Differential equations

Differential equations

Their solution using symmetries

HANS STEPHANI
Sektion Physik, Friedrich-Schiller-Universität, Jena, G. D. R.

Edited by
MALCOLM MACCALLUM
School of Mathematical Sciences, Queen Mary College, London

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Preface

Several years ago, when reading an old paper on a special class of solutions to Einstein's field equations, I became aware by pure chance of the existence of Lie's method of integrating ordinary differential equations when their symmetries are known. Since I had never heard of or read about this method, I consulted friends and textbooks to make sure that this was not only my personal fault and ignorance. Lie invented the theory of Lie groups when studying symmetries of differential equations. Lie groups became well known, but their original field of fruitful application remained hidden in the literature and either did not get into textbooks, or, if it did, only appeared in very mutilated versions. Hence many people who could have profited from this method simply were not aware of its existence. Even of the monographs then available, most contained only part of what I found when reading Lie's original papers (which fortunately enough were written in German). With the eagerness of a freshly baptized believer, I therefore decided to give a series of lectures on this subject, lectures that eventually developed into this book.

This book is intended to be an introduction. Almost nothing written in it is my own. But it is rather difficult to give proper credit to the origins of the various ideas and results – not only because of the gaps in my memory, but also because most of the material presented here is due to Lie, with only minor improvements and generalizations by later workers who rediscovered and used Lie's results.*

* Many remarks on the historical development of the subject can be found in Olver's textbook.

This book could not have been written without the help and advice given to me in many discussions, in particular by my colleagues Dietrich Kramer, Malcolm MacCallum, and Gernot Neugebauer, and could not have been printed without the innumerable amendments in style and grammar by which Malcolm MacCallum tried to make my English readable. I have to thank them all, and also Frau U. Kaschlik for the careful typing of the manuscript.

Jena/Crawinkel Hans Stephani