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Turbulent Impinging Jets into Porous Materials

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Chapter 2
Mathematical Modeling of Turbulence in Porous Media

As mentioned, the flow model here employed is described in Graminho and de Lemos [1] whereas thermal modeling is detailed in [2], including now the energy equation for heat transfer calculations. Therein, details can be found. As most of the theoretical development is readily available in the open literature, the governing equations will be just presented and details about their derivations can be obtained in the mentioned papers. Essentially, local instantaneous equations are volume-averaged using appropriate mathematical tools [3].

At the jet exit, a fully developed profile for velocity, k and ε was imposed. At the flow outlet through the clearance of width w, a zero diffusion flux condition was set. On the walls, a non-slip condition was applied and at the centerline of the cylinder, the symmetry condition was used.

2.1 Local Instantaneous Transport Equations

The governing equations for the flow and energy for an incompressible fluid are given by:

$$\text{Continuity: } \nabla \cdot \mathbf{u} = 0. \tag{2.1}$$

$$\text{Momentum: } \rho\left[\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u}\,\mathbf{u})\right] = -\nabla p + \mu \nabla^2 \mathbf{u}. \tag{2.2}$$

$$\text{Energy-Fluid Phase: } \left(\rho c_p\right)_f \left\{\frac{\partial T_f}{\partial t} + \nabla \cdot \left(\mathbf{u} T_f\right)\right\} = \nabla \cdot \left(k_f \nabla T_f\right) + S_f. \tag{2.3}$$

$$\text{Energy-Solid Phase (Porous Matrix)}: \ \left(\rho c_p\right)_s \frac{\partial T_s}{\partial t} = \nabla \cdot \left(k_s \nabla T_s\right) + S_s. \tag{2.4}$$

where the subscripts f and s refer to fluid and solid phases, respectively. Here, T is the temperature, k_f and k_s are the fluid and solid thermal conductivities,

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respectively, c_p is the specific heat and S is the heat generation term. If there is no heat generation either in the solid or in the fluid, one has $S_f = S_s = 0$.

2.2 Double-Decomposition of Variables

Macroscopic transport equations for turbulent flow in a porous medium are obtained through the simultaneous application of time and volume average operators over a generic fluid property φ. Such concepts are defined as.

$$\bar{\varphi} = \frac{1}{\Delta t}\int_{t}^{t+\Delta t} \varphi \, dt, \text{ with } \varphi = \bar{\varphi} + \varphi' \tag{2.5}$$

$$\langle \varphi \rangle^{i} = \frac{1}{\Delta V_f}\int\limits_{\Delta V_f} \varphi \, dV; \langle \varphi \rangle^{v} = \phi \langle \varphi \rangle^{i}; \phi = \frac{\Delta V_f}{\Delta V}, \text{ with } \varphi = \langle \varphi \rangle^{i} + {}^{i}\varphi \tag{2.6}$$

where ΔV_f is the volume of the fluid contained in a Representative Elementary Volume (REV) ΔV, intrinsic average and volume average are represented, respectively, by $\langle\ \rangle^{i}$ and $\langle\ \rangle^{v}$. The double decomposition idea fully described in [4], combines Eqs. (2.5) and (2.6) and can be summarized as:

$$\overline{\langle \varphi \rangle^{i}} = \langle \bar{\varphi} \rangle^{i}; {}^{i}\bar{\varphi} = \overline{{}^{i}\varphi}; \langle \varphi' \rangle^{i} = \langle \varphi \rangle^{i'} \tag{2.7}$$

and,

$$\left.\begin{aligned} \varphi' &= \langle \varphi' \rangle^{i} + {}^{i}\varphi' \\ {}^{i}\varphi &= \overline{{}^{i}\varphi} + {}^{i}\varphi' \end{aligned}\right\} \quad \text{where} \quad {}^{i}\varphi' = \varphi' - \langle \varphi' \rangle^{i} = {}^{i}\varphi - \overline{{}^{i}\varphi} \tag{2.8}$$

Therefore, the quantity φ can be expressed by either,

$$\varphi = \overline{\langle \varphi \rangle^{i}} + \langle \varphi \rangle^{i'} + \overline{{}^{i}\varphi} + {}^{i}\varphi' \tag{2.9}$$

or

$$\varphi = \langle \bar{\varphi} \rangle^{i} + {}^{i}\bar{\varphi} + \langle \varphi' \rangle^{i} + {}^{i}\varphi'. \tag{2.10}$$

The term ${}^{i}\varphi'$ can be viewed as either the temporal fluctuation of the spatial deviation or the spatial deviation of the temporal fluctuation of the quantity φ.

