Modern Birkhäuser Classics

Theory of Function Spaces II

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Preface

This book deals with the theory of function spaces of type B^s_{pq} and F^s_{pq} as it stands at the end of the eighties. These two scales of spaces cover many well-known spaces of functions and distributions such as Hölder-Zygmund spaces, Sobolev spaces, fractional Sobolev spaces (previously also often referred to as Bessel-potential spaces), Besov spaces, inhomogeneous Hardy spaces, spaces of BMO-type and local approximation spaces which are closely connected with Morrey–Campanato spaces.

The book is essentially self-contained, although it seems to be reasonable to call it part II of the author's book [Triß] with the same title, which dealt mainly with the same two scales of spaces B^s_{pq} and F^s_{pq} and which reflected the situation around 1980. But since that time many new discoveries changed the picture completely (this applies both to simpler proofs of known basic assertions and to new results and new applications). This may justify starting again from scratch and not simply continuing where we stopped in [Triß]. However those topics where we have nothing new to say will not again be treated in detail; we shall refer to [Triß] or other relevant sources. But in order to make the book self-contained we give in any case a detailed description of all needed results.

This book is the author's third monograph about spaces of this type (beside the two research reports [Tri4,5] and the joint monograph [ScT] with H.-J. Schmeisser which is related to the subject but not in a direct line). The two other directly related books are the above-mentioned [Triß] and [Triα]. The latter deals with some of these spaces from the standpoint of interpolation theory, whereas [Triß] is mainly based on Fourier-analytical techniques combined with maximal inequalities and applications to PDE's (partial differential equations). The present book is characterized by local means and local methods with applications to ψDE's (pseudodifferential equations). Although the book is mainly based on the author's results obtained in the last few years we try to give a sufficiently comprehensive picture of these spaces and of some of their applications, at least of those topics which are treated here in detail.

The book has seven chapters. Chapter 1 is a self-contained historically-orientated survey of those function spaces and their roots which are treated in the book, including the related devices for measuring smoothness (derivatives, differences of functions, boundary values of harmonic and thermic functions, local approximations, sharp maximal functions, interpolation methods, Fourier-analytical representations, atomic decompositions, etc.). This chapter surveys the rather different techniques developed in the last 50 years (some of them are quite recent) without proofs but with many references. The aim of this introductory chapter is twofold. First, it serves as an independent survey readable (so we hope) also for non-specialists who are not so much interested in technical details but who wish to learn recent trends in the theory of function spaces including some of their historical roots. Secondly, it prepares from a historical point of view what follows and it emphasizes the main goal of the book, that is, to show how at the end of the eighties all these apparently different devices come together, and to demonstrate that they are only different ways of characterizing the same function spaces.

In Chapters 2 and 3 we start again (now in earnest): We define the spaces B^s_{pq} and F^s_{pq} and prove equivalence assertions which cover the above-mentioned more or less classical spaces, including their related, seemingly different, devices for measuring smoothness. We develop the technical instruments which are the basis for the rest of the book. This includes atomic representations and local approximations. Morrey–Campanato spaces (including BMO-spaces) will be treated as a by-product of the developed techniques.

Based on the two preceding chapters we give in Chapter 4 new (simpler) proofs of some (more or less known) crucial theorems for the spaces B^s_{pq} and F^s_{pq}: invariance under diffeomorphic maps of $\mathbb{R}^n$ onto itself, pointwise multipliers, traces on hyperplanes, extensions of these spaces from $\mathbb{R}^n_+$ to $\mathbb{R}^n$.

The spaces $B^s_{pq}(\Omega)$, $F^s_{pq}(\Omega)$ and corresponding Morrey–Campanato spaces on bounded C^∞ domains Ω in $\mathbb{R}^n$ are treated in Chapter 5. We are mainly interested in intrinsic characterizations.

Mapping properties of pseudodifferential and Fourier integral operators of type $S^\mu_{\rho,\delta}$ will be studied in Chapter 6. Of special interest is the exotic class characterized by $\rho = \delta = 1$.

Chapter 7 deals with spaces of type F^s_{pq} and B^s_{pq} (including all the mentioned special cases) on Riemannian manifolds (bounded geometry, positive injectivity radius) and on Lie groups.

The reader is expected to have a working knowledge of functional analysis as presented in the classical textbooks (including the standard facts of the theory of distributions). A familiarity with the basic results of the spaces of differential functions, L_p-spaces and Sobolev spaces would be helpful.

The book is organized by the decimal system. "n.k.m" refers to subsection n.k.m, "Theorem n.k.m/l" means the theorem l in n.k.m etc. All unimportant positive numbers will be denoted by c (with additional indices if there are several c's in the same formula).

Jena, Summer 1990 Hans Triebel