

Preface

Although self-focusing and other self-action phenomena have been studied extensively for nearly 50 years (see, e.g., reviews [1–5] and the books [6–11]), this volume constitutes the first comprehensive treatment devoted entirely to the subject. It combines past and present results on both theory and experiments and includes interpretation of experiments with femtosecond light pulses and their comparison with nanosecond/picosecond self-focusing. The book is intended for scientists and engineers working with lasers and their applications. It consists of overview chapters on self-focusing in different media written by leading scientists from the United States, Russia, France, China, Canada, Israel, Italy, Lithuania, Mexico, and Brazil. Many of our authors participated in a symposium with the same title as this book during the International Quantum Electronics Conference (IQEC 2005) held during July 2005 in Tokyo, Japan.

Modern experiments with femtosecond laser pulses have provided new prospects for this rich field, promising numerous practical applications in terms of experiments that were not possible in the past. One of the current advances in femtosecond self-focusing at laser power levels of several TW is the observation of the generation of the collimated, coherent white light continuum propagating through the atmosphere up to an altitude of more than 10 km and its potential application for remote sensing and lightning control [12–14]. Attosecond pulse generation is another application of femtosecond filamentation [14].

New computer capabilities deepen our knowledge of self-focusing as well, for example, the *nonparaxial* approximation in nonlinear beam propagation showed *nonsingular* behavior at self-focus/self-foci even in the absence of nonlinear absorption and refractive index saturation mechanisms [15–17].

In this Preface, we provide a clarification of terminology, explain the importance of self-focusing studies, outline the book purpose, and briefly describe the main highlights of the chapters on self-focusing in the past and the present.

Terminology in the Classical Case

As described in textbooks on nonlinear optics (e.g., [10, 11]), *self-focusing*, *self-defocusing*, *self-trapping*, and *filamentation* are the major self-action effects in which a beam of light modifies its own propagation by means of the nonlinear response of a material medium. The early history of these concepts with relevant references is briefly outlined in [18].

Above a certain *critical power* P_{cr} , an intense light beam with an intensity gradient along its cross-section, propagating in a nonlinear medium, experiences *self-focusing* or *self-defocusing* depending of the sign of the nonlinear susceptibility $\chi^{(3)}$ [19]. For the stationary self-focusing case, a beam with a power above P_{cr} collapses into a focus [5, 7, 20] and/or multiple foci [2, 21] at the *self-focusing distance (length)* [20] within the material. For a pulsed laser, different parts of the pulse with different powers are focused at different distances, so an illusion of a *filament* is observed in the experiment (“moving focus” model [2, 4, 5, 18, 21]). At powers much higher than P_{cr} , the beam breaks up into many components causing *multiple filamentation* (“small-scale” *self-focusing*, *spatial modulation instability* [20, 22–24]). This process occurs as a consequence of the growth of both amplitude and phase perturbations of the beam wave front by means of forward four-wave mixing amplification of this noise.

In the *self-trapping* process a beam of light propagates with a constant diameter as a consequence of an exact balance between self-focusing and diffraction effects. In 1964, self-trapping (waveguiding) solutions of the wave equation in nonlinear Kerr media were independently found [25, 26]. For the two-dimensional case, the lowest-order solution for a beam with cylindrical symmetry was reported in [25] and is in the form of a bell-shaped curve called the “*Townes profile*.” *Higher-order solutions* with spatial profiles containing rings have been reported in [27, 28]. See also books [7, 11] and reviews [5, 17, 29]. Later papers showed that in reality this *spatial soliton* behavior can be stable only in special cases, for example, in a medium with saturation of nonlinearity [5] or in the case of a beam that varies in only one transverse dimension (planar waveguides) where these truly self-trapped filaments were observed experimentally [30–32]. So, in some early papers, the formation of self-focusing filaments in experiments is called self-trapping. For more details on self-trapping and spatial solitons see books [33, 34] and reviews [35–37].

Conical emission [36, 38] manifests itself as the occurrence of single or multiple concentric rings surrounding the pump laser beam. The generated cone can be either at the same frequency as the original beam [39, 40] or spectrally shifted [36, 38]. The physical process leading to conical emission is different under different circumstances [14, 36, 38–43].

Continuum (supercontinuum) generation [14, 38, 41, 44] was first observed in 1970. It is a nonlinear optical process for strong spectral broadening of light and

can span from the ultraviolet to the infrared (“white light” spectrum). It has been observed in bulk solids, liquids, gases and fibers including microstructured fibers [44]. An important property of such white light is its spatial coherence. The physical processes behind supercontinuum generation can be very different, for example, self-phase modulation, four-wave mixing, multiphoton ionization with the formation of free electrons, and so on.

Some other terminology used by the authors of this book including its modern development is explained later in this preface.