2.3 Macroscopic Flow Equations

When the average operators (2.5) and (2.6) are simultaneously applied over Eqs. (2.1) and (2.2), macroscopic equations for turbulent flow are obtained. Volume integration is performed over a Representative Elementary Volume (REV) [3], resulting in,

$$\text{Continuity: } \nabla \cdot \bar{\mathbf{u}}_D = 0. \tag{2.11}$$

where, $\bar{\mathbf{u}}_D = \phi\langle\bar{\mathbf{u}}\rangle^i$ and $\langle\bar{\mathbf{u}}\rangle^i$ identifies the intrinsic (liquid) average of the time-averaged velocity vector $\bar{\mathbf{u}}$.

Momentum:

$$\rho\left[\frac{\partial\bar{\mathbf{u}}_D}{\partial t} + \nabla\cdot\left(\frac{\bar{\mathbf{u}}_D\bar{\mathbf{u}}_D}{\phi}\right)\right] = -\nabla(\phi\langle\bar{p}\rangle^i) + \mu\nabla^2\bar{\mathbf{u}}_D - \nabla\cdot(\rho\phi\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i) - \left[\frac{\mu\phi}{K}\bar{\mathbf{u}}_D + \frac{c_F\phi\rho\,|\bar{\mathbf{u}}_D|\bar{\mathbf{u}}_D}{\sqrt{K}}\right] \tag{2.12}$$

where the last two terms in Eq. (2.12) represent the Darcy and Forchheimer or form drags. The symbol K is the porous medium permeability, c_F is the form drag or Forchheimer coefficient, $\langle\bar{p}\rangle^i$ is the intrinsic average pressure of the fluid and ϕ is the porosity of the porous medium.

The macroscopic Reynolds stress, $-\rho\phi\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i$, appearing in Eq. (2.12) is given as,

$$-\rho\phi\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i = \mu_{t_\phi}2\langle\bar{\mathbf{D}}\rangle^{\mathbf{v}} - \frac{2}{3}\phi\rho\langle k\rangle^i\mathbf{I} \tag{2.13}$$

where,

$$\langle\bar{\mathbf{D}}\rangle^v = \frac{1}{2}\left[\nabla(\phi\langle\bar{\mathbf{u}}\rangle^i) + [\nabla(\phi\langle\bar{\mathbf{u}}\rangle^i)]^T\right] \tag{2.14}$$

is the macroscopic deformation tensor, $\langle k\rangle^i = \langle\overline{\mathbf{u}'\cdot\mathbf{u}'}\rangle^i/2$ is the intrinsic turbulent kinetic energy, and μ_{t_ϕ}, is the turbulent viscosity, which is modeled similarly to the case of clear flow, in the form,

$$\mu_{t_\phi} = \rho f_\mu c_\mu \frac{\langle k\rangle^{i^2}}{\langle\varepsilon\rangle^i} \tag{2.15}$$

The intrinsic turbulent kinetic energy per unit mass and its dissipation rate are governed by the following equations,

$$\rho\left[\frac{\partial}{\partial t}\left(\phi\langle k\rangle^i\right) + \nabla\cdot\left(\bar{\mathbf{u}}_D\langle k\rangle^i\right)\right] = \nabla\cdot\left[\left(\mu + \frac{\mu_{t_\phi}}{\sigma_k}\right)\nabla\left(\phi\langle k\rangle^i\right)\right] - \rho\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i :\ \nabla\bar{\mathbf{u}}_D + c_k\rho\frac{\phi\langle k\rangle^i\,|\bar{\mathbf{u}}_D|}{\sqrt{K}} - \rho\phi\langle\varepsilon\rangle^i \tag{2.16}$$

$$\rho\left[\frac{\partial}{\partial t}\left(\phi\langle\varepsilon\rangle^i\right) + \nabla\cdot\left(\bar{\mathbf{u}}_D\langle\varepsilon\rangle^i\right)\right] = \nabla\cdot\left[\left(\mu + \frac{\mu_{t_\phi}}{\sigma_\varepsilon}\right)\nabla\left(\phi\langle\varepsilon\rangle^i\right)\right] + c_1\left(-\rho\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i :\ \nabla\bar{\mathbf{u}}_D\right)\frac{\langle\varepsilon\rangle^i}{\langle k\rangle^i} + c_2c_k\rho\frac{\phi\langle\varepsilon\rangle^i|\bar{\mathbf{u}}_D|}{\sqrt{K}} - c_2f_\mu\rho\phi\frac{\langle\varepsilon\rangle^{i^2}}{\langle k\rangle^t} \tag{2.17}$$

where, $\sigma_k = 1.4$, $\sigma_\varepsilon = 1.3$, $c_1 = 1.50$, $c_2 = 1.90$, $c_\mu = 0.09$ and $c_k = 0.28$ are non-dimensional constants tuned for the Low-Reynolds number k-ε model whereas f_2 and f_μ are damping functions given by [5]:

