Preface

During the last decades, geosciences and -engineering were influenced by two essential scenarios. First, the technological progress has changed completely the observational and measurement techniques. Modern high speed computers and satellite-based techniques are entering more and more all (geo) disciplines. Second, there is a growing public concern about the future of our planet, its climate, its environment, and about an expected shortage of natural resources. Obviously, both aspects, viz. (i) efficient strategies of protection against threats of a changing Earth and (ii) the exceptional situation of getting terrestrial, airborne as well as spaceborne, data of better and better quality explain the strong need for new mathematical structures, tools, and methods. In consequence, mathematics concerned with geoscientific problems, i.e., *geomathematics*, is becoming more and more important. Nowadays, geomathematics may be regarded as the key technology to build the bridge between real Earth processes and their scientific understanding. In fact, it is the intrinsic and indispensable *means* to handle geoscientifically relevant data sets of high quality within high accuracy and to improve significantly modeling capabilities in Earth system research.

From modern satellite-positioning, it is well known that the Earth's surface deviates from a sphere by less than 0.4% of its radius. This is the reason why spherical functions and concepts play an essential part in all geosciences. In particular, spherical polynomials and zonal functions constitute fundamental ingredients of modern (geo-)research – wherever spherical fields are significant, be they electromagnetic, gravitational, hydrodynamical, solid body, etc. Surprisingly enough, it turned out that essential features involving spherical vector and tensor structures were not available in the geosciences, when W. Freeden, first at the RWTH Aachen and later as head of the Geomathematics Group of the TU Kaiserslautern, started with the vector and/or tensor analysis of (Earth's) gravity field data obtained by satellite-to-satellite tracking (SST) and/or satellite gravity gradiometry (SGG). This is the reason why, based on results about Green's function with respect to the scalar Beltrami operator, a series of papers was initiated to establish vector and tensor counterparts of the Legendre polynomials, to verify vector and tensor extensions of the addition theorem, and to introduce vectorial and tensorial generalizations of the famous Funk-Hecke formula. Even more, the concept of zonal (kernel) functions (i.e., radial basis functions in the jargon of approximation theory), the theory of splines and wavelets etc could be generalized to the spherical vector/tensor case. All these new concepts were successfully applied in diverse areas such as climate and weather, deformation analysis, geomagnetics, gravitation, and ocean circulation.

This book collects all material developed by the Geomathematics Group, TU Kaiserslautern, during the last years to set up a theory of spherical functions of mathematical (geo-)physics. The work shows a twofold transition: First, the natural transition from the scalar to the vectorial and tensorial theory of spherical harmonics is given in coordinate-free representation, based on new variants of the addition theorem and the Funk–Hecke formulas. Second, the canonical transition from spherical harmonics via zonal (kernel) functions to the Dirac kernel is presented in close orientation to an uncertainty principle classifying the space/frequency (momentum) behavior of the functions for purposes of constructive approximation and data analysis. In doing so, the whole palette of spherical (trial) functions is provided for modeling and simulating phenomena and processes of the Earth system.

The main purpose of the book is to serve as a self-consistent introductory textbook for (graduate) students of mathematics, (geo-)physics, geodesy, and (geo-)engineering. In addition, the work should also be a valuable reference for scientists and practitioners facing spherical problems in their professional tasks. Essential ingredients of the work are the theses of W. Freeden (1979a), T. Gervens (1989), M. Schreiner (1994), S. Beth (2000), and H. Nutz (2002). Preliminary material can be found in the work by C. Müller (1952, 1966, 1998) and W. Freeden et al. (1998).

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