Importance of Self-Focusing

From a practical point of view, self-focusing effects impose a limit on the power that can be transmitted through an optical medium. Self-focusing also can reduce the threshold for the occurrence of other nonlinear optical processes [4]. Self-focusing often leads to damage in optical materials and is a limiting factor in the design of high-power laser systems [8, 9, 24], but it can be harnessed for the design of optical power limiters and switches.

At a formal level, the equations for self-focusing are equivalent to those describing Bose–Einstein condensates and certain aspects of plasma physics and hydrodynamics. The wave equation governing the self-focusing effect is the prototype of an important class of partial differential equations such as the Ginzburg–Landau equation for type II superconductors and the Schrödinger equation for particles with self-interactions. Important trends in self-focusing theory that have evolved over the past 40 years include *soliton formation* and *wave collapse*. Collapse is a self-compression of a light beam and/or pulse with the formation of a singularity. Analogous to a star undergoing gravitational collapse, nonlinear effects can cause a wave to collapse on itself. Such behavior is intrinsic to plasma physics, hydrodynamics, nonlinear optics, and Bose–Einstein condensates. Analysis of many different types of nonlinear wave equations indicates that a collapsing wave can transform into a universal transverse profile regardless of its initial shape [16, 29, 45, 46].

Remarkable self-focusing effects have been observed recently with femtosecond laser beams propagation in the atmosphere: light filaments (“light strings”) in the air with repeated filamentation over distances greater than 10 km [12–14] have been observed. A broad (ultraviolet to mid-infrared) spectrum of radiation is generated from such air filaments, permitting spectroscopy and localized remote sensing of chemical species and aerosols in the lower atmosphere [12–14]. The entire absorption spectrum can be determined by a single pulse from a portable femtosecond laser. Another exciting possibility of the use of these “light strings” containing plasma filaments is to guide lightning away from sensitive sites.

Intent and Outline of This Book

One of the goals of this book is to connect the extensive early literature on self-focusing, filamentation, self-trapping, and collapse with nanosecond and picosecond lasers with more recent studies since approximately the 1990s. These modern studies are aimed at issues such as self-focusing of femtosecond pulses, white light generation [38, 44], and the generation of long filaments in air [12–14, 41, 42, 47, 48].

It should be noted that in explaining femtosecond self-focusing and filamentation, some new terminology and new models were introduced that did not exist in the past. In this connection there has been some debate in the literature [21], so we tried to address this diversity of opinions. We hope that this book will help current and future researchers to understand whether or not this new terminology has analogs in the past or it is connected with purely new phenomena that cannot be deduced from scaling of similar experiments using nanosecond or picosecond pulses.

Because of book-length restrictions, we have omitted or only briefly outlined certain areas. Some of these topics have been discussed in other books, for example, spatial and temporal solitons [33–34]. We did not include thermal self-focusing, spatial self-phase modulation and solitons in liquid crystals, self-focusing in semiconductors, optical power limiting and switching effects, high $\chi^{(3)}$ materials, self-focusing in waveguides and fibers, and so on. The focus of our book is *filamentation* and *self-focusing collapse*.

In addition, we apologize in advance if somebody who made significant contribution to self-focusing is omitted from this book. Initially we planned to reprint the most significant papers on self-focusing, but while selecting such papers we realized that the size of this book would then have become enormous. It was decided to reprint only the first paper on self-focusing by Askar'yan (1962) [49]. To facilitate the understanding of Marburger's notes on early self-focusing theory, we have also reprinted his 1975 review with many citations of the papers of others [5]. Also, we were only able to reference some other excellent reviews of the past [1–4]. Two papers from 1964 that have been widely cited in the literature, but have never been published previously in English (Talanov) [26] or as a full journal publication (only 17 lines in an abstract by Hercher [50]), are included here as well.

The book contains two parts with 11 chapters in the first part, “Self-Focusing in the Past,” and 13 chapters in the second part, “Self-Focusing in the Present.” Sometimes this division is not rigid: the same chapter may contain the results both from the past and the present.

Next, we describe briefly the content of each of the chapters.

Part I: Self-Focusing in the Past

Part I consists of 11 chapters and contains surveys of experiments on whole-beam self-focusing, filamentation, self-trapping and small-scale self-focusing with nanosecond and picosecond laser pulses in liquids, solids, and atomic vapors. It also treats accompanying processes (e.g., conical emission and white-light supercontinuum generation) and methods of suppression of self-focusing effects. The main theoretical concepts and models from the past are also reviewed. These models are based on analytical approximation and/or numerical modeling of the nonlinear Schrödinger equation (NLSE) [51].

In the first chapter of this book, by Shen, a brief review is provided of early experimental research on self-focusing and filament formation of nanosecond and picosecond light pulses [4, 10]. Similarities and differences are discussed between nanosecond–picosecond self-focusing, and recent experiments with femtosecond pulses with the appearance of long filaments of light in air.

