Preface

Background of This Book

Chaos theory, once considered to be the third revolution in physics following relativity theory and quantum mechanics, has been studied extensively in the past thirty years. A lot of chaotic phenomena have been found and enormous mathematical strides have been taken. Nowadays, it has been agreed by scientists and engineers that chaos is ubiquitous in natural sciences and social sciences, such as in physics, chemistry, mathematics, biology, ecology, physiology, economics, and so on. Wherever nonlinearity exists, chaos may be found. For a long time, chaos was thought of as a harmful behavior that could decrease the performance of a system and therefore should be avoided when the system is running. One remarkable feature of a chaotic system distinguishing itself from other nonchaotic systems is that the system is extremely sensitive to initial conditions. Any tiny perturbation of the initial conditions will significantly alter the long-term dynamics of the system. This fact means that when one wants to control a chaotic system one must make sure that the measurement of the needed signals is absolutely precise. Otherwise any attempt of controlling chaos would make the dynamics of the system go to an unexpected state. With the development of chaos theory and practice in engineering, more and more people want to know the answers to the following questions:

(1) Can chaos be controlled?
(2) Can chaos be utilized?
(3) Can two chaotic systems be in resonance as in the case of periodic ones?
(4) If the answer to the second question is positive, then how to generate chaos in a nonchaotic system?

These questions have been partly answered by Ott, Pecora, and Chen in the 1990s, which has led a surge in the application study of chaos. From then on, a new research area, chaos control, including suppression, utilization, and generation of chaotic phenomena, came into being. Among these studies, three aspects attract more attention; that is, stabilization of chaos, synchronization of chaos, and anti-control of chaos.[3]

Why This Book?

Although several monographs on controlling chaos have been published, the present book has unique features which distinguish it from others.

First, the types of chaotic systems studied in this monograph are rather extensive. From the point of view of physics, readers can find not only well-known chaotic systems, such as the Lorenz system, the Rössler system, and the Hénon map, but also some new chaotic systems which appeared in recent years, such as the Liu hyperchaotic system, the Liao chaotic system, the Chen chaotic system, and the Lü chaotic system. From the point of view of models, one can find difference equations, ordinary differential equations, and time-delayed differential equations in this book, which are the main mathematical models describing chaos.

Second, since the monograph is a summary of the authors' previous research, the methods proposed here for stabilizing, synchronizing, and generating chaos in a great degree benefit from the theory of nonlinear control systems, and are more advanced than that appear in other introductory books. One example is that in order to stabilize a chaotic system to one of its equilibria, an inverse optimal control method is developed in this book. The controller designed according to this method not only stabilizes the system but also optimizes a meaningful cost functional. Therefore, the difficulty of solving the Hamilton–Jacobi–Bellman (HJB) equation is avoided. Another example is that in order to synchronize two discrete-time chaotic systems, the exact linearization method is used which provides a unified framework for controller design for both continuous-time and discrete-time chaotic systems. Yet a third example is that in order to chaotify a continuous-time nonchaotic system, a kind of impulsive control method is developed. A mathematical proof shows that the chaos induced by this method satisfies Devaney's definition of chaos.

Last but not least, some rather unique contributions are included in this monograph. One notable feature is the combination of fuzzy logic and chaos. Besides the famous Takagi–Sugeno (T–S) fuzzy model, a novel model, the fuzzy hyperbolic model (FHM), which was initially proposed by one of the authors and whose merits in modeling and control have been illustrated in our book[4] earlier, is also included in this book. In this monograph we combine chaos and fuzzy logic in many aspects: the T–S fuzzy model is used in Chaps. 4 and 8 for suppressing, modeling, and synchronizing chaotic systems, respectively; and the chaotification of the discrete-time FHM and the continuous-time FHM is studied in Chap. 9. Another notable feature is that the methods proposed in this monograph can be applied to a wide class of chaotic systems rather than a specific chaotic system. For example, in Chap. 4 a systematic method is proposed for stabilizing discrete-time chaotic systems and in

[3] Anticontrol of chaos is also known as chaotification. In this book, they are synonymous.

[4] H. Zhang and D. Liu, *Fuzzy Modeling and Fuzzy Control*. Birkhäuser, Boston, 2006.

