Preface

One of the findings of the 1988 Report by the Panel on Future Directions in Control Theory, chaired by Wendell H. Fleming, was:

> Many fundamental theoretical issues, such as control of nonlinear multivariable systems, or *control of nonlinear partial differential equations*, are not yet understood.

Nowadays, more than 20 years later, we believe we can say that a lot of fundamental issues concerning the latter topic have definitely been understood, thanks to the efforts of many researchers who produced a large body of results and techniques. And yet, this process has led to an enormous amount of open questions that will need to be addressed by new generations of scientists. Surveying the most important advances of the last two decades and outlining future research directions were the main motivations that led us to organize the CIME Course on Control of Partial Differential Equations that took place in Cetraro (CS, Italy), July 19–23, 2010. We hope this volume, which is one of the outcomes of that event, will provide an ultimate formative step for those who attended the course, and will represent an authoritative reference for those who were unable to do so.

The course consisted of five series of lectures, which are now the source of the chapters of this monograph. Specifically, the following topics were covered:

- *Stabilization of evolution equations* (by Fatiha Alabau-Boussouira): these lectures discussed recent advances, as well as classical methods, for the stabilization of wave-like equations. Special attention was paid to nonlinear problems, memory-damping, and indirect stabilization of coupled PDEs. All the problems were treated by a unified methodology based on energy estimates. It was shown how the introduction of optimal-weight convexity methods leads to easy computable upper energy decay estimates, and how these results can be completed by lower energy estimates for several examples.
- *Control of the Liouville equation* (by Roger Brockett): these equations describe the evolution of an initial density of points that move according to a given differential equation, and may depend on a control which can be chosen in order to satisfy some prescribed goals. This framework also allows to overcome

limitations of the classical theory: for example, the expense required to implement control laws. Several results (e.g., on ensemble control: controlling, with a single control, a finite but often large number of copies of a given system) as well as open problems were presented.
- *Control in fluid mechanics* (by Olivier Glass): the lectures treated various issues related to the controllability of two well-known equations in fluid mechanics, namely the Euler equation for perfect incompressible fluids in both Eulerian and Lagrangian coordinates, and the one-dimensional isentropic Euler equation for compressible fluids in the framework of entropy solutions. Special emphasis was put on the aspects of the theory that are connected with the nonlinear nature of the problem: linearization around an equilibrium gives here no information on the controllability of the nonlinear system.
- *Carleman estimates for elliptic and parabolic equations, with application to control* (by Jérôme Le Rousseau): these are weighted energy estimates for solutions of partial differential equations with weights of exponential type. The lectures derived Carleman estimates for elliptic and parabolic operators using several methods: a microlocal approach where the main tool is the Gårding inequality, and a more computational direct approach. It was also shown how Carleman estimates can be used to provide unique continuation properties, as well as approximate and null controllability results.
- *Control and numerics for the wave equation* (by Enrique Zuazua): these lectures provided a self-contained presentation of the theory that has been developed recently for the numerical analysis of the controllability properties of wave propagation phenomena. The methodology adopted the so-called discrete approach, which consists in analyzing whether the semidiscrete or fully discrete dynamics arising when discretizing the wave equation by means of the most classical scheme of numerical analysis share the property of being controllable, uniformly with respect to the mesh-size parameters, and the corresponding controls converge to the continuous ones as the mesh size tends to zero. All the results were illustrated by means of several numerical experiments.

Besides the above lectures, there were three seminars, given by Karine Beauchard (Some controllability results for the 2D Kolmogorov equation), Sylvain Ervedoza (Regularity of HUM controls for conservative systems and convergence rates for discrete controls), and Lionel Rosier (Control of some dispersive equations for water waves). There were also four communications given by Ido Bright (Periodic optimization suffices for infinite horizon planar optimal control), Khai Tien Nguyen (The regularity of the minimum time function via nonsmooth analysis and geometric measure theory), Camille Laurent (On stabilization and control for the critical Klein-Gordon equation on a 3-D compact manifold), and Vincent Perrollaz (Exact controllability of entropic solutions of scalar conservation laws with three controls). Seminars and communications are not reproduced in these notes.

One important point, contained in the 1988 Report we mentioned above, is that *advances in the control field are made through a combination of mathematics, modeling, computation, and experimentation*. Hoping the reader will find the present exposition in accord with such a basic principle, we wish to thank the lecturers and authors who designed their contributions in a detailed-yet-focussed form, for helping us realize this project. Overall, we are very grateful to all the 57 participants in the CIME course, for their enthusiasm that created a friendly and stimulating atmosphere in Cetraro. Finally, special gratitude is due to the GDRE CONEDP, for providing the essential support that allowed us to receive and accept a large number of applications, and to the C.I.M.E. Foundation, for making this event possible and for its very helpful assistance before and all along the lectures.

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