INTRODUCTION

The radar field is a mature discipline whose focus is electromagnetic systems for the detection of targets or aspects of the natural environment. A transmitted signal, for ease of referral: a TX, is radiated into space, which interacts with those targets or aspects, and a returning received signal, or echo, for ease of referral: an RX, is produced. The RX is captured by a receiver and, following suitable treatment, targets can be detected and other information about the target extracted from the return signal.

Conventional radars operate with TXs with frequencies ranging from the so-called high frequency (HF) range, (3–30 MHz), to the millimeter range (40–300 GHz). Conventionally, a radar system is constructed to produce TXs either at a set frequency, or over a limited frequency bandwidth, as in a linear frequency modulated (LFM) chirp, or as a very short duration pulse, the short duration of which produces a very wide bandwidth or ultrawideband (UWB) signal. In conventional approaches, only in a very general sense are the frequency and time duration attributes of the TX matched or aligned with similar attributes of the target. In contrast, the radar systems addressed by the present book are directly focused on matching as much as possible the attributes of the TX to the attributes of a designated target, class of targets, or medium-and-target. In a special emphasis, the major attributes of a target addressed are the target’s scattered RX radio frequency (RF) spectrum, and, when a target’s RX response is extended in time, i.e., if an extended target, the timing sequence of the separate returned wave packets constituting the target’s total RX.

Conventionally, it is either assumed that the RX of a target is qualitatively the same in response to all frequencies in the TX radar frequency bandwidth of a specific radar system, or the differences are not brought to full attention, and so are largely neglected. However, paradoxically, it is well known that the electromagnetic response of any
target is a function of target size and TX frequency. For example, there are three major so-called “scattering regions”: (i) the Rayleigh, (ii) the Mie\textsuperscript{1} or resonance, and (iii) the optical, regions. It is important to note, that these regions are not defined with respect to specific TX frequency regions. Rather, they are defined relative to the ratio of specific target lengths and TX wavelengths. Furthermore, these three (joint target-and-transmitted-signal-wavelength-dependent-) scattering regions have different underlying scattering mechanisms as a function of that ratio. In the case of optical scattering, the TX wavelength is less than a target’s dimensions, and the RX is due to parts of the target, and not due to the whole target or its volume or area. In the case of Rayleigh scattering, the TX wavelength is greater than a target’s dimensions, and the RX is due to its volume or area. At ratios in between these, Mie or resonance scattering occurs when the TX wavelength is approximately equal to both the specific target length and the lengths of subcomponents of the target. In these cases the whole target and its subcomponents oscillate at specific resonant frequencies and, significantly, the RX is somewhat greater in amplitude than the RX in the case of either optical or Rayleigh scattering. The distinction between the physical mechanisms underlying the three scattering regions is noteworthy, because the designs of the radar systems addressed by the present book are focused on techniques of matching, as much as possible, the wavelengths or frequencies of the TX to the size or resonances of designated targets and target subcomponents, in order to achieve Mie or resonance scattering.

One quantitative method of analysis describing the extent to which signals can be matched in time and frequency to receiver properties was available very early in the development of radar. The ambiguity function (Woodward, 1953; Cook & Bernfeld, 1967; Vakman, 1968) and its relative, the cross-ambiguity function, that relate properties of the RX signal to receiver properties, are analysis methods closely related to the matched filter concept. However, here we extend the matched filter concept by matching the TX signal in time and frequency to the known target RX, and the matched filter concept is expressed in both an optimum TX-target response relation as well as an optimum RX-receiver relation, rather merely in the conventional optimum RX-receiver relation regardless of the TX. In the conventional case of matching the receiver to a general RX, an optimal relation is only achieved by chance, because targets and their RX responses may vary in the extent of optimization, when the TX remains constant.

\textsuperscript{1}Gustav Adolf Feodor Wilhelm Ludwig Mie (1869–1957).
Conversely, in the case of matching the TX to a known target response, the target transfer response is optimally configured “in the channel” or “in-the-loop” between the transmitter and receiver as a transfer function, and an optimum response, RX, is due to matching the TX to that frequency dependent transfer function. Therefore the radar systems addressed by the present book are focused on both a TX-target response relation matched filtering as well as an RX-receiver relation matched filtering alone.

In the case of a conventional radar that transmits a signal unmatched to a designated target, there is both waste of transmitted energy at those frequencies unmatched to target resonances (because energy at those frequencies is minimally reflected or returned), or inadvertently matched to undesignated targets (i.e., “clutter”) with resonant frequencies other than target resonances. In the latter case the designated target RX is mixed with clutter returns, resulting in an undesirable signal-to-clutter ratio (SCR). However, when the transmit signal is matched to the designated target resonances, and, with the proviso that the resonances of the clutter do not coincide with the resonances of the designated target, an improved SCR is obtained.

The difference between the conventional approach to radar, and the approach taken in this book, is also mirrored in the difference in statistical decision theory between maximum-likelihood (ML) estimation detection, and maximum a posteriori (MAP) estimation detection. In the case of ML estimation, detection is optimal for decisions made on the basis of the magnitude of the RX given the TX, whatever the TX may be. This is similar to conventional radar detection. In the case of MAP estimation, detection is optimal for decisions made on the basis of the magnitude of the RX given that the TX was matched to known target attributes. Without that matching made possible by the availability of a priori information, MAP detection coincides with ML detection. The radar systems addressed by the present book implement MAP detection rather than ML detection, and we refer to such systems as MAP radars.

Since this book addresses MAP detection which requires target a priori information, how is this a priori information obtained? There are two ways. The first, and preferred, way provides class-of-target, and/or target,
and/or target subcomponent, frequency response information from either an anechoic chamber or “clutter-free” environment tests. The second, and more difficult, way provides the same information but gathered “on-the-fly” using heuristic transmit and filtering techniques. The obtainment of the (i) class-of-target, and/or (ii) target, and/or (iii) subcomponent of target, frequency response information permits the design of TXs to match the desired class-of-target, and/or target, and/or subcomponent of target. Furthermore, target identification by “question-and-answer” TX-RX protocol sequences are enabled. For example, if the target frequency response is known \textit{a priori} concerning (i) the major class of target sought; (ii) the minor class of target sought; and (iii) a critically identifying component of the specific target sought, then matched signals can be transmitted in a $TX_1 \rightarrow RX_1 \rightarrow TX_2 \rightarrow RX_2 \rightarrow TX_3 \rightarrow RX_3 \ldots$ sequence to confirm, or not, the identification of (a) the major class of target sought; (b) the minor target sought; and (c) a critically identifying component of the specific target sought. As a specific example: (i) might be all four-wheel vehicles of a certain size; (ii) might be all Humvee vehicles; and (iii) might be the antenna used by only certain Humvee vehicles. Furthermore, in the case of (i) we are concerned with major RX resonances shared by all targets in the class; in the case of (ii), with major resonances of the minor class of targets; and in the case of (iii), with minor resonances identifying specific targets in the minor class.

Due to the above considerations and with a focus of the present book on target major and minor resonances, a target’s \textit{a priori} frequency response information essentially amounts to a target’s radio frequency (RF) spectrum. Therefore, whereas conventional radar addresses \textit{ranging} and \textit{detection} of generally an unknown target — usually a point scatterer — and more recently, target imaging, the present book is focused on what might be called: RF spectroscopy, addressing the spectral characteristics of targets and target subcomponents.