$$f_2 = \left\{1 - exp\left[-\frac{(v\varepsilon)^{0.25}n}{3.1\,v}\right]\right\}^2 \times \left\{1 - 0.3exp\left[-\left(\frac{(k^2/v\varepsilon)}{6.5}\right)^2\right]\right\} \tag{2.18}$$

$$f_\mu = \left\{1 - exp\left[-\frac{(v\varepsilon)^{0.25}n}{14\,v}\right]\right\}^2 \times \left\{1 + \frac{5}{(k^2/v\varepsilon)^{0.75}}exp\left[-\left(\frac{(k^2/v\varepsilon)}{200}\right)^2\right]\right\} \tag{2.19}$$

where n is the coordinate normal to the wall.

2.4 One-Energy Equation Model

Similarly, macroscopic energy equations are obtained for both fluid and solid phases by applying time and volume average operators to Eqs. (2.3) and (2.4). As in the flow case, the integrations are performed over a Representative Elementary Volume (REV), resulting in,

$$\left(\rho c_p\right)_f \left[\frac{\partial \phi \langle \overline{T_f}\rangle^i}{\partial t} + \nabla \cdot \left\{\phi\left(\langle \bar{\mathbf{u}}\rangle^i \langle \overline{T_f}\rangle^i + \underbrace{\langle {}^i\bar{\mathbf{u}}^i \overline{T_f}\rangle^i}_{\text{thermaldisperson}} + \underbrace{\overline{\langle \mathbf{u}'\rangle^i \langle T_f'\rangle^i}}_{\text{turbulent heat flux}} + \underbrace{\langle {}^i\overline{\mathbf{u}'^i T_f'}\rangle^i}_{\text{turbulentthermal disperson}}\right)\right\}\right] \tag{2.20}$$

where A_i is the interfacial area between phases and the expansion,

$$\langle \overline{\mathbf{u}' T_f'}\rangle^i = \langle \overline{(\langle \mathbf{u}'\rangle^i + {}^i\mathbf{u}')\,(\langle T_f'\rangle^i + {}^iT')}\rangle^i = \overline{\langle \mathbf{u}'\rangle^i \langle T_f'\rangle^i} + \langle {}^i\overline{\mathbf{u}'\,{}^iT_f'}\rangle^i \tag{2.21}$$

has been used in light of the double decomposition concept given by Eqs. (2.7)–(2.10) [2]. For the solid phase, one has,

$$\left(\rho c_p\right)_s \left\{\frac{\partial (1-\phi)\langle \overline{T_s}\rangle^i}{\partial t}\right\} = \underbrace{\nabla \cdot \left\{k_s \nabla\left[(1-\phi)\langle \overline{T_s}\rangle^i\right] - \frac{1}{\Delta V}\int_{A_i} \mathbf{n}_i k_s \overline{T_s}\, dA\right\}}_{\text{conduction}} \underbrace{- \frac{1}{\Delta V}\int_{A_i} \mathbf{n}_i \cdot k_s \nabla \overline{T_s}\, dA}_{\text{interfacial heat transfer}} \tag{2.22}$$

An interfacial heat transfer coefficient is needed when corresponding terms in Eqs. (2.20) and (2.22) are modeled following the local thermal non-equilibrium assumption [6]. Here, however, we assume local thermal equilibrium between the fluid and solid phases, i.e., we add Eqs. (2.20) and (2.22) and consider $\langle\overline{T_f}\rangle^i = \langle\overline{T_s}\rangle^i = \langle\overline{T}\rangle^i$, giving further:

$$\begin{aligned}\left\{\left(\rho c_p\right)_f\phi + \left(\rho c_p\right)_s(1-\phi)\right\}\frac{\partial\langle\overline{T}\rangle^i}{\partial t} + \left(\rho c_p\right)_f\nabla\cdot\left(\mathbf{u}_D\langle\overline{T}\rangle^i\right)\\ = \nabla\cdot\left\{\left[k_f\phi + k_s(1-\phi)\right]\nabla\langle\overline{T}\rangle^i\right\} + \nabla\cdot\left[\frac{1}{\Delta V}\int\limits_{A_i}\mathbf{n}\left(k_f\overline{T_f} - k_s\overline{T_s}\right)dS\right]\\ - \left(\rho c_p\right)_f\nabla\cdot\left[\phi\left(\langle{}^i\bar{\mathbf{u}}^i\overline{T_f}\rangle^i + \langle\overline{\mathbf{u}'T_f'}\rangle^i\right)\right]\end{aligned} \tag{2.23}$$