The second chapter, by Marburger, contains the author's comments on early research in self-focusing theory, describing its major advances in light of his two papers and his review of the self-focusing theory before 1975 [5]. A reprint of this review is also provided.

The chapter by Fraiman, Litvak, Talanov, and Vlasov contains a review of self-focusing theory in the paraxial approximation as described in book [7], based on the NLS (parabolic) equation. Various manifestations of self-focusing are described: self-trapped solutions ("homogeneous waves" [7]), instability of plane waves, formation of nonlinear foci in cubic nonlinear media, self-similar collapse solution, suppression of self-focusing in periodic systems, and spectral broadening. Some questions of self-action effects involving femtosecond pulses are also discussed.

Chiao, Gustafson, and Kelley review self-focusing and related phenomena, for example, coupling of waves through the nonlinear index and spatial instability, beam breakup and multiple filament formation, spatial self-phase modulation and estimating the beam self-focusing distance, self-focusing intensity singularity and beam collapse, limitations on blow-up and collapse due to saturation of the nonlinear refractive index and other nonlinearities, self-focusing of pulses and "light bullets," and self-trapping (spatial solitons). They also discuss physical mechanisms that give rise to the refractive index nonlinearity.

Lugovoi and Manenkov address both the theory and experimental confirmation of the multifocus structure model for a stationary case and a moving focus model for laser radiation of nanosecond–picosecond pulse duration [2]. The influence of the beam's spatial profile (Gaussian or super-Gaussian) on the self-focusing threshold and length is discussed. Analysis of the adequacy of these models for self-focusing of femtosecond pulses, discussion of new terminology, and suggested future directions for the study of femtosecond self-focusing in air are provided.

Campillo describes in detail small-scale self-focusing as a result of transverse (spatial) modulational instability. He briefly surveys the essential physics, presents a linearized theory, and summarizes experimental examples illustrating the dependence between optimal spatial frequency, gain coefficient, small-scale self-focusing length, beam intensity, and beam shape including the modern advances in the theory and the experiments with femtosecond lasers. A number of applications are discussed as well as the impact of this field on other scientific disciplines.

Kuznetsov concentrates on a brief review of collapse theory in nonlinear optics described by the NLSE. Emphasis is placed on collapse classification (weak, strong, and black holes regime), self-similarity in collapse, correspondence between solitons and collapses, and the role of group velocity dispersion on the collapse of electromagnetic pulses.

Lukishova, Senatsky, Bykovsky, and Scheulin provide a brief outline of self-focusing as the primary nonlinear optical process limiting the brightness of high-peak-power Nd:glass laser systems. Both small-scale and large-scale self-focusing are considered. The role of Fresnel diffraction at apertures as the source of dangerous spatial scales for small-scale self-focusing is illustrated. Methods for the formation of super-Gaussian laser beams and their self-focusing are presented. Other methods for self-focusing suppression are described in addition to beam apodization.

Zerom and Boyd review self-focusing and other self-action effects in atomic vapors as a medium with a saturable nonlinearity. Experimental results on self-trapping (spatial solitons), conical emission, and filamentation are described. The models for the explanation of conical emission in atomic vapors are presented. Multiple filamentation is discussed in connection with optical pattern formation.

Ni and Alfano consider white-light supercontinuum generation [44] and periodic filamentation in glass controlled by Fresnel diffraction from a circular aperture or a straight edge. Also discussed are self-phase modulation (distortion in phase), self-steepening (distortion of pulse envelope shape), and conical emission in glass as the process of four-wave parametric generation with a nonlinear refractive index change at the surface of the filament. Coherence properties of supercontinuum sources accompanying the filaments generated at different spatial positions are reported.

The last chapter of Part I contains three papers from the past. Askar'yan's paper reprinted from 1962 [49] is the first paper where self-focusing was suggested. Talanov's paper from 1964 [26] is the first English translation of his work on the waveguide (self-trapped or soliton) solution. Hercher's paper is the first publication of his 1964 Optical Society of America Annual Meeting presentation [50] with the first photographs of damage resulting from self-focusing filaments. This paper and the photographs have never been published previously.

Part II: Self-Focusing in the Present

Extensive studies on femtosecond filamentation started after the first report in 1995 of the experimental observation of femtosecond filaments in air at a peak power exceeding a few GW [52]. A 20-m-long filament with an intense core of diameter of approximately 100 microns was observed.