In introducing an RF spectroscopy it is necessary to identify particular kinds of TXs and RXs. For ease of reference throughout this book we shall use some simplifying nomenclature. Apart from a target’s frequency response, there is the related impulse response. If the target \textit{a priori} information is obtained by means of pulsing the target with a very short monocycle pulse, we shall call that transmitted pulse: a PTX. The target echo, or returned signal elicited by the PTX, we shall call: a PRX. A PTX is identical to a UWB TX signal, and a PRX is identical to a UWB RX signal.
Similarly the target-matched TX we shall call: an MTX, which is the time reversal of a PRX target return echo pulse. The advantage to using an MTX is that it is matched (i) in frequency to the resonances of the target and also (ii) in time to the target’s orientation, or aspect angle, that determines when an extended target’s resonance responses are received. In the case of (i) resonance matching, we shall show that the return signal from the target, MRX, is target-and-transceiver-platform-position independent, or aspect independent, with respect to the target spectral frequency bands, while the amplitude of those frequency bands may vary with aspect. In the case of the (ii), the sequencing or time of arrival of RX packets, constituting an extended target’s subcomponent echo returns elicited by either PRXs or MRXs, is aspect dependent on target-transceiver-platform relative orientation. For these reasons, the radars addressed in this book are Resonance and Aspect Matched Adaptive Radars, or RAMARs.

An underlying assumption of a MAP RAMAR is that the target and its subcomponents, act as linear transfer functions of PTXs and MTXs. Hence PTX, MTX, PRX and MRX components are treated as statistically independent. Independent component analysis (ICA) processing is used, below, to indicate that the statistical independence assumption is justified — at least for the targets so far tested (Section 1.1.18).

The duration of a PTX is as short as equipment permits, and the duration of a MTX is dictated by the extended target’s response function — and also of short duration. A major disadvantage of using short duration transmit pulses of any kind is that the average energy per TX and RX pulse is low, and in order to detect targets at long range by returned pulse integration, short duration TX signals need to be transmitted at a high repetition rate. On the other hand, in the majority of situations, information about a target’s orientation is not required. In these cases a target’s a priori known frequency response information is sufficient to design a TX matched only to the target’s resonances (or RF spectrum), and not also to the target-platform relative orientation using the PRX timing information. In these majority of cases, the designed TX can be as long in time as target range requires and equipment limitations permit, and we refer to these TX signals as DTXs and STXs, below.

Two questions might be asked concerning the method used to obtain target a priori information: (1) If the targets tested so far in this program indicate that the target transfer function is linear, why not use an LFM TX signal, rather than a short monocycle PTX? (2) Why not just calculate
a target’s frequency response from its known dimensions and material composition?

In answer to the first question: As those targets tested so far have largely exhibited linear transfer function responses, LFM TXs, in which “all”, or a broad band of frequencies are applied, but applied *sequentially*, might indeed be substituted for a UWB signal, in which, in contrast, the same frequencies are applied “*instantaneously*”. However, statistically speaking, one cannot prove the null hypothesis. That is: one cannot extrapolate a conclusion of no differences (from linearity) in an examination of the few (targets), to a conclusion of no differences in the unexamined many (targets), so there always remains the possibility that some yet untested classes of targets are nonlinear, or partially nonlinear.\(^3\)

While the characterization of linear systems is an advanced discipline (cf. Bendat & Piersol, 1993, 2000), the characterization of nonlinear systems has just begun (cf. Bendat, 1990),\(^4\) and is hardly complete. The available techniques characterizing nonlinear systems apply to only a limited number of nonlinear models. The methods to detect nonlinear systems were formalized by Wiener (1958). These methods require the test application of *all* frequencies and *all* phases between those frequencies and if not with an infinite bandwidth, at least within a reasonably wide bandwidth, e.g., testing using white noise (cf. Barrett, 1975). The short ultrawideband pulse (PTX) approach that we used in both Ka-band and UHF prototype testing, described below, is a compromise approach. A radiated short pulse is not a Dirac delta function — which cannot be radiated — and is, at best, a monocycle. The substitution of a PTX short pulse for white noise means that as a short pulse is broad band, one can test with “all frequencies” within a reasonably wide bandwidth, but not all phases between those

\(^3\)There are many types of nonlinear systems (cf. Bendat, 1990), but there is no universally accepted definition of nonlinearity. When referring to a system that is nonlinear, the type of nonlinearity usually meant is one in which the principle of superposition does not apply to a systems’s individual frequency responses. There are, however, other definitions and meanings: e.g., an intensity-dependent nonlinearity; nonlinear feedback, etc. Furthermore, below we treat the whole target, with all subcomponent resonances excited, as a linear time variant (LTV) system. A nonlinearity in this case would be the target’s complete response not being the summation of subcomponent minor resonances acting in isolation.

\(^4\)The study has begun for third-order nonlinear polynomial systems consisting of linear systems in parallel with finite-memory square-law systems and finite-memory cubic systems.
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frequencies — because the phases between frequencies in a short pulse are static and time invariant.

Turning to the second question: Why not just calculate a target’s frequency response? — we answer this question referring to empirical tests conducted on designed subcomponent parts systematically joined (cf. Section 1.9, below). This question of whether a complex target’s RX signal is merely composed of the superposition of the minor resonances of separated subcomponents, that, when joined, compose the target, was answered in the negative. The tests conducted indicate that superposition of separate subcomponent resonances does not apply, and simulation of an empirically untested complex target’s RX spectrum is difficult due to this nonlinear addition. However, it is important to distinguish the difference between this nonlinear addition of subcomponent responses in physically forming the target, and a nonlinear addition of frequency responses after that target is physically formed — the latter having been addressed by the first question.

Another question then arises: given that a short pulse provides frequencies across a wide bandwidth, is the bandwidth an instantaneous bandwidth? That is, are all the frequencies instantaneous frequencies, i.e., all present at the same time? This question actually addresses the issue of frequency dispersion in antennas. The difficulties in designing a wide instantaneous bandwidth antenna (Anderson et al., 2003; Schantz, 2005; Gavami et al., 2007), as opposed to sequentially wide bandwidth antennas, are well-known, and we do not resolve these issues here. Whether a wide instantaneous bandwidth, or a wide sequential bandwidth antenna is used, the pulse method of obtaining a system’s frequency response is still, admittedly, a compromise.

Turning now to related previous work: Since this book addresses the optimized match of transmitted signals to designated targets using a priori information, it might asked: Have there been other proposals to do the same or similar? The answer is that there have been, but not exactly the same.

In the early years of radar, Ville (1948), Woodward & Davies (1950) and Woodward (1953) addressed optimum radar signals, but the search was for a single optimum signal for all targets. Cook & Bernfield (1967) mention that it might be possible to match a signal to its environment, but acknowledging the difficulties, looked for a signal optimum over a given number of situations and favored the LFM transmitted signal. Gjessig (1978, 1981, 1986; Apel & Gjessig, 1989) proposed a target adapted “matched illumination” radar. But whereas this radar is designed to achieve a target returned signal that is a “delta function” in the frequency domain,
the MAP systems addressed by the present book, on the other hand and quite in contrast, are designed to achieve a target returned signal (MRX) that, only under a specific aspect angle of target-and-platform (90°) approximates a “delta function” of the target’s impulse response (PRX) yet in the time — not in the frequency — domain. Thus, the systems addressed in the present book specifically address target resonances that are neglected in the matched illumination radars which address constructive interference by all the reflecting facets of the target.