The interface conditions at A_i are further given by,

$$\left.\begin{aligned}T_f &= T_s\\ \mathbf{n}\cdot\left(k_f\nabla T_f\right) &= \mathbf{n}\cdot\left(k_s\nabla T_s\right)\end{aligned}\right\}\ in\ A_i \tag{2.24}$$

Equation (2.23) express the one-equation model for heat transport in porous media. Further, using the double decomposition concept, Rocamora and de Lemos [2] have shown that the last term on the right hand side of Eq. (2.23) can be expressed as:

$$\langle\overline{\mathbf{u}'T_f'}\rangle^i = \langle\overline{\left(\langle\mathbf{u}'\rangle^i + {}^i\mathbf{u}'\right)\left(\langle T_f'\rangle^i + {}^iT'\right)}\rangle^i = \overline{\langle\mathbf{u}'\rangle^i\langle T_f'\rangle^i} + \langle\overline{{}^i\mathbf{u}'^iT_f'}\rangle^i \tag{2.25}$$

So, in view of Eqs. (2.21) and (2.23) can be rewritten as:

$$\begin{aligned}\left\{\left(\rho c_p\right)_f\phi + \left(\rho c_p\right)_s(1-\phi)\right\}\frac{\partial\langle\overline{T}\rangle^i}{\partial t} + \left(\rho c_p\right)_f\nabla\cdot\left(\mathbf{u}_D\langle\overline{T}\rangle^i\right)\\ = \nabla\cdot\left\{\left[k_f\,\phi + k_s(1-\phi)\right]\nabla\langle\overline{T}\rangle^i\right\} + \underbrace{\nabla\cdot\left[\frac{1}{\Delta V}\int\limits_{A_i}\mathbf{n}\left(k_f\overline{T_f} - k_s\overline{T_s}\right)dS\right]}_{I}\\ - \left(\rho c_p\right)_f\nabla\cdot\left[\phi\left(\underbrace{\overline{\langle\mathbf{u}'\rangle^i\langle T_f'\rangle^i}}_{II} + \underbrace{\langle{}^i\bar{\mathbf{u}}^i\overline{T_f}\rangle^i}_{III} + \underbrace{\langle\overline{{}^i\mathbf{u}'^iT_f'}\rangle^i}_{IV}\right)\right]\end{aligned} \tag{2.6}$$

where to the underscored terms in Eq. (2.26) the following physical significance can be attributed:

I. Tortuosity based on microscopic time averaged temperatures.
II. Turbulent heat flux due to the fluctuating components of macroscopic velocity and temperature ($\overline{\langle\mathbf{u}'\rangle^i\langle T_f'\rangle^i} = \overline{\langle\mathbf{u}\rangle^{i'}\langle T_f\rangle^{i'}}$).

III. Thermal dispersion associated with deviations of microscopic time average velocity and temperature. Note that this term is also present when analyzing laminar convective heat transfer in porous media.
IV. Turbulent thermal dispersion in a porous medium due to both time fluctuations and spatial deviations of both microscopic velocity and temperature.

In order to apply Eq. (2.26) to obtain the temperature field for turbulent flow in porous media, the underscored terms have to be modeled in some way as a function of the surface average temperature, $\langle \bar{T} \rangle^i$. To accomplish this, a gradient type diffusion model is used for all the terms, i.e., tortuosity (I), turbulent heat flux due to temporal fluctuations (II), thermal dispersion due to spatial deviations (III) and turbulent thermal dispersion due to temporal fluctuations and spatial deviations (IV).

Using these gradient type diffusion models, we can write:

$$\text{Tortuosity: } \left[\frac{1}{\Delta V}\int_{A_i} \mathbf{n}\left(k_f \overline{T_f} - k_s \overline{T_s}\right) dS\right] = \mathbf{K}_{tor} \cdot \nabla \langle \bar{T} \rangle^i \tag{2.27}$$

$$\text{Turbulent heat flux: } -\left(\rho c_p\right)_f \left(\phi \overline{\langle \mathbf{u}' \rangle^i \langle T_f' \rangle^i}\right) = \mathbf{K}_t \cdot \nabla \langle \bar{T} \rangle^i \tag{2.28}$$

$$\text{Thermal dispersion: } -\left(\rho c_p\right)_f \left(\phi \langle {}^i\bar{\mathbf{u}}^i \overline{T_f} \rangle^i\right) = \mathbf{K}_{disp} \cdot \nabla \langle \bar{T} \rangle^i \tag{2.29}$$

$$\text{Turbulent thermal dispersion: } -\left(\rho c_p\right)_f \left(\phi \overline{\langle {}^i\mathbf{u}'^i T_f' \rangle^i}\right) = \mathbf{K}_{disp,t} \cdot \nabla \langle \bar{T} \rangle^i \tag{2.30}$$

