Chapter 1

A Historical Introduction: Sir George Biddell Airy

George Biddell Airy was born 27 July 1801 at Alnwick in Northumberland (North of England). His family was rather modest, but thanks to the generosity of his uncle Arthur Biddell, he went to study at Trinity College, University of Cambridge. Although a sizar,[1] he was a brilliant student and finally graduated in 1823 as a senior wrangler. Three years later he was appointed to the celebrated Lucasian chair of mathematics. However, his salary as Lucasian professor was too small to marry Richarda, the young lady's father objected, so he applied for a new position. In 1828, Airy obtained the Plumian chair, becoming professor of Astronomy and director of the new observatory at Cambridge. His early work at this time concerned the mass of Jupiter and the irregular motions of the Earth and Venus.

In 1834, Airy started his first mathematical studies on the diffraction phenomenon and optics. Due to diffraction, the image of a point through a telescope is actually a spot surrounded by rings of smaller intensity. This spot is now called the *Airy spot*; the associated Airy function, however, has nothing to do with the purpose of this book.

In June 1835, Airy became the 7^{th} Royal Astronomer and director of the Greenwich observatory, succeeding John Pond. Under his administration, modern equipment was installed, leading the observatory to worldwide fame assisted by the quality of its published data. Airy also introduced the study of sun spots and of the Earth's magnetism, and built new apparatus for the observation of the Moon, and for cataloguing the stars. The question of absolute time was also a major challenge: Airy defined the *Airy Transit Circle*, which in 1884 became the Greenwich Mean Time. However the renown of Airy is also due to the *Neptune affair*. During the decade 1830–

[1]Meaning that he paid a reduced fee in exchange for working as a servant to richer students.

40, astronomers were interested in the perturbations of Uranus, which had been discovered in 1781. In France, François Arago suggested to Urbain Le Verrier that he should seek for a new planet that might have caused the perturbations of Uranus. In England, the young John Adams was doing the same calculations with a slight advance. Airy however was dubious about the outcome of his work. Adams tried twice to meet Airy in 1845 but was unsuccessful: the first time Airy was away, the second time Airy was having dinner and did not wish to be disturbed. Finally, Airy entrusted the astronomer James Challis with the observation of the new planet from the calculations of Adams. Unfortunately, Challis failed in his task. At the same time, Le Verrier asked the German astronomer Johann Galle in Berlin to locate the planet from his data: the new planet was discovered on 20 September 1846. A controversy then started between Airy and Arago, between France and England, and also against Airy himself. The dispute became more acrimonious concerning the name of the planet itself, Airy wanting to name the new planet Oceanus. The name Neptune was finally given. The story goes that, in the end, Adams and Le Verrier became good friends.

In 1854 Airy attempted to determine the mean density of the Earth by comparing the gravity forces on a single pendulum at the top and the bottom of a pit. The experiment was carried out near South Shields in a mine 1250 feet in depth. Taking into account the elliptical form and the rotation of the Earth, Airy deduced a density of 6.56, which is not so far — considering the epoch — from the currently accepted density 5.42.

Airy was knighted in 1872, and so became Sir George Biddell.[2] At this time, Airy started a lunar theory. The results were published in 1886, but in 1890 he found an error in his calculations. The author was then 89 years old and was unwilling to revise his calculations. Late in 1881, Sir George retired from his position as astronomer at Greenwich. He died January 2, 1892.

The autobiography of Sir George, edited by his son Wilfred, was published in 1896 [W. Airy (1896)]. The name of Airy is associated with many phenomena such as the Airy spiral (an optical phenomenon visible in quartz crystals), the Airy spot in diffraction phenomena or the Airy stress function which he introduced in his work on elasticity, which is different again from the Airy functions that we shall discuss in this book. Among the most well-known books he wrote, we may mention *Mathematical tracts on physical astronomy* (1826) and *Popular astronomy* (1849) [W. Airy (1896)].

[2]After having declined the offer on three occasions, objecting to the fees.

Fig. 1.1 Sir George Biddell Airy (after the Daily Graphic, January 6, 1892).

Airy was particularly involved in optics: for instance, he made special glasses to correct his own astigmatism. For the same reason, he was also interested in the calculation of light intensity in the neighbourhood of a caustic [Airy (1838), (1849)]. For this purpose, he introduced the function defined by the integral

$$W(m) = \int_0^\infty \cos\left[\frac{\pi}{2}\left(\omega^3 - m\omega\right)\right] \mathrm{d}\omega,$$

which is now called the Airy function. This is the object of the present book. W is the solution to the differential equation

$$W'' = -\frac{\pi^2}{12} m W.$$

The numerical calculation of Airy functions is somewhat tricky, even today! However in 1838, Airy gave a table of the values of W for m varying from -4.0 to $+4.0$. Thence in 1849, he published a second table for m varying from -5.6 to $+5.6$, for which he employed the ascending series. The problem is that this series is slowly convergent as m increases. A few years later, Stokes (1851, 1858) introduced the asymptotic series of $W(m)$, of its derivative and of the zeros. Practically no research was undertaken on the Airy function until the work by Nicholson (1909), Brillouin (1916) and Kramers (1926) who contributed significantly to our knowledge of this function.

In 1928 Jeffreys introduced the notation used nowadays

$$Ai(x) = \frac{1}{\pi} \int_0^\infty \cos\left(\frac{t^3}{3} + xt\right) \mathrm{d}t,$$

which is the solution of the homogeneous differential equation, called the Airy equation

$$y'' = xy.$$

Clearly, this equation may be considered as an approximation of the differential equation of the second order

$$y'' + F(x)y = 0,$$

where F is any function of x. If $F(x)$ is expanded in the neighbourhood of a point $x = x_0$, we have to the first order ($F'(x_0) \neq 0$)

$$y'' + [F(x_0) + (x - x_0)F'(x_0)]\, y = 0.$$

Then with a change of variable, we find the Airy equation. This method is particularly useful in the neighbourhood of a zero of $F(x)$. The point x_0 defined by the relation $F(x_0) = 0$ is called a transition point by mathematicians and a turning point by physicists. Turning points are involved in the asymptotic solutions to linear differential equations of the second order [Jeffreys (1942)], such as the stationary Schrödinger equation.

Finally we can note that Airy functions are Bessel functions (or linear combinations of these functions) of order 1/3. The relation between the Airy equation and the Bessel equation is performed with the change of variable $\xi = \frac{2}{3}x^{3/2}$, leading Jeffreys (1942) to say: "*Bessel functions of order* 1/3 *seem to have no application except to provide an inconvenient way of expressing this function*!"