Conjugate Duality in Convex Optimization

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Preface

The results presented in this book originate from the last decade research work of the author in the field of duality theory in convex optimization. The reputation of duality in the optimization theory comes mainly from the major role that it plays in formulating necessary and sufficient optimality conditions and, consequently, in generating different algorithmic approaches for solving mathematical programming problems. The investigations made in this work prove the importance of the duality theory beyond these aspects and emphasize its strong connections with different topics in convex analysis, nonlinear analysis, functional analysis and in the theory of monotone operators.

The first part of the book brings to the attention of the reader the perturbation approach as a fundamental tool for developing the so-called conjugate duality theory. The classical Lagrange and Fenchel duality approaches are particular instances of this general concept. More than that, the generalized interior point regularity conditions stated in the past for the two mentioned situations turn out to be particularizations of the ones given in this general setting. In our investigations, the perturbation approach represents the starting point for deriving new duality concepts for several classes of convex optimization problems. Moreover, via this approach, generalized Moreau–Rockafellar formulae are provided and, in connection with them, a new class of regularity conditions, called closedness-type conditions, for both stable strong duality and strong duality is introduced. By stable strong duality we understand the situation in which strong duality still holds whenever perturbing the objective function of the primal problem with a linear continuous functional. The closedness-type conditions constitute a class of regularity conditions recently introduced in the literature. They experience at present an increasing interest in the optimization community, as they are widely applicable than the generalized interior point ones, a fact that we also point out in this work.

We employ the conjugate duality in establishing biconjugate formulae for different classes of convex functions and, in the special case of Fenchel duality, we offer some deep insights into the existing relations between the notions strong and stable strong duality. Moreover, we enlarge the class of generalized interior point regularity conditions given for both Fenchel and Lagrange duality approaches by formulating corresponding sufficient conditions expressed via the quasi-interior and quasi-relative interior.

The convex analysis and, especially, the duality theory have surprisingly found in the last years applications in rediscovering classical results and also in giving new powerful ones in the field of monotone operators. Among others, we provide a regularity condition of closedness-type for the maximality of the sum of two maximal monotone operators in reflexive Banach spaces, which proves to be weaker than all the other generalized interior point conditions introduced in the literature with the same purpose.

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