Chap. 5 a method based on nonlinear geometric control theory is proposed which provides a systematic procedure to synchronize two or more chaotic systems.

The Content of This Book

The whole book involves nine chapters. As indicated by the title of the book, the main content of the book is composed of three parts: suppressing chaos, synchronizing chaos, and generating chaos. To make the book self contained, additional materials are added to provide readers with a brief review of the history of chaos control and some necessary mathematical preliminaries on dynamical systems.

In Chap. 1, we briefly review the history of chaos theory and chaos control. We first review the history of chaos by following the important events in the development of chaos theory. We start the review from the last decade of the 19th century to the 1980s in the 20th century. The work of many distinguished scientists, such as Poincaré, Birkhoff, van de Pol, Littlewood, Andoronov, Lorenz, Smale, Kolmogorov, Arnol'd, Feigenbaum, Li, Yorke, and May, is summarized. After that, we review the development of chaos control from three different aspects, i.e., from the points of view of suppression, synchronization, and chaotification. For each aspect, not only the main methods are introduced but also the ideas behind those methods are mentioned. Some representative methods are introduced, such as the Ott–Grebogi–Yorke (OGY) method and its extensions, the entrainment and migration method, the time-delay feedback method, and some state feedback methods. Chaos synchronization is introduced according to different synchronization patterns, such as complete synchronization, phase synchronization, lag synchronization, and generalized synchronization. Chaotification was proposed by Chen in 1997 and has attracted a lot of attention since then. Methods for chaotification will be reviewed, including the state feedback method, the state delay feedback method, the impulsive control method, and the Smale horseshoe method.

In Chap. 2, necessary mathematical background materials on nonlinear dynamics and chaos are introduced. Dynamical system theory is a powerful tool for chaos study. For the completeness of the book we provide a brief introduction to nonlinear dynamical systems. This chapter is rather difficult for readers with engineering background since many mathematical concepts, definitions, and theorems are involved. The content of this chapter includes two parts. Some concepts and definitions about nonlinear ordinary equations and dynamical systems are introduced first, such as the concepts of flow, fixed point, equilibrium state, invariant set, attractor, stable (unstable) manifold, Floquet index, Lyapunov exponents, and Smale horseshoe. We also state some important theorems, such as the theorem about existence and uniqueness of solutions, the Hartman-Grobman theorems, and the Lyapunov stability theorems. Some concepts and theorems about retarded functional differential equations (RFDEs) are introduced next, such as the definitions of solutions and the initial problem, existence and uniqueness of solutions, and stability of solutions. After that, some stability criteria for RFDEs are introduced.

In Chap. 3, the entrainment and migration control of chaos is introduced. It is usually the case that several attractors coexist in a complex dynamical system. These attractors can be stable or unstable. When a key parameter is changed the attractor may be altered either in appearance or in spatial position, or in both. One of the goals of chaos control is to steer the system's trajectory to the expected state. The background of entrainment and migration control is based on two facts: a multi-attractor chaotic system is sensitive to both initial conditions and parameters and each stable attractor has its basin of attraction. After introducing the basics of entrainment and migration control, a modified entrainment and migration control method: the open-plus-closed-loop (OPCL) control method, is introduced. The OPCL method is an effective method for polynomial chaotic systems when the order of the polynomial is less than two. To overcome this restriction, based on the OPCL method, we propose an improved method, the open-plus-nonlinear-closed-loop (OPNCL) control method. The OPNCL method can be applied to polynomial chaotic systems with arbitrary order and greatly improves the control accuracy. Finally, the OPNCL method is employed for controlling continuous-time polynomial chaotic systems and discrete-time polynomial chaotic systems. By simulating the Chua circuit, the logistic map, and the Hénon map, we validate the effectiveness of the OPNCL method.

The parameters of a chaotic system play an important role, whose variation will lead to completely different dynamics. Sometimes, we want to design a controller which is optimal in a certain sense. However, it is a difficult task to solve the HJB equation when one designs an optimal controller directly along the conventional route. To solve the aforementioned problems, in Chap. 4, we focus on two kinds of methods of suppressing chaos: the adaptive control method and the inverse optimal control method. We develop two new methods of parametric adaptive control for a class of discrete-time chaotic systems and a class of continuous-time chaotic systems with multiple parameters, respectively. The systems are assumed to be linearly parameterized in the adaptive control algorithm. Then, systems with nonlinear distributed parameters and uncertain noise are considered. We apply the inverse optimal control method to stabilize a new four-dimensional chaotic system. The merit of this approach is that it does not need to solve the complicated HJB equation.