For the above shown expressions, Eq. (2.26) can be rewritten as:

$$\left\{\left(\rho c_p\right)_f \phi + \left(\rho c_p\right)_s (1-\phi)\right\} \frac{\partial \langle \overline{T} \rangle^i}{\partial t} + \left(\rho c_p\right)_f \nabla \cdot \left(\mathbf{u}_D \langle \overline{T} \rangle^i\right) = \nabla \cdot \left\{\mathbf{K}_{eff} \cdot \nabla \langle \overline{T} \rangle^i\right\} \tag{2.31}$$

where, $\mathbf{K}_{eff}$, given by:

$$\mathbf{K}_{eff} = \underbrace{\left[\phi k_f + (1-\phi) k_s\right]}_{k_{eff}} \mathbf{I} + \mathbf{K}_{tor} + \mathbf{K}_t + \mathbf{K}_{disp} + \mathbf{K}_{disp,t} \tag{2.32}$$

is the effective overall conductivity tensor.

In order to be able to apply Eq. (2.31), it is necessary to determine the conductivity tensors in Eq. (2.32), i.e., $\mathbf{K}_{tor}$, $\mathbf{K}_t$, $\mathbf{K}_{disp}$ and $\mathbf{K}_{disp,t}$. Following Kuwahara and Nakayama [7], this can be accomplished for the tortuosity and thermal dispersion conductivity tensors, $\mathbf{K}_{tor}$ and $\mathbf{K}_{disp}$, by making use of a unit cell subjected to periodic boundary conditions for the flow and a linear temperature gradient imposed over the domain. The conductivity tensors are then obtained directly from the microscopic results (see [8] for detail). Nevertheless, for simplicity, the tortuosity and dispersion mechanisms are here neglected.

The turbulent heat flux and turbulent thermal dispersion terms, $\mathbf{K}_t$ and $\mathbf{K}_{disp,t}$, which can not be determined from such a microscopic calculation, are modeled through the Eddy diffusivity concept, similarly to Kuwahara and Nakayama [7]. It should be noticed that these terms arise only if the flow is turbulent, whereas the tortuosity and the thermal dispersion terms exist for both laminar and turbulent flow regimes.

The macroscopic version of the 'turbulent heat flux' is given by:

$$-\left(\rho c_p\right)_f \langle \overline{\mathbf{u}' T_f'} \rangle^i = \left(\rho c_p\right)_f \frac{\nu_{t_\phi}}{\sigma_{t_\phi}} \nabla \langle \bar{T}_f \rangle^i \tag{2.33}$$

where ν_{t_ϕ} is the macroscopic cinematic eddy viscosity related to dynamic Eddy viscosity, $\mu_{t_\phi} = \rho \nu_{t_\phi}$ given by Eq. (2.15) and $\sigma_{t_\phi} = 0.9$ is the macroscopic turbulent Prandtl number.

According to Eqs. (2.21) and (2.33), the macroscopic heat flux due to turbulence is taken as the sum of the turbulent heat flux and the turbulent thermal dispersion found by Rocamora and de Lemos [2]. In view of the arguments given above, the turbulent heat flux and turbulent thermal dispersion components of the conductivity tensor, $\mathbf{K}_t$ and $\mathbf{K}_{disp,t}$, respectively, will be expressed as:

$$\mathbf{K}_t + \mathbf{K}_{disp,t} = \phi \left(\rho c_p\right)_f \frac{\nu_{t_\phi}}{\sigma_{t_\phi}} \mathbf{I} \tag{2.34}$$

2.5 Two-Energy Equation Model

Similarly, macroscopic energy equations are obtained for both fluid and solid phases by applying time and volume average operators to local instantaneous energy equations, resulting in,

$$\left(\rho c_p\right)_f \left[\frac{\partial \phi \langle \overline{T_f} \rangle^i}{\partial t} + \nabla \cdot \left\{ \phi \left(\langle \bar{\mathbf{u}} \rangle^i \langle \overline{T_f} \rangle^i + \underbrace{\langle {}^i\bar{\mathbf{u}}\, {}^i\overline{T_f} \rangle^i}_{\text{thermal disperson}} + \underbrace{\overline{\langle \mathbf{u}' \rangle^i \langle T_f' \rangle^i}}_{\text{turbulent heat flux}} + \underbrace{\langle \overline{{}^i\mathbf{u}'\, {}^iT_f'} \rangle^i}_{\text{turbulent thermal disperson}} \right) \right\} \right]$$

$$= \underbrace{\nabla \cdot \left[k_f \nabla \left(\phi \langle \overline{T_f} \rangle^i \right) + \frac{1}{\Delta V} \int_{A_i} \mathbf{n}_i k_f \overline{T_f} \, dA \right]}_{\text{conduction}} + \underbrace{\frac{1}{\Delta V} \int_{A_i} \mathbf{n}_i \cdot k_f \nabla \overline{T_f} \, dA}_{\text{interfacial heat transfer}}$$