For the explanation of femtosecond filament formation, the following principal models have been suggested: self-channeling, moving focus model, and dynamic spatial replenishment. We do not consider here relativistic and charge displacement self-channeling [53] at incident intensities $\sim 10^{19}$ W/cm² when strong ponderomotive forces cause electron cavitation. In the *self-channeling* model [52], self-trapping occurs when self-focusing is counteracted by defocusing owing to medium ionization and diffraction. However, this balance was never identified in numerical simulations showing several cycles of *focusing–defocusing–refocusing*. This process can occur more than once for high-power input pulses. Refocusing events on the trailing edge of the pulse are called “*dynamic spatial replenishment of light*” [54]. In the *moving focus* model [41, 42], which was suggested in the past for nanosecond and picosecond pulses [2, 4, 5], a filament arises from the continuous succession of foci arising from the self-focusing of the various longitudinal slices of a pulse, producing the illusion of long-distance propagation of one self-guided pulse (filament). The same illusion is created in the dynamic spatial replenishment model. The concept of an “*energy reservoir*” was introduced for the background beam that surrounds the filament core and provides energy to it [41–43, 47]. Many practical applications need *regularization* and *control* of the propagation distance of *multiple filaments*. It is achieved by diffraction at apertures, lens arrays, and beam ellipticity [14, 23, 38, 42, 43].

The formation of *spatiotemporal solitons* (STS), sometimes called “*light bullets*,” is one of the major goals in the field of nonlinear waves. Although STS in all three space dimensions and time have not been observed experimentally, enormous progress was made in lower space-dimension STS formation in media with cascaded quadratic nonlinearity [55]. Related to STS are *nonlinear X-waves* that are only weakly localized, but manifest a kind of spatiotemporal trapping [55, 56]. X-waves are nondiffractive and nondispersive waves with an X-shaped axial cross-section both in the far-field and the near-field. In the linear case, they can be formed by axicons. In media with normal group-velocity dispersion, the nonlinearity acts as the driving mechanism that reshapes a narrow beam and ultrashort pulse into conical elements with features of X-waves.

Part II consists of 13 chapters. Six chapters contain results on self-focusing and filamentation of *femtosecond* laser pulses and their interpretation by each of several groups actively working in this field. Filamentation both in air and in condensed matter is considered with the emphasis on femtosecond filament

formation and on the models explaining its origin and behavior. Four chapters describe the formation of nonlinear X-waves, spatial and temporal dynamics of self-focusing collapse of ultrashort pulses, some aspects of modern self-focusing theory including nonparaxial approximations of the wave equation and its application to the theory of collapse, high-order self-trapped solutions, and spatial modulation instability. Three additional chapters review self-focusing in media with a quadratic nonlinearity, self-trapping in photorefractive crystals, and measurements of nonlinear refraction and its dispersion.

In particular, Couairon and Mysyrowicz review both simulations and experimental results on self-action effects that occur during femtosecond filamentation in gases, liquids, and solids. They outline the evolution of the modeling of femtosecond filamentation in transparent media, and they consider a simple self-trapping model, the moving focus model, and saturation of self-focusing and self-channeling with formation of several focusing–defocusing cycles at intensities sufficient for multiphoton absorption and plasma-induced defocusing. Recently discovered self-action effects are outlined: the generation of single cycle pulses by filamentation in gases, the beam self-cleaning effect, filamentation in an amplifying medium, and the organization of multiple filaments by various methods. They briefly outline the current state of the art of long-distance filamentation in air. Conical emission accompanying a self-guided pulse, continuum generation, and nonlinear X-waves are also discussed.

Zhang, Hao, Xi, Lu, Zhang, Yang, Jin, Wang, and Wei describe experiments on filaments of nearly 160 m in length under propagation of intense femtosecond pulses in air, and they describe in detail their diagnostic techniques. The spatial evolution of the filaments, interactions between filaments, third harmonic generation, lifetime prolongation of the filament by use of a second, sub-ns laser pulse, and laser-guided discharge and laser propulsion are also considered. The dependence of long-distance filamentation on chirp and divergence angle, the role of an energy reservoir, and the comparison of filamentation in focused and unfocused beams are discussed.

Chin, Liu, Kosareva, and Kandidov discuss physical concepts underlying femtosecond laser filamentation as deduced from experiment. These include slice-by-slice self-focusing according to the moving focus model, intensity clamping by plasma-induced defocusing effect, supercontinuum generation and conical emission, the background reservoir concept, multiple refocusing, and multiple filamentation competition. Some important potential applications are also briefly mentioned.

Kandidov, Dormidonov, Kosareva, Chin, and Liu outline the theory of the moving-focus model for the analysis of femtosecond filament formation and the ring formation of a ring pattern in the beam's transverse cross-section near the filament. The authors also provide a comparison of the femtosecond focusing–defocusing–refocusing model with the earlier multifocus model for the stationary case [2]. Their model of filament formation includes the initial stage of

filamentation, estimation of the critical power, and the generalization of Marburger's formula for the self-focusing length for femtosecond pulses with elliptical cross-section and for the chirped pulses. The influence of turbulence and of aerosols in the atmosphere on chaotic pulse filamentation is considered as well as the spatial regularization of filaments by the introduction of initial regular perturbation of amplitude and phase. This chapter also contains the first published photograph (1965) of self-focusing filaments [57]. See also [50] and [58, 59] of 1965 for filament photographs.