In Chap. 5, the synchronization problems of both continuous-time systems and discrete-time chaotic systems are studied. After introducing some necessary preliminaries, a method is proposed to make the single output signal of the response system synchronized with that of the drive system with a scalar controller. The method is based on nonlinear geometric control theory and an exact linearization technique. Then, this method is generalized to the case of multiple output signals. An adaptive method is also proposed to synchronize two different continuous-time chaotic systems. Finally, for problems of synchronizing discrete-time chaotic systems with parametric perturbations, an adaptive controller is designed. This method is based on the theory of exact linearization of discrete-time systems. The methods developed in this chapter have a systematic procedure and can be used with a rather wide class of chaotic systems.

In Chap. 6, we study how to synchronize two identical or different chaotic systems by impulsive control methods. Impulsive control is an efficient method to deal with dynamical systems which cannot be controlled by continuous control. Additionally, in the synchronization process, the response system receives information from the drive system only at discrete time instants, which drastically reduces the amount of synchronization information transmitted from the drive system to the response system and makes this method more efficient in a great number of real-life applications. We first study the complete synchronization of a class of chaotic systems and, after that, we develop synchronization methods for the unified systems with channel time delay in the sense of practical stability. Then, robust synchronization schemes are studied for the chaotic systems with parametric uncertainty and parametric mismatch. Our goal is to develop practical impulsive control methods for different synchronization schemes.

In engineering applications, time delays always exist, and parameters of the system are inevitably perturbed by external disturbances. Moreover, the values of delays and parameters are often unknown in advance. In some special cases, the structure or parameters of the drive system are even unknown in advance. In Chap. 7, we study how to synchronize chaotic systems when time delay exists and the synchronized systems have different structures. Synchronization methods are developed for a class of delayed chaotic systems when the drive system and the response system have the same structures but different parameters. After that, the problem of synchronizing different chaotic systems is studied. Some concrete examples are presented to show how to design the controller. Based on that, a more general case, synchronizing two different delayed chaotic neural networks with unknown parameters, is studied.

In recent years, fuzzy logic systems have received much attention from control theorists as a powerful tool for nonlinear control. A motivation for using fuzzy systems and fuzzy control stems in part from the fact that they are particularly suitable in industrial processes when the physical systems or qualitative criteria are too complex to model and they have provided an efficient and effective way in the control of complex uncertain nonlinear or ill-defined systems. Among various kinds of fuzzy control or system methods, the T–S fuzzy system is widely accepted as a powerful tool for fuzzy control. Chap. 8 focuses on the modeling and synchronizing of chaotic and hyperchaotic systems. We first introduce fuzzy modeling methods for some classical chaotic systems via the T–S fuzzy models. Next, we model some hyperchaotic systems with T–S fuzzy models and then, based on these fuzzy models, we develop an H_∞ synchronization method for two different hyperchaotic systems. Finally, the problem of synchronizing a class of time-delay chaotic systems based on the T–S fuzzy model is considered.

As a reverse process of suppressing or eliminating chaotic behaviors in order to reduce the complexity of an individual system or a coupled system, chaotification aims at creating or enhancing the system's complexity for some special applications. In recent years, many conventional control methods and special techniques were applied to the chaotification of discrete-time dynamical systems or continuous-time dynamical systems. However, most of them are based on the assumption that the analytical representations of the nonlinear dynamical systems to be chaotified are known exactly. For an unknown or uncertain nonlinear dynamical system, the above methods are ineffective. Chap. 9 is devoted to the chaotification of dynamical systems which are originally nonchaotic. We develop a simple nonlinear state feedback controller to chaotify the discrete-time FHM with uncertain parameters. After that, we use an impulsive and nonlinear feedback control method to chaotify the continuous-time FHM. We believe that if a nonchaotic system can be approximated by a fuzzy model with adequate accuracy, then the chaotifying controller of the fuzzy model will also make the nonchaotic system chaotic. Finally, we chaotify two classes of continuous-time linear systems via a sampled data control approach.

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