Furthermore, this book provides evidence that in many, if not most, instances, the target acts as a linear time varying or dependent (LTV) system, rather than a linear time invariant or independent (LTI) system. Thus the systems addressed in the present book differ from those systems treated as LTI systems (Bell, 1988, 1993; Haykin, 2006; Guerci, 2010). Bell, although exploiting target resonance effects and addressing waveform design, assumes a static target, i.e., an LTI system. Guerci (2010) likewise addresses LTI targets, even although this form of analysis cannot be extrapolated to real world non-static LTV targets. Although a so-called knowledge based radar (Haykin, 2006) uses a priori information, that information appears to be collected from three generic conventional radars of limited TX pulse capability and which cannot provide an estimate of target’s frequency response. In summary, the closest surveillance/signaling systems to the MAP systems addressed in this book are not in the field of radar, but rather in the fields of photonics and acoustics.

The radar systems described in this book require a priori information concerning the time and frequency characteristics of the target’s transfer function, and although such information can be obtained with more difficulty, on-the-fly, here we concentrate on the collection of that information by pulsing the target in a clutter-free environment to obtain empirical knowledge of a target’s transfer function. The information in the returned signal is then used to design other kinds of transmitted signals with optimum properties. As an aid to exposition, from now forward we shall use the following abbreviations and mnemonics to describe these signals, some of which have been already introduced:

- **A MAP signal**: a target-matched adaptive time-frequency wave packet or signal using a priori information and permitting maximum a posteriori estimation detection according to Bayesian statistics.
- **PTX**: a transmitted (TX) short duration packet/signal, modulating an arbitrary carrier, generally of 1 or 2 nanosecond duration, and that is
used to obtain a priori information concerning the target in a specified aspect to the transmitter. A PTX is equivalent to an UWB transmitted pulse and functions as a “δ function” surrogate or approximation.\textsuperscript{5}

- **PRX:** the received (RX) return target echo, modulating an arbitrary carrier that is elicited by a PTX. A PRX is equivalent to an UWB return echo signal, and approximates the target impulse response at a specified target aspect to the transmitter.
- **MTX:** a MAP transmitted (TX) short duration packet/signal, envelope or amplitude modulation of an arbitrary carrier. An MTX envelope is a PRX but time reversed resulting in a matching of the envelope of the MTX transmitted signal to the designated target.
- **MRX:** the envelope of the received (RX) return target echo, modulating an arbitrary carrier, that is elicited by an MTX.
- **DTX:** a PRX-derived transmitted (TX) arbitrarily long duration transmitted signal, modulating an arbitrary carrier, and which addresses some selected resonance or collection of resonances identified in the PRX spectrum.
- **DRX:** the envelope of the received (RX) return target echo, modulating an arbitrary carrier, that is elicited by a DTX.
- **STX:** A transmitted (TX) arbitrarily long duration TX signal that is a collection or bundle of DTXs.
- **SRX:** the received (RX) return target echo, modulating an arbitrary carrier, that is elicited by an STX.

1. **A Priori and A Posteriori Information Captures**

The generic protocol for the PRX-MRX relation where there is access to a clutter-free environment involves two “captures” of target information:

1. **The A Priori Capture.** A PTX is transmitted in a clutter and multipath-free environment with the target in a known orientation or aspect to the transceiver. With respect to this transmitted signal, an extended target acts as a temporal scattering matrix dispersing the resonance response of individual subcomponents in wave packets (Fig. 1) — the target resonances being temporally dispersed as a function of target aspect angle (Figs. 3–5), and an extended target

\textsuperscript{5}A true δ function is of infinitely short duration and of infinite bandwidth, and even an approximation could not be propagated. What is meant in this context is a very short pulse — usually a monocycle.
A. Test Pulse: PTX-PRX

Fig. 1  The A Priori Capture. A short duration pulse or wave packet, PTX — ideally, shorter in length than any feature of interest on the target — is used to obtain the scattering matrix of a target which is here arbitrarily assigned 4 subcomponent resonances in this figure and is arbitrarily oriented head-on to the transmitter. The PTX is used as an approximation to a “δ function”. A PTX is also an ultrawideband UWB signal. The target does not act as a point scatterer, but decomposes into its individual scattering components — i.e., is an extended target. Each of these individual components is scattered back to the transmitter (1) with specific frequency and amplitude modulation packets, the spectral characteristics of which are aspect independent; and (2) at separate phasing or time intervals of arrival that are aspect dependent. This is a feature of extended targets. The impulse response of the target can be used for target imaging, but in the case of interest is here used as a priori information for the A Posteriori Capture. The total response is the target’s impulse response and is referred to as PRX which is the target’s Green’s function, \( G(t, f; s) \) — \( t \), time, \( f \), frequency, \( s \), spatial position. The target’s whole body response is not addressed in this illustration.

response being composed of a collection of subcomponent-related wave packets, as well as a whole body response. The PRX encodes both (i) resonances, the frequency band spectral positions of which are aspect independent, as well as (ii) their temporal dispersion, which is aspect dependent (Fig. 1):

(2) The A Posteriori Capture. An MTX is constructed by time reversing the PRX and transmitting (Fig. 2). With respect to this transmitted
B. Matched Pulse: MTX-MRX

Fig. 2 The *A Posteriori* Capture. A *MAP TX* pulse, or *MTX*, is constructed by time reversing the *PRX* and is thereby matched to the amplitude and frequency modulation of the target at a specific target aspect angle, i.e., the *MTX* is the complex conjugate of the impulse response of the target at set aspect angle, or $G^{-1}(t, f; s)$. The *MTX* signal excites each target subcomponent (1) with the appropriate resonance frequency of the subcomponent; (2) with the appropriate relative amplitude modulation, and also, in the instance shown, (3) at the appropriate timing of TX packets for the target’s aspect angle. This technique permits selective enhancement of component parts of the target, if required, and both the avoidance of clutter returns — if the clutter does not share resonant frequencies with the target — as well as confining TX energies to those frequencies to which the target is responsive. The returned signal, *MRX*, can be a short duration signal, i.e., acts as a loose approximation to a $\delta$ function, if the target orientation is “head-on” (0° aspect agreeing with the *A Priori* Capture). 

$$MRX = G(t, f; s)G^{-1}(t, f; s) = \delta(t, f; s).$$

The target’s whole body response is also not addressed in this illustration.
Resonance and Aspect Matched Adaptive Radar (RAMAR)

signal, an extended target acts as a temporal compressor matrix, the target resonance reflections being almost simultaneously received and adding, if the target is at the identical orientation as in the A Priori Capture.

If the target is at a changed orientation, the spectral positioning of the resonance bands identified in a first capture do not change, but there can be changes in (a) the amplitude of the MRX resonances; and (b) the temporal compression (Figs. 3–5). For example, if the target (TGT) is position at an angle, e.g., 00°, or head-on, with respect to the TX-RX transceiver, or TGT(00), and the PRX and MRX received signals for this orientation are PRX(00) and MRX(00), a maximum compressed returned MRX is obtained for:

(1st capture) PTX(00) → TGT(00) → PRX(00),
(2nd capture) MTX(00) → TGT(00) → MRX(00).
Fig. 4 Here a target is arbitrarily represented with 3 wave packet subcomponent resonances. As the target changes its aspect angle, the 3 resonance RX signals arrive back at the transmitting platform at different times of arrival — upper figure. However, this aspect dependence is only reflected in the spectrum by differences in amplitude of the 3 spectral bands. Spectral band occupancy, but not band amplitude, is an aspect independent target signature.