Matvienko, Bagaev, Zemlyanov, Geints, Kabanov, and Stepanov present an analysis of femtosecond self-focusing in terms of the evolution of the so-called effective parameters (energy transfer coefficient, effective beam radius, effective pulse duration, limiting angular divergence, and effective intensity) describing the regime of formation of a single axial filament. Filamentation behavior of femtosecond laser pulses in the air in the presence of aerosols is also considered.

Gaižauskas, Dubietis, Kudriašov, Sirutkaitis, Couairon, Faccio, and Di Trapani focus on the experimental observations and theoretical investigations of the propagation of intense femtosecond pulses in water and fused silica. They point out that nonlinear losses cause beam distortion which influences self-focusing. Explaining fringe formation during beam propagation with nonlinear losses (see also book [7]), they introduce the concept of a conical (Bessel-like) wave. In the non-waveguiding regime, they find a steady-state solution of the high-intensity central core of the beam containing $\sim 20\%$ of total beam energy. This core takes the energy from the rest of the beam (energy reservoir) surrounding it. Multiple filaments from elliptical beams and nonlinear X-waves are observed experimentally in both water and fused silica.

Conti, Di Trapani, and Trillo overview the main experimental and theoretical results on nonlinear X-waves during filamentation and trapping in normally dispersive media, starting from linear X-waves and X-waves in second-harmonic generation. The role of group velocity dispersion (GVD) during filamentation in Kerr-like media on nonlinear X-wave formation is emphasized.

Gaeta considers recent theoretical and experimental work on spatial and temporal dynamics of self-focusing collapse of ultrashort laser pulses, for example, self-similar evolution, modulation instability versus Townes collapse, and the collapse of super-Gaussian beams. He also discusses pulse splitting in the normal GVD regime, self-focusing in the anomalous GVD regime, optical shock formation and supercontinuum generation, femtosecond filamentation and light strings in air.

Fibich reviews some aspects of modern self-focusing theory, highlighting some pre-1975 results, describes the current understanding of some old concepts, and describes new theoretical challenges. He emphasizes universal and new blow-up profiles of collapsing beams, the blow-up rate, super-Gaussian beam collapse, self-focusing of "low power beams" ($\sim$ several or tens of P_{cr}), self-focusing distance, partial beam collapse, the arresting of collapse by

nonparaxiality with focusing–defocusing oscillations (multiple foci in [15]), multiple filamentation, and the effect of normal GVD.

Chávez-Cerda, Itube-Castillo, and Hickmann consider the “nonparaxial NLSE” and effects of nonparaxiality on whole-beam self-focusing (see also the chapter by Fibich and [7, 15]). This chapter also shows that nonparaxiality arrests collapse, allowing further beam propagation and providing the opportunity to examine the beam behavior beyond the predicted catastrophic focus. In addition, this chapter provides nonparaxial analogs of stationary (self-trapped) solutions of the NLSE. It is shown that the generation of these modes can be induced by a diffracted beam at a circular aperture, propagating in a self-focusing medium and that, under perturbations, they may break up into hot spots. Modulation-instability patterns are presented for both Gaussian and flat-top beams and are compared with classical experiments of the past (see Campillo’s chapter in Part I).

Wise and Moses review self-focusing and self-defocusing of femtosecond pulses in media with cascaded quadratic nonlinearities. Quadratic media appear to be unique in offering a means of impressing a self-defocusing nonlinear phase shift on ultrashort pulses. Self-defocusing nonlinearities are discussed in connection with modelocking in a femtosecond laser, pulse compression, nonlinear polarization rotation and compensation of self-focusing. Saturable self-focusing and space–time solitons (“light bullets”) are considered as well.

DelRe and Segev review the current state of the art on both experiments and theory of self-focusing and self-trapping (spatial solitons) in photorefractive media. The study of photorefractive solitons has a significant impact on soliton science. The chapter includes a description of the mechanisms of photorefractive nonlinearity. Some important breakthroughs obtained with solitons in photorefractive crystals are described: soliton interaction and collisions, two-dimensional solitons, vector and composite solitons, and incoherent solitons. Optical induction methods for nonlinear photonic lattices and a series of new solitons are mentioned, for example, 2D lattice (“discrete”) solitons, spatial gap solitons, and others.

Van Stryland and Hagan in their chapter on measuring nonlinear refraction and its dispersion describe the beam distortion and Z-scan methods and include a discussion of some possible pitfalls in measuring absolute values of nonlinearities with the Z-scan method. They also describe a method for determining the spectral dependence of the nonlinear change in the refractive index using femtosecond white-light continua as the source for Z-scan. Physical mechanisms leading to nonlinear refraction are also briefly discussed.

