Preface

The present monograph is a revised and augmented edition to Direct Methods in the Calculus of Variations [179] which is now out of print. The core and the structure of the present book are essentially the one of [179], although it has now almost doubled its size. While writing the present volume, it clearly appeared to me that a new subject has emerged and that it deserves to be called "*quasiconvex analysis*". This name, of course, refers to "convex analysis", although the new subject is still in its infancy when compared with the classical one.

The calculus of variations is an immense and very active field. It is therefore, when writing a book, necessary to make a severe selection. This was already the case for [179] and is even more so for this new edition. Rather than superficially covering a lot of materials, I preferred to privilege only some aspects of the field. Here are some main features of the book. I strongly emphasized the resemblances between convex and quasiconvex analysis as well as the "algebraic" aspect of the field, notably through the determinants and singular values. Besides the classical results on lower semicontinuity and relaxation, an important feature of the monograph is the emphasis on the existence of minimizers for non convex problems.

In doing so I missed several important aspects of the calculus of variations such as regularity theory, study of stationary points, existence and relaxation in BV spaces, minimal surfaces, Young measures and the mathematical study of microstructures, Γ convergence and homogenization. However there are already several excellent books on these subjects, some of them very classical, such as: Almgren [18], Ambrosio-Fusco-Pallara [25], Braides-Defranceschi [101], Buttazzo [112], Buttazzo-Giaquinta-Hildebrandt [117], Dal Maso [217], Dierkes-Hildebrandt-Küster-Wohlrab [248], Dolzmann [249], Ekeland [263], Ekeland-Temam [264], Evans [271], Fonseca-Leoni [284], Giaquinta [307], Giaquinta-Hildebrandt [309], Giaquinta-Modica-Soucek [312], Gilbarg-Trudinger [313], Giusti [315], [316], Ladyzhenskaya-Uraltseva [388], Mawhin-Willem [440], Morrey [455], Müller [462], Nitsche [476], Pedregal [492], Roubicek [517] or Struwe [546], [547]. I have also added in the bibliography several articles which present important developments that I did not discuss in the present monograph, but are still closely related.

For a reader not very familiar with the calculus of variations, it might be advisable to start with an introductory book such as [180], which could be considered as a companion to the present one. Nevertheless, the present monograph,

which is essentially a reference book on the subject of quasiconvex analysis, can be used, as was [179], for an advanced course on the calculus of variations.

I would next like to reiterate my thanks to the people who helped me while writing the earlier version [179], namely J.M. Ball, L. Boccardo, P. Ciarlet, I. Ekeland, J.C. Evard, B. Kawohl, P. Marcellini, J. Moser, C.A. Stuart, E. Zehnder and B. Zwahlen.

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