However if the target aspect angle is changed to 90°, e.g., \( TGT(90) \), but the \( MTX(00) \) is transmitted, i.e., the situation is:

(1st capture) \( PTX(00) \rightarrow TGT(00) \rightarrow PRX(00) \),

(2nd capture) \( MTX(90) \rightarrow TGT(00) \rightarrow MRX(00/90) \).

In this latter case the \( MTX \) will still have addressed the target resonances, but (a) the amplitude of the resonances at target aspect angle 90° may have changed from those at target aspect angle 00°. Therefore, (b) the temporal compression will be less than optimum. Yet the \( MTX \) will still only have transmitted energy at frequencies matched to target resonances, and in many instances clutter resonances will still not have been excited.

As illustration of a (target) matched adaptive time-frequency packet-signal (MAP) system we consider 3 simplified model \( MTX \) signals using an assortment of 4 minor resonances, bandwidths, and differences in time
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A: Target as a Resonance Dispersion Matrix, and
B: After TX Equalization to Target: Correlator-Compressor.

A.

B. Precoding (Equalization)

Fig. 5 In the A Priori Capture (Fig. 1), the target linear distribution operator, \( L \), acts as a frequency dispersion matrix to the PTX according to \( \delta = L(t, f; s) \ast G(t, f; s) \).

In the A Posteriori Capture (Fig. 2), the target linear distribution operator, \( L \), acts as a correlator-compressor to this different input, according to \( G^{-1}(t, f; s) = L(t, f; s) \ast \delta(t, f; s) \). Here, \( PTX = \delta; PRX = G(t, f; s); MTX = G^{-1}(t, f; s); MRX = G(t, f; s)G^{-1}(t, f; s) = \delta(t, f; s) \), where, in all cases, the \( \delta \) function is loosely defined.

of MRX arrival at the receiver in the case of an extended target, and for 2 different target aspect angles: 00° (Head-on — Fig. 7A) and 90° (Side-on — Fig. 7B). In Fig. 7, the 3 model MTXs are shown in the left columns — top to bottom — and the MRXs — autocorrelations/selfconvolutions of the MTXs — in the right columns.

Time-frequency spectra for the MRXs (right columns) of Fig. 7 are shown in Fig. 8. Figure 8A shows the plots for the case of target aspect angle 00° (Head-on) and Fig. 8B for the case of target aspect angle 90° (Side-on). In Fig. 8A it can be seen that the time-frequency spectrum for MRX B-00 in which the four minor resonances are all at \( f = 75 \) Hz is
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1.3 GHz 99% BW Envelope on Miscellaneous Carriers

- UHF @ 650 MHz, Q = 0.5
- C Band @ 6 GHz, Q = 4.6
- X Band @ 10 GHz, Q = 7.7
- Ku Band @ 15 GHz, Q = 11.5
- Ka Band @ 33.5 GHz, Q = 25.8

Fig. 6 Target scattering can be a function of transmitted pulse envelope modulation and independent of the carrier. Here are shown 5 possible carriers for the same target TX/RX resonance bandwidth. If the channel medium between platform and target permits transmission without unacceptable penalties, the MTX can be matched to the target with envelope modulation, rather than carrier modulation. This has the important consequence that as the TX envelope matching is carrier independent, the size of the antenna can be small if the carrier chosen is of high frequency. In the example shown here, 5 possible antennas can be used, for 5 different carriers for matching the same TX envelope modulation to the same target (1.3 GHz, 99% bandwidth), with the difference in size between the Ka band and the UHF band antenna being of orders of magnitude.

Of most importance, the required $Q$ (center (carrier) frequency/bandwidth) increases as the center frequency increases: $Q = 0.5$ (UHF), 4.6 (C band), 7.7 (X band), 11.5 (Ku band), and 25.8 (Ka band).

spread in time. There is also time spread for MRX C-00 ($f_1 = 60; f_2 = 60; f_3 = 30; f_4 = 30$ Hz) and minimum spread for MRX A-00. In Fig. 8B it can be seen that all MRX minor resonances arrive at the receiver at approximately the same time and thus the time-frequency spectra show minimum spread in time ($f_1 = 100; f_2 = 75; f_3 = 50; f_4 = 25$). We see, therefore, that in the time domain the MRX’s show aspect dependence in the uniqueness of the spread in time of the individual packets constituting the total response of the target. We can also remark on the symmetrical nature of all MRX returns, and a typical MRX obtained in empirical testing is shown in Fig. 8C, agreeing with the model approach.

When the frequency marginals are calculated for Fig. 8, as shown in Fig. 9, target aspect independence is indicated. These model signals are illustrative. It should be cautioned that at different aspect angles the minor
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Aspect Angle: 00 deg (Head-on); $f = 100, 75, 50, 25$; $bw = 0.002$

$\text{MTX}$

$\text{MRX (AUTOCORRELATIONS)}$

Aspect Angle : 00

$\text{MRX}$ Signal A

MRX Signal B

MRX Signal C

Time (Normalized)

Fig. 7A (Continued)
resonance features of a target may be partially or completely hidden, in which case the amplitude of the target spectral bands will change or even be zero. Yet while the amplitude of a spectral band may vary with aspect, the resonance frequencies will not. With this proviso, we return to the scenario of the MTX-MRX spectra shown in Fig. 4, and the dynamic of MTX design based on the time reversal of a PRX providing a MRX that has a target-specific spectrum permitting the interpretation of the empirical results to be shown in succeeding chapters.

As indicated above, although in the frequency domain there is target aspect independent information in the MRX signal, in the time domain there is extended target aspect dependent information in the differences in time of arrival of the sequences of packets in the MTX. However, if information concerning target orientation is not required, then DTX and STX signals can be used of any temporal length.

In the case of DTXs and STXs information concerning target orientation is discarded and target orientation is considered of no consequence. The discarded information is in the time variance of the target’s frequency response with the target considered as a linear time variant (LTV) system and we now discuss further the distinction between an LTV system, and a linear time invariant (LTI) system.

Fig. 7A With reference to the TX/RX and target situation shown in Fig. 2, here the receiver-target aspect angle is at “Target Head-on” (00°) aspect angle. There are 3 hypothetical MTX signals shown on the right (top to bottom) that are matched to 3 targets. Each target has 4 subcomponent resonances. Each subcomponent is spread out along the target so that due to the “Target Head-on” aspect angle, the 4 returned MRX minor resonance signals from the target arrive back at the receiver at different times — Left 3 figures. The MTXs convolve with the targets providing the MRXs shown in the Right 3 figures. The details of the normalized frequencies of the minor resonances (f’s), normalized bandwidths (bw’s) and normalized time of arrival at the receiver differences (Δt’s) for the 3 hypothetical MRXs are:

(A - 00) \[ f_1 = 100; f_2 = 75; f_3 = 50; f_4 = 25; \]
\[ bw_1 = 0.002; bw_2 = 0.002; bw_3 = 0.002; bw_4 = 0.002; \]
\[ Δt_1 = 0.2; Δt_2 = 0.4; Δt_3 = 0.6; Δt_4 = 0.8; \]

(B - 00) \[ f_1 = 75; f_2 = 75; f_3 = 75; f_4 = 75; \]
\[ bw_1 = 0.002; bw_2 = 0.002; bw_3 = 0.002; bw_4 = 0.002; \]
\[ Δt_1 = 0.2; Δt_2 = 0.4; Δt_3 = 0.6; Δt_4 = 0.8; \]

(C - 00) \[ f_1 = 60; f_2 = 60; f_3 = 30; f_4 = 30; \]
\[ bw_1 = 0.008; bw_2 = 0.002; bw_3 = 0.006; bw_4 = 0.005; \]
\[ Δt_1 = 0.2; Δt_2 = 0.4; Δt_3 = 0.6; Δt_4 = 0.8; \]
Resonance and Aspect Matched Adaptive Radar (RAMAR)

**Fig. 7B (Continued)**
2. LTI versus LTV Systems

If we take a difference equation approach to a system’s filtering, then an \(i\)’th LTI system, is described by:

\[ y_i(n) = \alpha_0 x(n) + \alpha_1 x(n-1) + \alpha_2 x(n-2) + \cdots \]

where each \(\alpha_i\) refers to an RX wavepacket in the present target metaphor and where the coefficients \(\alpha\)’s can be real or complex, but are shift invariant. Shift invariance translates in the present target metaphor, to time of arrival invariance — which we do not see in empirical testing.