Conclusion

We believe that this book will prove useful for academics, researchers, engineers, and students in various disciplines who require a broad introduction to

this subject and who would like to learn more about the state-of-the art and upcoming trends in self-focusing, self-trapping, filamentation, and self-focusing collapse. We thank all the contributors who found the time, energy and enthusiasm to write these chapters. We thank Elsevier, Science Direct, American Institute of Physics, and Izvestia Vuzov, Radiophysica for permission to reprint certain papers. We are also grateful to the U.S. Air Force Office for Scientific Research for its support of the IQEC 2005 symposium on this topic, which provided the motivation for the creation of this book.

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References

1. S.A. Akhmanov, R.V. Khokhlov, A.P. Sukhorukov: Self-focusing, self-defocusing and self-modulation of laser beams, *Laser Handbook*, **2**, E3, 1151–1228, F.T. Arecchi, E.O. Schulz-Dubois (Eds.), North-Holland (1972). See also S.A. Akhmanov, A.P. Sukhorukov, R.V. Khokhlov: Self-focusing and diffraction of light in a nonlinear medium, *Sov. Phys. Uspekhi* **10**, 609–636 (1968).
2. V.N. Lugovoi, A.M. Prokhorov: Theory of the propagation of high-power laser radiation in a nonlinear medium, *Sov. Phys. Uspekhi* **16**, 658–679 (1974).
3. O. Svelto: Self-focusing, self-trapping, and self-phase modulation of laser beams: Progress in Optics XII, E. Wolf (ed.), pp. 3–51, North-Holland, Amsterdam (1974).
4. Y.R. Shen: Self-focusing: experimental, *Prog. Quant. Electr.* **4**, 1–34 (1975).
5. J.H. Marburger: Self-focusing: theory, *Prog. Quant. Electr.* **4**, 35–110 (1975). See also Chapter 2 of this book by Marburger.
6. M.S. Sodha, A.K. Ghatak, V.K. Tripathi: *Self-Focusing of Laser Beams in Dielectrics, Plasma, and Semiconductors*, Tata McGraw-Hill, New Delhi (1974).
7. S.N. Vlasov, V.I. Talanov: *Wave Self-Focusing*, Institute of Applied Physics of the Russian Academy of Science, Nizhny Novgorod, 220 pp (1997).
8. D.C. Brown: *High-Peak-Power Lasers*. Chapter 7: Nonlinear effects in high-peak-power Nd: glass laser systems, pp. 188–235, Springer-Verlag, New York (1981).
9. A.A. Mak, L.N. Soms, V.A. Fromzel, V.E. Yashin: *Nd:glass Lasers*. Chapter 6.1: Main physical restrictions of laser power and brightness; Chapter 6.2: Methods of self-focusing suppression, pp. 242–259, Nauka, Moscow (1990).
10. Y.R. Shen: *The Principles of Nonlinear Optics*. Chapter 17: Self-focusing, pp. 303–333, John Wiley, New York (1984).
11. R.W. Boyd, *Nonlinear Optics*. Chapter 7: Processes resulting from the intensity-dependent refractive index, pp. 311–370, Academic Press, San Diego (2003).
12. A.L. Gaeta: Collapsing light really shines, *Science*, **301**, 54–55 (2003).
13. J. Kasparian, M. Rodriguez, G. Méjean, J. Yu, E. Salmon, H. Wille, R. Bourayou, S. Frey, Y.-B. André, A. Mysyrowicz, R. Sauerbrey, J.-P. Wolf, L. Wöste: White-light filaments for atmospheric analysis, *Science*, **301**, 61–64 (2003).

