

Preface

Hemodynamics is the study of the forces and physical mechanisms associated with blood flow in the cardiovascular system. Due to the fact that blood is a suspension of flexible particles in plasma and to the coupling between motion of blood and the vessel wall, necessarily this subject includes both fluid and solid mechanical processes. Hemodynamic features such as flow separation, flow recirculation, and low and oscillatory wall shear stress are believed to play important roles in the localization and development of vascular diseases such as atherosclerosis, cerebral aneurysms, post-stenotic dilations and arteriovenous malformations. Therefore, modeling, mathematical analysis and numerical simulation of these processes can ultimately contribute to improved clinical diagnosis and therapeutic planning.

However, the circulatory system is extremely complex and so researchers are faced with the need to formulate the numerical or mathematical problem in a form which is sufficiently simple to be tractable, yet maintains enough complexity to be relevant. For example, rather than modeling the entire circulatory system, isolated segments of the circulation are studied, introducing the need to choose appropriate inflow and outflow boundary conditions and possibly take a multi-scale approach. The blood vessel wall is an inhomogeneous, nonlinear, material capable of growth and remodeling, and blood is a concentrated suspension of deformable cellular elements in plasma. The modeler needs to choose suitable constitutive models for the wall and blood. The diameter of vessels in the circulatory system ranges from the order of centimeters in the larger arteries to microns in the capillaries. It is therefore appropriate to model blood as a single phase continuum in some parts of the circulatory system, while in others, it is necessary to model blood as a suspension. The chapters in this book address these and other topics from different perspectives.

The present volume is a collection of six chapters which are based on a series of lectures delivered by Anne M. Robertson (University of Pittsburgh), Giovanni P. Galdi (University of Pittsburgh), Rolf Rannacher (University of Heidelberg), and Stefan Turek (University of Dortmund) at the Oberwolfach Seminar “Hemodynamical Flows: Aspects of Modeling, Analysis and Simulation”, during the period November 20–26, 2005.

These lectures focused on various aspects of hemodynamics from different angles, including physical modeling, mathematical analysis and numerical simulation. Accordingly, this volume addresses the following main topics:

- General background in continuum mechanics;
- Multiphase nature of blood;
- Rheological data for blood;
- Newtonian and non-Newtonian constitutive models for blood;
- Mechanical models for blood vessel walls;
- Numerical methods for flow simulation;
- Aspects of mesh and model adaptivity;
- Particle transport in viscous flows;

- Flows through systems of pipes;
- Fluid-structure interaction in blood vessels.

The above topics are organized as follows:

In the first chapter, *Review of Relevant Continuum Mechanics*, by A.M. Robertson, the basic kinematical and dynamical issues that are at the foundation of continuum mechanics used in the book are surveyed. The constitutive theory for Newtonian fluids, general nonlinear viscous fluids, yield stress "fluids", viscoelastic fluids and thixotropic fluids are covered. In preparation for a discussion of experimental data on blood in the second chapter, viscometric flows and commonly used rheometers are discussed. Finally, the fundamentals of nonlinear elastic solids are introduced.

The second chapter, *Hemorheology*, by A.M. Robertson, A. Sequeira, and M.V. Kameneva, is dedicated to constitutive models for blood, based on phenomenological considerations. Experimental data on the multiphase properties of blood are considered as well as the relationship between these properties and the mechanical behavior of blood. These mechanical properties include shear thinning viscosity, yield stress behavior and viscoelasticity. The significance of these non-Newtonian behaviors in the circulation are addressed. The subject of blood coagulation is considered, motivated by its importance in cardiovascular device design. The chapter concludes with sections on the effect of gender and certain diseases states on the mechanical response of blood.

The third chapter, *Mathematical Problems in Fluid Mechanics*, by Giovanni P. Galdi, discusses some, of the many, topics which are at the foundation of the analysis of models for blood flow, and points out directions for future research. Specifically, it focuses on the following three different problems: pipe flow of a Navier-Stokes liquid, flow of non-Newtonian and, in particular, viscoelastic liquids, and liquid-particle interaction. This analysis has two main objectives. The first is the study of the well-posedness of the relevant problems, whereas the second is to provide a rigorous explanation of some fundamental experiments. In particular, special attention is given to the investigation of the dependence of even qualitative features of the on the non-Newtonian properties of the liquid.

The fourth chapter, *Methods for Numerical Flow Simulation*, by Rolf Rannacher, introduces the computational methods for the simulation of PDE based models of laminar hemodynamical flows. Space and time discretization is discussed with emphasis on operator-splitting and finite-element Galerkin methods because of their flexibility and rigorous mathematical basis. Special attention is paid to the simulation of pipe flow and the related question of artificial outflow boundary conditions. Further topics include efficient methods for the solution of the resulting algebraic problems, techniques of sensitivity-based error control and mesh adaptation, as well as flow control and model calibration. The analysis is restricted to laminar flows, where all relevant spatial and temporal scales can be resolved, and no additional modeling of turbulence effects is required. This covers most of the relevant situations of hemodynamical flows.

In the fifth chapter, *Numerics of Fluid-Structure Interaction*, by Sebastian Bönisch, Thomas Dunne, and Rolf Rannacher, numerical methods for simulating the interaction of viscous liquids with rigid or elastic bodies are described. General examples of fluid-solid/structure interaction (FSI) problems are flow transporting rigid or elastic particles (particulate flow), flow around elastic structures (airplanes, submarines) and flow in elastic structures (hemodynamics, transport of fluids in closed containers). A common variational description of FSI is developed as the basis of a consistent Galerkin discretization with a-posteriori error control and mesh adaptation, as well as the solution of optimal control problems based on the Euler-Lagrange approach.

The sixth chapter, *Numerical Techniques for Multiphase Flow with Liquid-Solid Interaction*, by Jaroslav Hron and Stefan Turek, discusses numerical methods for simulating multiphase flows with liquid-solid interaction based on the incompressible NavierStokes equations combined with constitutive models for nonlinear solids. More precisely, it addresses the following three topics. The first concerns finite-element discretization and corresponding solver techniques for the resulting algebraic systems. The second regards a fully monolithic finite-element approach for fluid-structure interactions with elastic materials which is applied to several benchmark configurations. Finally, the third section introduces the concept of FEM fictitious boundary techniques, together with operator-splitting approaches for particulate flow. This latter is especially designed for the efficient simulation of systems with many solid particles of different shapes and sizes.

This work is aimed at a diverse readership. For this reason, an effort was made to keep every topic as self-contained as possible. In fact, whenever details are not explicitly given, the reader is referred to the appropriate literature.

Last, but not least, the authors would like to convey their sincere thanks to all participants, who, with their questions and insights, helped to maintain a lively and stimulating scientific atmosphere, typical of all Oberwolfach meetings.