigrant born in England of Eastern European parents.) Later I wrote two graduate textbooks on variational approaches to solid mechanics with Irving H. Shames, a long-time University Professor at the State University of New York at Buffalo and now at George Washington University. Professor Steven J. Fenves encouraged me to think about new approaches to teaching structures to undergraduates while we were colleagues at Carnegie Mellon University in the early 1970s. I am grateful to all, but especially to Joe, Nicholas, Irv, and Steve.

I have profited enormously from a careful, critical reading of the manuscript as it unfolded by a friend and colleague at Harvey Mudd College, Professor Harry E. Williams. Harry has an uncanny knack for reminding one of what the issues and principles really are, and for providing support and intellectual rigor and honesty at the same time.

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Professors Peter A. Chang (University of Maryland), Steve Fenves, and Victor Kaliakin (University of Delaware) have each read the penultimate version of this book and provided detailed and constructive reviews. As a result of what I learned from these very civil engineers, I made major changes that have significantly improved the book's organization and format. However, I alone remain responsible for errors, oversights, and infelicities.

There are twenty-three photographs of structures in this book. They are partly about structural history and structural function, yet they are also a small personal reflection on some structures that I find beautiful. The photographers of each are identified, but I do want to thank several people who provided both inspiration and very practical help. Professor David P. Billington of Princeton University wrote a wonderful book, *The Tower and The Bridge*, gave me access to his photographs, and intoduced me to his close colleague, J. Wayman Williams of Basking Ridge, New Jersey. Wayman is both a structural engineer and a photographer of structures, and he provided me with photographs, ideas, and very enthusiastic encouragement. Emeritus Professor Colin O'Connor of the University of Queensland, Australia, author of the evocative book *Roman Bridges*, kindly provided me with prints. Professor E. C. Ruddock of the University of Edinburgh directed me to the Royal Commission on the Ancient and Historical Monuments of Wales, who provided the photograph of a beautiful Welsh bridge.

Florence Padgett of the Press is once again my wonderfully supportive editor. This is our fourth collaboration, including two projects at another publisher (to remain nameless), and our professional relationship has blossomed into a valued friendship.

Florence also encouraged me to involve my younger daughter, Miriam, an artist, in my work. As one result, Miriam did the cover art for an earlier book on engineering design. For this book, she designed the entire cover and drew the figures that appear throughout. I am very grateful to Miriam for her work.

Finally, Joan Elizabeth Wilson Anderson provided encouragement, friendship, love, and oft-needed distraction while this book was being completed and produced. It is a great pleasure to acknowledge what this has meant to me.