Preface

This book[1] introduces the main ideas and fundamental methods of differential and integral calculus for functions of several variables.

In Chapter 1 we discuss differential calculus for functions of several variables with a short excursion into differential calculus in Banach spaces.

In Chapter 2 we present some of the most relevant results of the Lebesgue integration theory, including the limit and approximation theorems, Fubini's theorem, the area and coarea theorems, and Gauss–Green formulas. The aim is to provide the reader with all that is needed to use the power of Lebesgue integration. For this reason some details as well as some proofs concerning the formulation of the theory are skipped, as we think they are more appropriate in the general context of measure theory.

In Chapter 3 we deal with potentials and integration of differential 1-forms, focusing on solenoidal and irrotational fields.

Chapter 4 provides a sufficiently wide introduction to the theory of holomorphic functions of one complex variable. We present the fundamental theorems and discuss singularities and residues as well as Riemann's theorem on conformal representation and the related Schwarz and Poisson formulas and Hilbert's transform.

In Chapter 5, we discuss the notions of immersed and embedded surface in $\mathbb{R}^n$, and we present the implicit function theorem and some of its applications to vector fields, constrained minimization, and functional dependence. The chapter ends with the study of some notions of the local theory of curves and surfaces, such as of curvature, first variation of area, the Laplace–Beltrami operator, and distance function.

In Chapter 6, after a few preliminaries about systems of linear ordinary differential equations, we discuss a few results concerning the stability of nonlinear systems and the Poincaré–Bendixson theorem in order to show that dynamical systems with one degree of freedom do not present chaos, in contrast with the one-dimensional discrete dynamics or the higher-dimensional continuous dynamics.

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[1] This book is a translated and revised edition of M. Giaquinta, G. Modica, *Analisi Matematica, IV. Funzioni di più variabili*, Pitagora Ed., Bologna, 2005.

The study of this volume requires a stronger effort compared to that of [GM1],[GM2],and [GM3][2] both because of intrinsic difficulties and broad scope of the themes we present. We think, in fact, that it is useful for the reader to have a wide spectrum of contexts in which these ideas play an important role and wherein even the technical and formal aspects play a role. However, we have tried to keep the same spirit, always providing examples, illustrations, and exercises to clarify the main presentation, omitting several technicalities or developments that we thought to be too advanced.

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Note: We have tried to avoid misprints and errors. But, as most authors, we are imperfect. We will be very grateful to anybody who wants to inform us about errors or just misprints, or wants to express criticism or other comments. Our e-mail addresses are

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We shall try to maintain any errata and corrigenda at the following web pages:

$$\texttt{http://www.sns.it/\textasciitilde giaquinta}$$

$$\texttt{http://www.dma.unifi.it/\textasciitilde modica}$$

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[2]   We shall refer to the following sources as [GM1], [GM2], and [GM3], respectively: [GM1]: M. Giaquinta, G. Modica, *Mathematical Analysis, Functions of One Variable*, Birkhäuser, Boston, 2003; [GM2]: M. Giaquinta, G. Modica, *Mathematical Analysis, Approximation and Discrete Processes*, Birkhäuser, Boston, 2004; [GM3]: M. Giaquinta, G. Modica, *Mathematical Analysis, Linear and Metric Structures and Continuity*, Birkhäuser, Boston, 2007.

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