In contrast, a difference equation for an \(i\)’th LTV system is described by:

\[ u_i(n) = \beta_0(n)x(n) + \beta_1(n-1)x(n-1) + \beta_2(n-2)x(n-2) + \cdots \]

where the \(\beta\) coefficients are shift variant, which translates in the present target model, to aspect angle variance and time variance — which we in fact see in empirical testing. A mixed LTV-LTI system will then be:

\[ z(n) = \beta_0(n)y_0(n) + \beta_1(n-1)y_1(n-1) + \beta_2(n-2)y_2(n-2) + \cdots \]

This describes targets in which target minor features are \(y_i\) LTI systems, but the total composite target is a \(z\) LTV system. There is also the possibility

Fig. 7B  Here the receiver-target aspect angle is at “Target Side-on” (90°) aspect. As in Fig. 7A, 3 hypothetical MTX signals are shown on the right (top to bottom) that are matched to the same three 3 targets but at the “Target Side-on” aspect. Each target has the same 4 subcomponent resonances and each subcomponent is again spread out along the target but now due to the “Target Side-on” aspect angle, the 4 returned MRX minor resonance signals from the target arrive back at the receiver at the same time — Left 3 figures. The MTXs convolve with the targets providing the MRXs shown in the Right 3 figures. The details of the normalized frequencies of the minor resonances (\(f\)’s) — same as for Fig. 7A, normalized bandwidths (\(bw\)’s) — same as for Fig. 7A and normalized time of arrival at the receiver differences (\(\Delta t\)’s) — different from Fig. 7A — for the 3 hypothetical MRXs are:

\[
\begin{align*}
(A-90) & \quad f_1 = 100; f_2 = 75; f_3 = 50; f_4 = 25; \\
& \quad bw_1 = 0.002; bw_2 = 0.002; bw_3 = 0.002; bw_4 = 0.002; \\
& \quad \Delta t_1 = 0.5; \Delta t_2 = 0.5; \Delta t_3 = 0.5; \Delta t_4 = 0.5; \\
(B-90) & \quad f_1 = 75; f_2 = 75; f_3 = 75; f_4 = 75; \\
& \quad bw_1 = 0.002; bw_2 = 0.002; bw_3 = 0.002; bw_4 = 0.002; \\
& \quad \Delta t_1 = 0.5; \Delta t_2 = 0.5; \Delta t_3 = 0.5; \Delta t_4 = 0.5; \\
(C-90) & \quad f_1 = 60; f_2 = 60; f_3 = 30; f_4 = 30; \\
& \quad bw_1 = 0.008; bw_2 = 0.002; bw_3 = 0.006; bw_4 = 0.005; \\
& \quad \Delta t_1 = 0.5; \Delta t_2 = 0.5; \Delta t_3 = 0.5; \Delta t_4 = 0.5; 
\end{align*}
\]
Resonance and Aspect Matched Adaptive Radar (RAMAR)

Fig. 8A  Time-frequency spectra for target aspect angle 0° (Head-on) MRXs: A-00, B-00, C-00. Left column: using WH-0 wavelet scaling function (averaging) filter set. Right column: using WH-1 wavelet (differentiating) filter set. Noticeably, the time-frequency spectrum for MRX B-00 in which the four minor resonances are all at $f = 75$ Hz is spread in time. There is also time spread for MRX C-00 and minimum spread for MRX A-00.

of further mixtures, e.g.:

$$z_T(n) = \beta_0(n)u_0(n) + \beta_1(n-1)y_1(n-1) + \beta_2(n-2)z_2(n-2) + \cdots$$

where each component may be LTI or LTV and with subcomponents which also may be LTI or LTV. Because in the present instance any variance is due to changes in transceiver-target spatial orientation (aspect angle) that determine the relative time of arrival of RX packets at the receiver, if the aspect angle is held constant, or eliminated from consideration (by use of DTXs or STXs), these LTV or mixed LTV-LTI system will reduce to a conventional LTI system.

Now, the differences in MRX time of subcomponent signal arrivals shown in Figs. 7A & B and 8A, B & C for the model MTX signals A-00, B-00, C-00, A-90, B-90 and C-90, reflect targets that are LTV due to these differences in arrival time being a function of target aspect angle.
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Aspect Angle: 90 deg (Side-on); f = 100, 75, 50, 25; BW = 0.002, 0.002, 0.002, 0.002; t = 0.5, 0.5, 0.5, 0.5; WH0 Hz

Aspect Angle: 90 deg (Side-on); f = 75, 75, 75, 75; BW = 0.002, 0.002, 0.002, 0.002; t = 0.5, 0.5, 0.5, 0.5; WH0 Hz

Aspect Angle: 90 deg (Side-on); f = 60, 60, 30, 30; BW = 0.008, 0.002, 0.006, 0.005; t = 0.5, 0.5, 0.5, 0.5; WH0 Hz

**Wavelet filter:** WH-0 (averager)  
**Wavelet filter:** WH-1 (differentiator)

Fig. 8B Time-frequency spectra for target aspect angle 90° (Side-on). *MRXs: A-90, B-90, C-90. Left column: using WH-0 wavelet scaling function (averaging) filter set. Right column: using WH-1 wavelet (differentiating) filter set. As to be expected, with target aspect angle 90° all MRX minor resonances arrive at the receiver at approximately the same time and thus the time-frequency spectra show minimum spread in time.

Equally, the differences in arrival time are a function of the spatial distance of each resonating target subcomponent from the receiver. Changes in relative spatial transceiver-target orientation produce changes in relative time of minor signal arrival intervals, i.e., in the β’s, but not the α’s. These differences are evident in the time domain, but not evident in the frequency domain. Thus they are important when detection of target aspect angle is required and the radar is in an MTX-MRX short pulse mode. However, when knowledge of target aspect angle is of no importance and the radar is in a DTX-DRX or STX-SRX mode, the target can be treated as a conventional LTI system.

However, in the MTX-MRX short pulse mode the distinction between an LTV and an LTI system has consequences. A variable network is defined as one in which one or more element-values are dependent in a specified way upon a combination of three variables: time, input and output. Whereas in LTI systems a transfer or system function is defined as the Fourier transform
of the response to a unit impulse and hence is a function of frequency and phase but independent of time, in the case of LTV systems a transfer or system function is defined as a function of frequency and phase with time as a parameter (Zadeh, 1950a, b). In the limiting case of time independence (which, in the present instance is aspect angle independence), an LTV transfer function reduces to that of a LTI transfer function and, with a specific time defined, a LTV transfer function is an instantaneous transfer function. We turn now to the relation of matched filtering and LTV systems.