14. A. Couairon et al.: Self-focusing and filamentation of femtosecond pulses in air and condensed matter: Simulations and experiments. *Self-focusing: Past and Present*, R. W. Boyd, S.G. Lukishova, Y.R. Shen (Eds.), (this volume, Chapter 12).
15. M.D. Feit, J.A. Fleck, Jr.: Beam nonparaxiality, filament formation, and beam breakup in the self-focusing of optical beams, *J. Opt. Soc. Am. B* **5**, 633–640 (1988).
16. G. Fibich: Some modern aspects of self-focusing theory. *Self-Focusing: Past and Present*, R.W. Boyd, S.G. Lukishova, Y.R. Shen (Eds.), (this volume, Chapter 17).
17. S. Chávez-Cerda et al.: Diffraction-induced high-order modes of the $(2 + 1)$ -D nonparaxial nonlinear Schrödinger equation. *Self-Focusing: Past and Present*, R.W. Boyd, S.G. Lukishova, Y.R. Shen (Eds.), (this volume, Chapter 22).
18. Y.R. Shen: Self-focusing and filaments of light: Past and present. *Self-Focusing: Past and Present*, R.W. Boyd, S.G. Lukishova, Y.R. Shen (Eds.), (this volume, Chapter 1).
19. E.W. Van Stryland, D.J. Hagan: Measuring nonlinear refraction and its dispersion. *Self-Focusing: Past and Present*, R.W. Boyd, S.G. Lukishova, Y.R. Shen (Eds.), (this volume, Chapter 24).
20. R.Y. Chiao et al.: Self-focusing of optical beams. *Self-Focusing: Past and Present*, R. W. Boyd, S.G. Lukishova, Y.R. Shen (Eds.), (this volume, Chapter 4). For the self-focusing distance see P. L. Kelley: Self-focusing of optical beams, *Phys. Rev. Lett.*, **15**, 1005–1008 (1965).
21. V.N. Lugovoi, A.A. Manenkov: Multi-focus structure and moving nonlinear foci – adequate model of self-focusing of laser beams in nonlinear media. *Self-Focusing: Past and Present*, R.W. Boyd, S.G. Lukishova, Y.R. Shen (Eds.), (this volume, Chapter 5).
22. V.I. Bespalov, V.I. Talanov: Filamentary structure of light beams in nonlinear liquids, *JETP Lett.*, **3**, 307–310 (1966).
23. A.J. Campillo: Small-scale self-focusing. *Self-Focusing: Past and Present*, R.W. Boyd, S.G. Lukishova, Y.R. Shen (Eds.), (this volume, Chapter 6).
24. S.G. Lukishova et al.: Beam shaping and suppression of self-focusing in high-peak-power Nd:glass laser systems. *Self-Focusing: Past and Present*, R.W. Boyd, S.G. Lukishova, Y.R. Shen (Eds.), (this volume, Chapter 8).
25. R.Y. Chiao, E. Garmire, C.H. Townes: Self-trapping of optical beams, *Phys. Rev. Lett.* **13**, 479–482 (1964). Errata, *Phys. Rev. Lett.*, **14** (25), 1056 (1965).
26. V.I. Talanov: Self-focusing of electromagnetic waves in nonlinear media. *Izv. Vuzov, Radiophysica*, **7**, 564–565 (1964). See also Chapter 11.2. *Self-Focusing: Past and Present*, R.W. Boyd, S.G. Lukishova, Y.R. Shen (Eds.), (this volume).
27. H.A. Haus: Higher-order trapped light beam solutions, *Appl. Phys. Lett.*, **8**, (5), 128–129 (1966).
28. Z.K. Yankauskas: Radial field distributions in a self-focusing light beam, *Radio-physics and Quantum Electronics* (Springer, NY), **9**, 261–263 (1966).
29. G.M. Fraiman et al.: Optical self-focusing: Stationary beams and femtosecond pulses. *Self-Focusing: Past and Present*, R.W. Boyd, S.G. Lukishova, Y.R. Shen (Eds.), (this volume, Chapter 3).
30. J.E. Bjorkholm, A. Ashkin: CW self-focusing and self-trapping of light in sodium vapor, *Phys. Rev. Lett.*, **32**, 129–132 (1974).
31. A. Barthelemy, S. Maneuf, C. Froehly: Propagation soliton et auto-confinement de faisceaux laser par non linearité optique de kerr, *Opt. Commun.*, **55**, 201–206 (1985).

