

Preface

Mathematical control theory of applied partial differential equations is built on linear and nonlinear functional analysis and many existence theorems in control theory result from applications of theorems in functional analysis. This makes control theory inaccessible to students who do not have a background in functional analysis.

Many advanced control theory books on infinite-dimensional systems were written, using functional analysis and semigroup theory, and control theory was presented in an abstract setting. This motivates me to write this text for control theory classes in the way to present control theory by concrete examples and try to minimize the use of functional analysis. Functional analysis is not assumed and any analysis included here is elementary, using calculus such as integration by parts. The material presented in this text is just a simplification of the material from the existing advanced control books. Thus this text is accessible to senior undergraduate students and first-year graduate students in applied mathematics, who have taken linear algebra and ordinary and partial differential equations.

Elementary functional analysis is presented in Chapter 2. This material is required to present the control theory of partial differential equations. Since many control concepts and theories for partial differential equations are transplanted from finite-dimensional control systems, a brief introduction to feedback control of these systems is presented in Chapter 3. The topics covered in this chapter include controllability, observability, stabilizability, pole placement, and quadratic optimal control. Theories about the feedback stabilization of linear reaction-convection-diffusion equations are presented in Chapter 4. Both interior and boundary control problems are addressed. The methods employed to handle the problems include eigenfunction expansions, integral transforms, and optimal control. Finally, theories about feedback stabilization of linear wave equations are presented in Chapters 5 and 6. First, the one-dimensional wave equations are considered, then higher dimensional wave equations follow, since it is easier to use the one-dimensional equations to illustrate the theories and methods. The perturbed energy method is emphasized to deal with both interior and boundary control problems while the optimal control technique is used.

This text can be used as a textbook for an introductory one-semester graduate course on control theory of partial differential equations. If students do not have a background in finite-dimensional control systems, one can cover Sections 3.1-3.7 of Chapter 3, Sections 4.1-4.3 of Chapter 4, all sections of Chapter 5, and Sections 6.1- 6.3 of Chapter 6. Material from Chapter 2 can be introduced whenever needed, and all material about optimal control can be skipped. If students have had a background in both functional analysis and finite-dimensional control systems, one can cover all of Chapters 4, 5, 6 and give a brief review of Chapters 2 and 3.

Inevitably, the material is selected from the topics I have worked on or am most familiar with, and many other important topics are not covered. Control theory for other applied partial differential equations, such as elastic equations, thermoelastic equations, viscoelastic equations, Schrödinger's equations, and Navier-Stokes equations, is not included. In addition, exact and approximate controllability is not discussed since it is difficult to present it in an elementary way.

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