Preface

Two traditional mathematical concepts, classical in their own fields, are brought forward in this brief volume. Reviewing these concepts separately, with no connection to each other, would definitely look natural, but bringing them together into a single book format is quite a different story. The point is that the concepts are drawn from subject areas of mathematics that have no evident points of contiguity. That is why the reader might be intrigued with our intention in this book to explore their mutual fusion. This endeavor provides a basis for a challenging and nontrivial investigation.

The first of the two concepts is the *Green's function*. It represents an important topic in standard courses of differential equations and is customarily covered in most texts in the field. The second concept, of *infinite product*, belongs, in turn, to classical mathematical analysis. As to Green's functions for partial differential equations, it is not a common practice in existing textbooks for careful consideration to be given to procedures used for their construction. On the other hand, the standard texts on mathematical analysis do not usually confront the infinite product representation of elementary functions. A simultaneous review of just these two subject areas (the construction of Green's functions and the infinite product representation of elementary functions) constitutes the context of the present book.

Green's functions for the two-dimensional Laplace equation are most widely represented in relevant texts. They are conventionally constructed using the method of images, conformal mapping, or eigenfunction expansion. The present volume focuses on the construction of such Green's functions for a wide range of boundary-value problems. A comprehensive review of the traditional methods is provided, with emphasis on the infinite-product-containing expressions of Green's functions, which are obtained by the method of images. This provides a background for the central theme in this book, which is the development of an innovative approach to the representation of elementary functions in terms of infinite products.

The intention in the present volume is not just to familiarize the reader in the traditional manner with the state of things in the area, but rather to reach beyond traditions. That is, we plan not only to introduce the classical topics of the construction of Green's functions and the infinite product representation of elementary functions, but also to present a challenging investigation into the intersection of these fields.

To be well prepared for the presentation in this book, the reader is required to have a reasonably solid background in the standard undergraduate courses of calculus and differential equations. In addition, the reader would definitely benefit from a superficial knowledge of the basics of numerical analysis.

There is good reason to believe that this piece of work is original. To the author's best knowledge, there are no analogous books available on the market. That is why we anticipate that the book will not be overlooked by the professional community. It might, for example, be adopted as supplementary reading for an undergraduate course or as a seminar topic within the scope of a pure or applied mathematics curriculum. *Infinite Product Representation of Elementary Functions*, *A Further Linking of Differential Equations with Calculus*, or *Broadening the Use of Green's Functions* might be the title for such a course or seminar topic.

Very initial results on the Green's-function-based approach to the infinite product representation of elementary functions were reported not long ago. The first printed publications on progress in this field appeared just recently. It then took us over three years to ultimately come up with this book, which was originally intended as a text for an elective course within the computational sciences Ph.D. program just launched at Middle Tennessee State University.

It is with pleasure and gratitude that the author acknowledges the editorial services provided by the staff of Birkhäuser Boston, with special thanks to Tom Grasso, senior mathematics editor, for his professional treatment of nontrivial situations. Although the editing process was not fast, smooth, and painless, it has significantly improved the quality of the presentation and definitely made this book a much better read.

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