With a single channel time series, a matched filter is conventionally derived to maximize the output signal-to-noise ratio (SNR) at a specified time (Trees, 1968; Papoulis, 1984). In the case of beam-forming or multichannel signal processing, maximizing the SNR results in the minimum mean-square error beam former (Trees, 2002). In a further development, Chambers et al. (2004) showed that for multiple radiators identified as a complex Green’s function and with a multiple set of excitations identified as a filter set, an SNR is defined as a function of that Green’s function, and the matched filter is the integral over time of the product of the Green’s function and its complex conjugate. Moreover, the complex Green’s function obtained by transmitting a pulse at time $t_0$ and time reversing the result
**Introduction**

Wavelet filter: WH-0 (averager)

**MRXs Overlaid Frequency Marginals: Aspect Angles : 00 & 90**

Fig. 9A Overlap of the marginal spectra for the left columns (using averager scaling function WH-0) of the time-frequency plots of Fig. 7A (target aspect angle 00°) and of Fig. 7B (target aspect angle 90°) indicating MRX aspect independence in the frequency domain.

(i.e., to obtain MTX = PRX\(^{-1}\)) is equal to the complex conjugate of the complex Green’s function at a later time (Kuperman et al., 1998). In other words, in the time domain time, time reversal and matched filtering are equivalent for LTV systems. In the frequency domain, time reversal is phase conjugation and there is also a similar equivalence to matched filtering.

It is well known that LTV matched filters are used in the detection of signals in dispersive media and adaptive equalizers are used in wireless communications to cancel channel characteristics that vary over time. There are also many techniques for dealing with the response of LTI systems. However, these LTI techniques are not strictly valid for LTV systems. In contrast, the discrete wavelet transform is valid for LTV systems, because it is naturally time variant due to the use of the decimation operation, which permits the local (as opposed to global) time-frequency analysis of signals and systems. With this local wavelet analysis approach an input signal is expanded into a linear combination of orthogonal basis signals, and an LTV filter is constructed by replacing each basis signal with a new basis signal. Other methods involve the matching pursuit algorithm (Moll & Fritzen, 2010), or a time-varying autoregressive process, to construct an LTV filter. In the empirically testing examined later, the target’s impulse
response (PRX) at a known aspect angle is obtained empirically, and the matched signal (MTX) is constructed from the time reversal of that impulse response.

Just as the concept of LTI transfer functions can be extended to LTV, a Green’s function and a time-reversal mirror concept can be extended from time invariance to time variance (Chambers et al., 2004), producing an optimal matched filter (Tanter et al., 2000). The steps demonstrating this extension (from LTI to LTV systems) are as follows, and we generalize them here to MAP radar systems.

Fink and Dorme showed that time reversal of fields in an array added coherently at the focus of the array (Fink, 1992; Dorme & Fink, 1995). If \( G(p_1, t_1; p_2, t_2) \) is the Green’s function describing the target impulse response with the transmitter at position \( p_1 \), the target at position, \( p_2 \), signal transmission to target at time \( t_1 \) and signal reception from target at time \( t_2 \), then time invariance implies \( G(p_1, t_1; p_2, t_2) = G(p_1, t_1 - \tau; p_2, t_2 - \tau) \) and reciprocity implies \( G(p_1, t_1; p_2, t_2)^{-1} = G(p_2, t_2; p_1, t_1) \) — or, in the present instance, \( \text{PRX}^{-1} = \text{MRX} \).
Introduction

The general observation is that the target response is represented by:

\[ RX(t) = \sum_{n=1}^{N} \int_{0}^{t} h_{n}(t-t') TX_{n}(t') dt', \]

where \( \{ TX_{n}(t): n = 1, 2, \ldots, N \} \) is the set of input signals (e.g., subcomponents of PTX, MTX), and \( \{ h_{n}(t): n = 1, 2, \ldots, N \} \) is the set of target filters producing \( n \) minor and major target resonances.

The MRX time response is then:

\[ MRX = G * G^{-1} \]

or

\[ MRX = PRX * PRX^{-1} = PRX * MTX \]

where: \( G = h_{n}(t-t') = PRX \) and \( G^{-1} = PRX^{-1} = MTX \) as in Figs. 1 and 2, above.

Setting constant the inter-packet time of arrival influences (due to aspect angle influences), if the time reversed signal PRX\(^{-1} = MTX\) is associated with the complex Green’s function \( G(TX_{n}, t; RX_{n}, t') \) where \( RX_{n} \) specifies the return signal from a local resonator due to a target subcomponent, \( n \), and \( TX_{n} \) specifies a transmitted signal matched to a local resonance \( n \), in the case of the spatio-temporal matched filter for an LTI system the SNR is maximum when:

\[
SNR(TX_{s}, t_{s}) = \frac{\left( \sum_{n=1}^{N} \int_{0}^{t_{s}} |G(TX_{s}, t_{s}; RX_{n}, t)|^2 dt \right) \left( \sum_{n=1}^{N} \int_{0}^{t_{s}} |G^*(TX_{s}, t_{s}; RX_{n}, t)|^2 dt \right)}{\sum_{n=1}^{N} \int_{0}^{t_{s}} |G^*(TX_{s}, t_{s}; RX_{n}, t)|^2 dt} = \sum_{n=1}^{N} \int_{0}^{t_{s}} |G(TX_{s}, t_{s}; RX_{n}, t)|^2 dt
\]

which shows that the SNR is the squared ratio of the energy of a specific transmitted signal \( TX_{s} \) at a specific time \( t_{s} \) to the energy in the total received signal \( RX \).

For an LTI system, the MRX or return signal from a target in response to a matched transmitted signal, MTX, is:

\[ RX_{MF}(t) = \sum_{n=1}^{N} \int_{0}^{t} G(TX_{s}, t; RX_{n}, t') G^*(TX_{s}, t_{s}; RX_{n}, t') dt' \]
In the case of a time reversal mirror and an LTV system, and for the case of (i) an impulse being transmitted at time \( t = 0 \), (ii) the target response being received, time reversed, and (iii) retransmitted, the returned signal, MRX, is:

\[
RX_t = \sum_{n=1}^{N} \int_{0}^{t} G(TX_s, t; RX_n, t') \times G(RX_n, t_s - t'; TX_s, 0) dt', \quad 0 < t < t_s.
\]

Moreover, if the Green’s function satisfies:

\[
G(RX_n, t_s - t; TX_s, 0) = G^*(TX_s, t_s; RX_n, t), \quad 0 < t < t_s,
\]

i.e., if the time reversed response of a minor resonance to a specific transmitted signal at a specific time (aspect angle) is equal to the complex conjugate of the impulse response of that minor resonance for that specific time (aspect angle), then under these conditions \( RX_t = RX_{MF}(t) = MRX \) and the LTV response is also a matched filter response.

Thus the difference between an LTV and LTI system is in the constancy of specific TX times, \( t_s \) (of LTI systems) relating to the minor resonances and that are a function of the target aspect angle or relative times of arrival of extended target wave packets. Elimination of \( t_s \) by constancy of time, i.e., constancy of aspect angle, reduces total target LTV condition to that of a minor resonance featured LTI system. Furthermore,

1. It is well known that the SNR is maximized for an RX signal, when the impulse response of the optimum filter (i.e., in this instance, the target) is a reversed and a time delayed copy of the TX signal. But more can be said due to the self-convolution operation being commutative. In terms of, e.g., a matrix representation, the TX signal and the filter (represented by the target) can switch places. Therefore,

2. With the switch made, we can state that the SNR is maximized for an RX signal, when the TX signal (as opposed to the filter) is a reversed and a time delayed copy of the impulse response of the filter (as opposed to the TX signal).