where the expansion,

$$\langle \overline{\mathbf{u}' T_f'} \rangle^i = \overline{\langle (\langle \mathbf{u}' \rangle^i + {}^i\mathbf{u}')(\langle T_f' \rangle^i + {}^iT') \rangle^i} = \overline{\langle \mathbf{u}' \rangle^i \langle T_f' \rangle^i} + \langle \overline{{}^i\mathbf{u}'\, {}^iT_f'} \rangle^i \tag{2.36}$$

has been used in light of the double decomposition concept given by [2]. For the solid phase, one has,

$$\begin{aligned}\left(\rho c_p\right)_s\left\{\frac{\partial(1-\phi)\langle\overline{T_s}\rangle^i}{\partial t}\right\} &= \underbrace{\nabla\cdot\left\{k_s\nabla\left[(1-\phi)\langle\overline{T_s}\rangle^i\right]-\frac{1}{\Delta V}\int\limits_{A_i}\mathbf{n}_i k_s\overline{T_s}dA\right\}}_{\text{conduction}} \\ &\underbrace{-\frac{1}{\Delta V}\int\limits_{A_i}\mathbf{n}_i\cdot k_s\nabla\overline{T_s}dA}_{\text{interfacial heat transfer}}\end{aligned} \tag{2.37}$$

In (2.20) and (2.22), $\langle\overline{T_s}\rangle^i$ and $\langle\overline{T_f}\rangle^i$ denote the intrinsic average temperature of solid and fluid phases, respectively, A_i is the interfacial area within the REV and $\mathbf{n}_i$ is the unit vector normal to the fluid–solid interface, pointing from the fluid towards the solid phase. Equations (2.20) and (2.22) are the macroscopic energy equations for the fluid and the porous matrix (solid), respectively.

In order to use Eqs. (2.20) and (2.22), the underscored terms have to be modeled in some way as a function of the intrinsically averaged temperature of solid phase and fluid, $\langle\overline{T_s}\rangle^i$ and $\langle\overline{T_f}\rangle^i$. To accomplish this, a gradient type diffusion model is used for all the terms, in the form,

$$\text{Turbulent heat flux: } -\left(\rho c_p\right)_f\left(\phi\overline{\langle\mathbf{u}'\rangle^i\langle T_f'\rangle^i}\right)=\mathbf{K}_t\cdot\nabla\langle\bar{T}_f\rangle^i \tag{2.38}$$

$$\text{Thermal dispersion: } -\left(\rho c_p\right)_f\left(\phi\langle^i\bar{\mathbf{u}}^i\overline{T_f}\rangle^i\right)=\mathbf{K}_{disp}\cdot\nabla\langle\bar{T}_f\rangle^i \tag{2.39}$$

$$\text{Turbulent thermal dispersion: } -\left(\rho c_p\right)_f\left(\phi\langle^i\overline{\mathbf{u}'^i T_f'}\rangle^i\right)=\mathbf{K}_{disp,t}\cdot\nabla\langle\bar{T}_f\rangle^i \tag{2.40}$$

$$\begin{aligned}\text{Local conduction: }\nabla\cdot\left[\frac{1}{\Delta V}\int\limits_{A_i}\mathbf{n}_i k_f\overline{T_f}dA\right]&=\mathbf{K}_{f,s}\cdot\nabla\langle\bar{T}_s\rangle^i \\ -\nabla\cdot\left[\frac{1}{\Delta V}\int\limits_{A_i}\mathbf{n}_i k_s\overline{T_s}dA\right]&=\mathbf{K}_{s,f}\cdot\nabla\langle\bar{T}_f\rangle^i\end{aligned} \tag{2.41}$$

where $\mathbf{n}_i$ in (2.27) is the unit vector pointing outwards of the fluid phase. In this work, for simplicity, one assumed that for turbulent flow the overall thermal resistance between the two phases is controlled by the interfacial film coefficient, rather than by the thermal resistance within each phase. As such, the tortuosity coefficients $\mathbf{K}_{f,s}, \mathbf{K}_{s,f}$ are here neglected for the sake of simplicity.

The heat transferred between the two phases can be modeled by means of a film coefficient h_i such that,

$$h_i a_i\left(\langle\overline{T_s}\rangle^i - \langle\overline{T_f}\rangle^i\right) = \frac{1}{\Delta V}\int_{A_i} \mathbf{n}_i \cdot k_f \nabla\overline{T_f}\, dA = \frac{1}{\Delta V}\int_{A_i} \mathbf{n}_i \cdot k_s \nabla\overline{T_s}\, dA \tag{2.42}$$

where $a_i = A_i/\Delta V$ is the surface area per unit volume.