32. J.S. Aitchison, Y. Silberberg, A.M. Weiner, D.E. Leaird, M.K. Oliver, J.L. Jackel, E. M. Vogel, P.W.E. Smith: Spatial optical solitons in planar glass waveguides, *J. Opt. Soc. Am. B* **8**, 1290–1297 (1991).
33. S. Trillo, W. Torruellas (Eds): *Spatial Solitons*, 474 pp, Springer-Verlag, New York (2001).
34. Yu.S. Kivshar, G.P. Agrawal: *Optical Solitons: From Fibers to Photonic Crystals*, 544 pp, Elsevier Science, New York (2003).
35. G.I.A. Stegeman, D.N. Christodoulides, M. Segev: Optical spatial solitons: historical perspectives, *IEEE J. Selected Topics in Quant. Electron.*, **6**, 1419–1427 (2000).
36. P. Zerom, R.W. Boyd: Self-focusing, conical emission and other self-action effects in atomic vapors. *Self-Focusing: Past and Present*, R.W. Boyd, S.G. Lukishova, Y.R. Shen (Eds.), (this volume, Chapter 9).
37. E. DelRe, M. Segev: Self-focusing and solitons in photorefractive media. *Self-Focusing: Past and Present*, R.W. Boyd, S.G. Lukishova, Y.R. Shen (Eds.), (this volume, Chapter 23).
38. X. Ni, R.R. Alfano: Periodic filamentation and supercontinuum interference. *Self-Focusing: Past and Present*, R.W. Boyd, S.G. Lukishova, Y.R. Shen (Eds.), (this volume, Chapter 10).
39. E. Garmire, R.Y. Chiao, C.H. Townes: Dynamics and characteristics of the self-trapping of intense light beam, *Phys. Rev. Lett.*, **16**, (9), 347–349 (1966).
40. D. Grishkowsky: Self-trapping of light by potassium vapor, *Phys. Rev. Lett.*, **24**, 866 (1970).
41. S.L. Chin et al.: The Physics of intense femtosecond laser filamentation. *Self-Focusing: Past and Present*, R.W. Boyd, S.G. Lukishova, Y.R. Shen (Eds.), (this volume, Chapter 14).
42. V.P. Kandidov et al.: Self-focusing and filamentation of powerful femtosecond laser pulses. *Self-Focusing: Past and Present*, R.W. Boyd, S.G. Lukishova, Y.R. Shen (Eds.), (this volume, Chapter 15).
43. E. Gaĩzauskas et al.: On the role of conical waves in self-focusing and filamentation of femtosecond pulses with nonlinear losses. *Self-Focusing: Past and Present*, R.W. Boyd, S.G. Lukishova, Y.R. Shen (Eds.), (this volume, Chapter 19).
44. R.R. Alfano (Ed.): *The Supercontinuum Laser Source: Fundamentals with Updated References*, 2nd ed., 568 pp, Springer, New York (2006).
45. E.A. Kuznetsov: Wave collapse in nonlinear optics. *Self-Focusing: Past and Present*, R.W. Boyd, S.G. Lukishova, Y.R. Shen (Eds.), (this volume, Chapter 7).
46. A. Gaeta: Spatial and temporal dynamics of collapsing ultrashort laser pulse. *Self-Focusing: Past and Present*, R.W. Boyd, S.G. Lukishova, Y.R. Shen (Eds.), (this volume, Chapter 16).
47. J. Zhang et al.: Self-organized propagation of femtosecond laser filamentation in air. *Self-Focusing: Past and Present*, R.W. Boyd, S.G. Lukishova, Y.R. Shen (Eds.), (this volume, Chapter 13).
48. G.G. Matvienko et al.: Effective parameters of high-power laser femtosecond radiation at self-focusing in gas and aerosol media. *Self-Focusing: Past and Present*, R.W. Boyd, S.G. Lukishova, Y.R. Shen (Eds.), (this volume, Chapter 21).
49. G.A. Askar'yan: Effects of the gradient of a strong electromagnetic beam on electrons and atoms, *Sov. Phys. JETP* **15**, 1088–1090 (1962). See also Chapter 11. 1. *Self-Focusing: Past and Present*, R.W. Boyd, S.G. Lukishova, Y.R. Shen (Eds.), (this volume).

50. M. Hercher: Laser-induced damage in transparent media, *J. Opt. Soc. Am.* **54**, 563 (1964). See also Chapter 11.3. *Self-Focusing: Past and Present*, R.W. Boyd, S.G. Lukishova, Y.R. Shen (Eds.), (this volume).
51. C. Sulem, P.L. Sulem: *The Nonlinear Schrödinger Equation*, Springer-Verlag New York (1999).
52. A. Brown, G. Korn, X. Liu, D. Du, J. Squier, G. Moorou: Self-channeling of high-peak-power femtosecond laser pulses in air, *Opt. Lett.*, **20**, 73–75 (1995).
53. A.V. Borovsky, A.L. Galkin, O.B. Shiryayev, T. Auguste: *Laser Physics at Relativistic Intensities*, 218 pp, Springer-Verlag, New York (2003).
54. M. Mlenek, E.M. Wright, J.V. Moloney: Dynamic spatial replenishment of femtosecond pulses propagating in air, *Opt. Lett.*, **23**, 382–384 (1998).
55. F. Wise, J. Moses: Self-focusing and self-defocusing of femtosecond pulses with cascaded quadratic nonlinearities. *Self-Focusing: Past and Present*, R.W. Boyd, S.G. Lukishova, Y.R. Shen (Eds.), (this volume, Chapter 20).
56. C. Conti et al.: X-waves in self-focusing of ultra-short pulses. *Self-Focusing: Past and Present*, R.W. Boyd, S.G. Lukishova, Y.R. Shen (Eds.), (this volume, Chapter 18).
57. N.F. Pilipetskii, A.R. Rustamov: Observation of self-focusing of light in liquids, *JETP Lett.* **2**, 55–56 (1965).
58. P. Lallemand, N. Bloembergen: Self-focusing of laser beams and stimulated Raman gain in liquids, *Phys. Rev. Lett.*, **15**, 1010–1012 (1965).
59. G. Hauchecorne, G. Mayer: Effects de l'anisotropie moléculaire sur la propagation d'une lumière intense, *Comptes Rendus, Acad. Sci. Paris*, **261**, 4014–4017, 15 November (1965).