This book addresses the condition (2).

3. **Signal Envelope Match vs Carrier Match**

A major observation in empirical testing described later is that the matching of a TX pulse to the target’s transfer characteristics need not be
a match of the carrier of the signal to the characteristics, but can also be a
match of only the amplitude modulation of a carrier — provided that the
channel medium through which the transmit signal passes is transparent to
the carrier (Fig. 6). The restriction to carrier transparent channel conditions
in this instance is necessary because of the need of, e.g., foliage penetrating
radars, which require that the carrier penetrate foliage, and ground/wall
penetrating radars, which require that the carrier penetrate ground/walls.
Both these cases require both carrier and envelope to be at medium-
penetrating frequencies. But even in these cases, if a medium-penetrating
carrier is chosen, the transmitted signal match to a designated target-and-
medium need only be required of the carrier envelope, and the carrier need
only be medium-penetrating.

This circumstance has major system benefits in that the size of
the transmit-receive antenna is determined by the carrier and less by
its bandwidth set by the carrier’s envelope modulation, as antenna size
scales with wavelength. Thus, absent medium-penetration requirements,
the transmit-receive antenna can be at a high frequency and of smaller
size — as it is for the Ka-band prototype results reported here — while
the target-matched envelope modulation of the carrier can be at much
lower frequencies, as opposed to transmittance of those same envelope lower
frequencies without a carrier, this latter configuration requiring a large
antenna. The advantages of this circumstance for antenna size reduction
are therefore considerable (see Fig. 6).

4. Target Modeling and Identification by Coherence
Functions

Addressing resonance (Mie) scattering with MAP captures provides tar-
get aspect-independent information in the frequency domain, and target
aspect-dependent information in the time domain. A system designed to
capture and exploit this information is a resonance- and aspect-matched
adaptive radar (RAMAR). A RAMAR addresses target/subcomponent
resonances and therefore offers identification of targets by RX resonance
spectral profiles. Taking resonance spectral profile identification farther:
target identification can be based on the degree of coherence-match to a
target’s transfer function treated as LTV, and there are multiple methods
that model a linear target transfer function. One such method is coherence
function profiling, which we describe here.

Coherence is a measure of the consistency of phase relations between
two time series (Marple, 1987). For purposes of exposition, a model target
Resonance and Aspect Matched Adaptive Radar (RAMAR)

Fig. 10A The frequency responses of a synthetic target (composed of FIR filters), with resonances (passbands) at \( f_c = 310, 525 & 650 \text{ MHz} \) and \( BW = 20 \text{ MHz} \) for each band. \( F_s = 2 \text{ GHz} \).

can be created that has, e.g., three resonances — at 310, 525 & 650 MHz — and whose filter frequency response and impulse response autocorrelation are shown in Figs. 10A & B. Suppose now that DTX signals (i.e., single frequency or constant wavelength (cw) signals) compose the following two STX bundles:

1. An “on-resonance” STX composed of DTX frequencies at the target resonances 310, 525 & 650 MHz (Fig. 11A).
2. An “off-resonance” STX composed of DTX frequencies 290, 545 & 630 MHz (Fig. 11B).

Then the SRXs elicited by either of these STX long duration bundles of DTX signals are obtained by convolving the STXs with the target’s transfer function (Fig. 12). The DTXs, together with the SRXs provide the “on-resonance” and “off-resonance” cases, represented by matrices, both composed of 3 DTXs (i.e., three DTX frequencies = the STX bundle) and 1 SRX. These 2 matrices are each composed of 3 system inputs and 1 output.

Coherence functions are defined by (Thomson, 1982; Xu et al., 1999):

\[
\gamma_{xy}^2(f) = \frac{\left| \sum_{k=0}^{K-1} x_k(f) y_k(f) \right|^2}{\left| \sum_{k=0}^{K-1} x_k(f) \right|^2 \left| \sum_{k=0}^{K-1} y_k(f) \right|^2}
\]
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**Fig. 10B** The autocorrelation of the impulse response of the synthetic target, with resonances (passbands) at 310, 525 & 650 MHz and BW = 20 MHz for each band; $F_s = 2$ GHz.

**Fig. 11** The “On resonance” STX composed of DTXs at 310, 525 & 650 MHz and the “Off resonance” STX composed of DTXs at 290, 545 & 630 MHz. Both 2000 data points in duration and $F_s = 2e9$. 
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where the $x$’s and the $y$’s are power spectra obtained by taking the Fourier transform of the autocorrelation, and $k$ ranges from 1 to 4, the first 3 being the STX’s and the 4th being the DRX.

The results of coherence function measurements for the on-resonance and off-resonance model cases are shown in Figs. 13–16. Examination of these figures shows that the coherence function method, in principle, is a promising approach to MAP target identification.

In Fig. 13 are shown coherence functions for SRX frequencies and target resonances, with target resonances at 310, 525 & 650 MHz; STX (on resonance) frequencies at 310, 525 & 650 MHz; and DTX (off resonance) frequencies at 290, 545 & 630 MHz. In (A) $\gamma_{12}^{\text{on resonance}} = 310$ MHz (on resonance), and in (B) $\gamma_{14}^{\text{off resonance}} = 290$ MHz (off resonance).

Notice in (A) that $\gamma_{12}^{\text{on resonance}}$, $\gamma_{13}^{\text{on resonance}}$, and $\gamma_{14}^{\text{on resonance}}$ are minimum at all frequencies (lack of coherence), as expected. $\gamma_{14}^{\text{on resonance}}$ is high at 310 MHz (maximum coherence) and minimum at 525 & 650 MHz (lack of coherence), also as expected.

Notice in (B) that $\gamma_{12}^{\text{off resonance}}$, $\gamma_{13}^{\text{off resonance}}$, and $\gamma_{14}^{\text{off resonance}}$ are all minimum (lack of coherence) and broad band, as expected.

Fig. 12 The SRXs obtained by convolving the on-resonance and off-resonance STXs (Figs. 11A & 11B) with the target’s frequency response (Fig. 10A).

Fig. 13 Coherence Functions for DRX frequencies and target resonances. Target resonances: 310, 525 & 650 MHz; STX (on resonance) frequencies: 310, 525 & 650 MHz; DTX (off resonance) frequencies: 290, 545 & 630 MHz. In (A) $\gamma_{12}^{\text{on resonance}} = 310$ MHz (on resonance), and in (B) $\gamma_{14}^{\text{off resonance}} = 290$ MHz (off resonance). Notice in (A) that $\gamma_{12}^{\text{on resonance}}$ and $\gamma_{13}^{\text{on resonance}}$ are minimum at all frequencies (lack of coherence), as expected. $\gamma_{14}^{\text{on resonance}}$ is high at 310 MHz (maximum coherence) and minimum at 525 & 650 MHz (lack of coherence), also as expected. Notice in (B) that $\gamma_{12}^{\text{off resonance}}$, $\gamma_{13}^{\text{off resonance}}$, and $\gamma_{14}^{\text{off resonance}}$ are all minimum (lack of coherence) and broad band, as expected.
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Coherence Function: TX On Resonance: \( f_1 = 310e6; f_2 = 525e6; f_3 = 650e6 \)

Coherence Function: TX Off Resonance: \( f_1 = 290e6; f_2 = 545e6; f_3 = 630e6 \)

Fig. 13
Coherence Function: TX On Resonance: $f_1 = 310e6$; $f_2 = 525e6$; $f_3 = 650e6$

(A)

Coherence Function: TX Off Resonance: $f_1 = 290e6$; $f_2 = 545e6$; $f_3 = 630e6$

(B)

Fig. 14 (Continued)
resonance), and in (B) $\text{DTX}_1 = 290$ MHz (off resonance). It can be seen that in (A) that $\gamma_{12}^2$ & $\gamma_{13}^2$ are minimum at all frequencies (lack of coherence), as expected, but $\gamma_{14}^2$ is high at 310 MHz (maximum coherence) and minimum at 525 & 650 MHz (lack of coherence), also as expected. It can be seen in (B) that $\gamma_{12}^2$, $\gamma_{13}^2$ & $\gamma_{14}^2$ are all minimum (lack of coherence) and broad band, as expected.

