Foundations in Signal Processing, Communications and Networking 4

# Advanced Topics in System and Signal Theory

A Mathematical Approach

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# Preface

The requirement of causality in system theory is inevitably accompanied by the appearance of certain mathematical operations, namely the Riesz projection, the Hilbert transform, and the spectral factorization mapping. A classical example illustrating this is the determination of the so-called Wiener filter (the linear, minimum means square error estimation filter for stationary stochastic sequences [88]). If the filter is not required to be causal, the transfer function of the Wiener filter is simply given by $H(\omega) = \Phi_{xy}(\omega)/\Phi_{xx}(\omega)$, where $\Phi_{xx}(\omega)$ and $\Phi_{xy}(\omega)$ are certain given functions. However, if one requires that the estimation filter is causal, the transfer function of the optimal filter is given by

$$H(\omega) = \frac{1}{[\Phi_{xx}]_{+}(\omega)} \mathfrak{P}_{+} \left( \frac{\Phi_{xy}(\omega)}{[\Phi_{xx}]_{-}(\omega)} \right) , \qquad \omega \in (-\pi, \pi] .$$

Here $[\Phi_{xx}]_{+}$ and $[\Phi_{xx}]_{-}$ represent the so called spectral factors of $\Phi_{xx}$, and $\mathfrak{P}_{+}$ is the so called Riesz projection. Thus, compared to the non-causal filter, two additional operations are necessary for the determination of the causal filter, namely the *spectral factorization mapping* $\Phi_{xx} \mapsto ([\Phi_{xx}]_{+}, [\Phi_{xx}]_{-})$, and the *Riesz projection* $\mathfrak{P}_{+}$.

In applications the two functions $\Phi_{xx}(\omega)$ and $\Phi_{xy}(\omega)$ are usually not perfectly known but disturbed by measurement errors, or their values are only given at a finite number of sampling points $\{\omega_k\}_{k=1}^{N}$. The question arises, how these errors in the given data influence the calculation of the optimal filter $H(\omega)$. The answer will depend strongly on the metric in which the errors are measured, i.e. on the function spaces on which these problems are considered, and an answer requires the investigation of the continuity and boundedness of the involved operations (Riesz projection and spectral factorization) on the desired function spaces.

This monograph is intended primarily for engineers working on such robustness problems under a causality constraint. At the beginning, it presents the mathematical methods, necessary to approaching these problems. Then some related classical results concerning the boundedness and continuity of the Hilbert transform and Riesz projection are presented. Finally, these methods and results are applied to selected topics from signal processing.

The first part of the this monograph gives a very brief introduction to the main mathematical methods used later in the book. This part serves primarily as a review of results needed in later chapters, so that the work becomes essentially self-contained. The different topics are only covered as far as they will be needed and proofs are sometimes omitted. Appropriate reference are given for those who want a more detailed introduction to the different topics. This work presupposes a working knowledge of real and complex analysis (roughly as contained in [70]) as well as some basic elements of functional analysis (e.g. as in [54]).

The second part collects the basic abstract results concerning the continuity and the boundedness of the Hilbert transform and the Riesz projection on different Banach spaces. These results are the basis for the applications discussed in the third part of this monograph. Here four applications from signal processing are investigated in some detail, namely the expansions of transfer functions in orthonormal bases in Chapter 7, the linear approximation from measured data in Chapter 8, the calculation of the Hilbert transform in Chapter 9, and the spectral factorization in Chapter 10. All these topics are essentially problems of recovery or approximation of causal function from measured data, which are generally corrupted by small errors. It is investigated how these errors influence the possibility of recovering the desired signal from the measurements under the restriction that this recovery is based only on past and present measurements, i.e. under the requirement of causality.

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