For the above shown expressions, Eqs. (2.20) and (2.22) can be written as:

$$\left\{\left(\rho c_p\right)_f \phi\right\}\frac{\partial\langle\overline{T_f}\rangle^i}{\partial t} + \left(\rho c_p\right)_f \nabla\cdot\left(\mathbf{u}_D\langle\overline{T_f}\rangle^i\right) = \nabla\cdot\left\{\mathbf{K}_{eff,f}\cdot\nabla\langle\overline{T_f}\rangle^i\right\} + h_i a_i\left(\langle\overline{T_s}\rangle^i - \langle\overline{T_f}\rangle^i\right) \tag{2.43}$$

$$\left\{(1-\phi)\left(\rho c_p\right)_s\right\}\frac{\partial\langle\overline{T_s}\rangle^i}{\partial t} = \nabla\cdot\left\{\mathbf{K}_{eff,s}\cdot\nabla\langle\overline{T_s}\rangle^i\right\} - h_i a_i\left(\langle\overline{T_s}\rangle^i - \langle\overline{T_f}\rangle^i\right) \tag{2.44}$$

where, $\mathbf{K}_{eff,f}$and $\mathbf{K}_{eff,s}$ are the effective conductivity tensor for fluid and solid, respectively, given by:

$$\mathbf{K}_{eff,f} = [\phi k_f]\mathbf{I} + \mathbf{K}_{f,s} + \mathbf{K}_t + \mathbf{K}_{disp} + \mathbf{K}_{disp,t} \tag{2.45}$$

$$\mathbf{K}_{eff,s} = [(1-\phi)k_s]\mathbf{I} + \mathbf{K}_{s,f} \tag{2.46}$$

and $\mathbf{I}$ is the unit tensor.

2.5.1 Turbulence

In order to apply Eqs. (2.31)–(2.44) for the Two-Energy Equation Model (2EEM), or Eq. (2.31) for the One-Energy Equation Model (1EEM), it is necessary to determine the components of the effective conductivity tensors in Eq. (2.32), i.e., $\mathbf{K}_{f,s}$, $\mathbf{K}_t$, $\mathbf{K}_{disp}$ and $\mathbf{K}_{disp,t}$ for the 2EEM, and $\mathbf{K}_{tor}$, $\mathbf{K}_t$, $\mathbf{K}_{disp}$ and $\mathbf{K}_{disp,t}$ in Eq. (2.32), for the 1EEM. Here, for simplicity, all mechanisms are neglected except turbulence, which is here explicitly accounted for (see [7, 8] for a discussion on the determination of such coefficients).

The turbulent heat flux and turbulent thermal dispersion terms, $\mathbf{K}_t$ and $\mathbf{K}_{disp,t}$, are here modeled through the Eddy diffusivity concept, as:

$$\mathbf{K}_t + \mathbf{K}_{disp,t} = \phi\left(\rho c_p\right)_f \frac{\nu_{t_\phi}}{\sigma_T}\mathbf{I} \tag{2.47}$$

2.5.2 Interfacial Heat Transfer, h_i

Wakao et al. [9] proposed a correlation for h_i for closely packed bed and compared results with their experimental data. This correlation reads,

$$\frac{h_i D}{k_f} = 2 + 1.1 Re_D^{0.6} Pr^{1/3}. \tag{2.48}$$

Kuwahara et al. [10] also obtained the interfacial convective heat transfer coefficient for laminar flow, as follows,

$$\frac{h_i D}{k_f} = \left(1 + \frac{4(1-\phi)}{\phi}\right) + \frac{1}{2}(1-\phi)^{1/2} \mathrm{Re}_D \overset{1/3}{\mathrm{Pr}}, \text{ valid for } 0.2 < \phi < 0.9 \tag{2.49}$$

Equation (2.49) is based on porosity dependency and is valid for packed beds of particle diameter D. Following this same methodology, in which the porous medium is considered an infinite number of solid square rods, Saito and de Lemos [6] proposed a correlation for obtaining the interfacial heat transfer coefficient for turbulent flow as,

$$\frac{h_i D}{k_f} = 0.08 \left(\frac{Re_D}{\phi}\right)^{0.8} Pr^{1/3}; \text{for } 1.0 \times 10^4 < \frac{Re_D}{\phi} < 2.0 \times 10^7, \quad \text{valid for } 0.2 < \phi < 0.9, \tag{2.50}$$

2.5.3 Non-Dimensional Parameters

The local Nusselt number for the one-energy equation model used by [11] is given by:

$$Nu = \left(\frac{\partial \langle \bar{T} \rangle^i}{\partial y}\right)_{y=H} \frac{H}{T_1 - T_0} \tag{2.51}$$

Equation (2.51) assumes the local thermal equilibrium hypothesis, i.e., $\langle T \rangle^i = \langle T_s \rangle^i = \langle T_f \rangle^i$. When the Local Non-thermal Equilibrium model is applied, that are distinct definitions for the Nusselt number associated to each phase, as follows [12],

Fluid phase Nusselt number:

$$Nu_f = \left(\frac{\partial \langle \bar{T}_f \rangle^i}{\partial y}\right)_{y=H} \frac{H}{T_1 - T_0} \tag{2.52}$$

Solid phase Nusselt number:

$$Nu_s = \left(\frac{\partial \langle \bar{T}_s \rangle^i}{\partial y} \right)_{y=H} \frac{H}{T_1 - T_0} \tag{2.53}$$

2.6 Boundary Conditions and Numerical Details

2.6.1 Flow Boundary and Interface Conditions

At the interface between the porous layer and the clear region the macroscopic velocity, intrinsic pressure, turbulence kinetic energy and its dissipation rate, as well as their respective diffusive fluxes, were assumed to be continuous functions so that,

$$\bar{\mathbf{u}}_D|_{0<\phi<1} = \bar{\mathbf{u}}_D|_{\phi=1} \tag{2.54}$$

$$\langle \bar{p} \rangle^i|_{0<\phi<1} = \langle \bar{p} \rangle^i|_{\phi=1} \tag{2.55}$$

$$\langle k \rangle^v|_{0<\phi<1} = \langle k \rangle^v|_{\phi=1} \tag{2.56}$$

$$\left(\mu + \frac{\mu_{t_\phi}}{\sigma_k} \right) \frac{\partial \langle k \rangle^v}{\partial y}\bigg|_{0<\phi<1} = \left(\mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial \langle k \rangle^v}{\partial y}\bigg|_{\phi=1} \tag{2.57}$$

$$\langle \varepsilon \rangle^v|_{0<\phi<1} = \langle \varepsilon \rangle^v|_{\phi=1} \tag{2.58}$$

$$\left(\mu + \frac{\mu_{t_\phi}}{\sigma_\varepsilon} \right) \frac{\partial \langle \varepsilon \rangle^v}{\partial y}\bigg|_{0<\phi<1} = \left(\mu + \frac{\mu_t}{\sigma_\varepsilon} \right) \frac{\partial \langle \varepsilon \rangle^v}{\partial y}\bigg|_{\phi=1} \tag{2.59}$$

2.6.2 Heat Boundary and Interface Conditions

For the planar geometries in Fig. 1.2, temperature of the incoming fluid was specified at jet inlet. At the flow exit ($x = L$), a zero diffusion flux condition was imposed and a symmetry plane was considered at the left boundary ($x = 0$). On the bottom wall, a constant given temperature was assumed and, on the top wall, thermal isolation condition prevailed.

At the macroscopic interface shown in Fig. 1.2b ($y = h$), the volume-time-averaged fluid temperature was continuous as well as the total transverse heat flux (laminar plus turbulent). As a consequence of the imposed continuity of the fluid phase heat flux at the interface, the heat conducted by the solid matrix in the

y-direction at the macroscopic boundary was of null value. As will be shown in the chapters to follow, solid temperature gradients attained zero value at $y = h$.

2.6.3 Numerical Details

Equations (2.11), (2.12) and (2.31), subject to interface and boundary conditions were discretized in a two-dimensional control volume involving both clear and porous media. The finite volume method was used for discretizing the equation set and the SIMPLE algorithm was applied to handle the pressure-velocity coupling (Patankar [13]). The discretized form of the two-dimensional conservation equation for a generic property φ in permanent regime reads,

$$I_e + I_w + I_n + I_s = S_\varphi \tag{2.60}$$

where I_e, I_w, I_n and I_s represent, respectively, the fluxes of φ in the faces east, west, north and south faces of the control volume and S_φ its term source.

Standard source term linearization is accomplished by using,

$$S_\varphi \approx S_\varphi^{**} \langle \varphi \rangle_p^i + S_\varphi^* \tag{2.61}$$

Discretization in the x-direction momentum equation gives,

$$S^{*x} = \left(S_e^{*x}\right)_P - \left(S_w^{*x}\right)_P + \left(S_n^{*x}\right)_P - \left(S_s^{*x}\right)_P + S_P^* \tag{2.62}$$

$$S^{**x} = S_\phi^{**} \tag{2.63}$$

where, S^{*x} is the diffusive part, here treated in an explicit form. The second term, S^{**x}, entails the additional drag forces due to the porous matrix, last two terms in Eq. (2.12), are here treated explicitly.

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