In Fig. 14 (A) $\text{DTX}_2 = 525$ MHz (on resonance), and in (B) $\text{DTX}_2 = 545$ MHz (off resonance). It can be seen that in (A) that $\gamma_{21}^2$ & $\gamma_{23}^2$ are minimum at all frequencies (lack of coherence), as expected, but $\gamma_{24}^2$ is high at 525 MHz (maximum coherence) and minimum at 310 & 650 MHz (lack of coherence), also as expected. It can be seen in (B) that $\gamma_{21}^2$, $\gamma_{23}^2$ & $\gamma_{24}^2$ are all minimum (lack of coherence) and broad band, as expected.

In Fig. 15 (A) $\text{DTX}_3 = 650$ MHz (on resonance), and in (B) $\text{DTX}_3 = 630$ MHz (off resonance). It can be seen in (A) that $\gamma_{31}^2$ & $\gamma_{32}^2$ are minimum at all frequencies (lack of coherence), as expected, but $\gamma_{34}^2$ is high at 650 MHz (maximum coherence) and minimum at 310 & 525 MHz (lack of coherence), also as expected. It can be seen in (B) that $\gamma_{31}^2$, $\gamma_{32}^2$ & $\gamma_{34}^2$ are all minimum (lack of coherence) and broad band, as expected.

In Fig. 16 in the case of both (A) (on resonance), and (B) (off resonance), $\text{SRX} = 310, 525 & 650$ MHz. It can be seen in (A) that $\gamma_{41}^2$ is minimum at 525 & 650 MHz (lack of coherence), $\gamma_{42}^2$ is minimum at 310 & 650 MHz (lack of coherence), $\gamma_{43}^2$ is minimum at 310 & 525 MHz (lack of coherence), but $\gamma_{44}^2$ is at all frequencies (maximum coherence) as expected; It can be seen in (B) that $\gamma_{41}^2$, $\gamma_{42}^2$ & $\gamma_{43}^2$ are all minimum at 310, 525 & 650 MHz (lack of coherence), but $\gamma_{41}^2$ is at all frequencies (maximum coherence) as expected.

Given an LTV target transfer function, this exercise indicates that target identification can be based quite accurately on the degree of coherence-match.

Fig. 14 Coherence Functions for SRX frequencies and target resonances.

Target resonances: 310, 525 & 650 MHz;

DTX (on resonance) frequencies: 310, 525 & 650 MHz;

DTX (off resonance) frequencies: 290, 545 & 630 MHz.

In (A) $\text{DTX}_2 = 525$ MHz (on resonance), and in (B) $\text{DTX}_2 = 545$ MHz (off resonance).

Notice in (A) that $\gamma_{21}^2$ & $\gamma_{23}^2$ are minimum at all frequencies (lack of coherence), as expected, but $\gamma_{24}^2$ is high at 525 MHz (maximum coherence) and minimum at 310 & 650 MHz (lack of coherence), also as expected.

Notice in (B) that $\gamma_{21}^2$, $\gamma_{23}^2$ & $\gamma_{24}^2$ are all minimum (lack of coherence) and broad band, as expected.
Resonance and Aspect Matched Adaptive Radar (RAMAR)

Coherence Function: TX On Resonance: $f_1 = 310e6; f_2 = 525e6; f_3 = 650e6$

Fig. 15 (Continued)
5. The WH Transform & WHWFs

As the appropriate analysis method for both LTI and LTV system signals is time-frequency analysis, a local wavelet decomposition of RX signals was used to analyze test results. The specific wavelet analysis method used provides a solution to the well-known “energy confinement problem”, or time-bandwidth product limiting problem, addressed by Slepian and collaborators (Landau & Pollak, 1961, 1962; Slepian, 1964, 1978; Slepian & Pollak, 1961). We provide a short introduction here, and an expanded version in the following chapters.

Slepian et al.’s solution to the energy confinement problem was the prolate spheroidal wavefunction (PSWF) series, that can be recursively, but not analytically, generated. An alternative solution is the parabolic cylinder or Weber-Hermite wave function (WHWF) series (Barrett, 1972, 1973a,b), that are analytic.

The WHWFs are related to Weber’s equation (Weber, 1869):
\[
\frac{d^2\psi_n(x)}{dx^2} + \left(n + \frac{1}{2} - \frac{1}{4}x^2\right)\psi_n(x) = 0,
\]
for which the general Weber equation, or parabolic cylinder differential equation, is (Abramowitz & Stegun, 1972):
\[
\frac{d^2\psi_n(x)}{dx^2} + (ax^2 + bx + c)\psi_n(x) = 0,
\]
The solutions of this equation are the parabolic cylinder or Weber-Hermite wave functions (WHWFs):
\[
\psi_n(x) = 2^{-n/2} e^{-x^2/4} H_n(x/\sqrt{2}), \quad n = 0, 1, 2, \ldots,
\]
where the $H_n$ are Hermite polynomials. When $n$ is an integer, the Weber-Hermite functions become proportional to the Hermite polynomials. Other

---

Fig. 15  Coherence Functions for SRX frequencies and target resonances.
Target resonances: 310, 525 & 650 MHz;
DTX (on resonance) frequencies: 310, 525 & 650 MHz;
DTX (off resonance) frequencies: 290, 545 & 630 MHz.
In (A) DTX$_3 = 650$ MHz (on resonance), and In (B) DTX$_3 = 630$ MHz (off resonance).
Notice in (A) that $\gamma_{31}^2$ & $\gamma_{32}^2$ are minimum at all frequencies (lack of coherence), as expected, but $\gamma_{34}^2$ is high at 650 MHz (maximum coherence) and minimum at 310 & 525 MHz (lack of coherence), also as expected.
Notice in (B) that $\gamma_{31}^2$, $\gamma_{32}^2$ & $\gamma_{34}^2$ are all minimum (lack of coherence) and broad band, as expected.
Resonance and Aspect Matched Adaptive Radar (RAMAR)

Coherence Function: TX On Resonance: \( f_1 = 310\text{e}^6; f_2 = 525\text{e}^6; f_3 = 650\text{e}^6 \)

Coherence Function: TX Off Resonance: \( f_1 = 290\text{e}^6; f_2 = 545\text{e}^6; f_3 = 630\text{e}^6 \)

Fig. 16 (Continued)
names are used for the Weber-Hermite wave functions, e.g., Hermite-Gaussian functions. I prefer the designation Weber-Hermite wave function because (a) Hermite-Gaussian implicates Gaussian in all polynomials, $n = 0, 1, 2, \ldots$ but the Gaussian is only just the first, for the case $n = 0$; (b) Weber’s equation is more general than Hermite’s equation; (c) the name “Weber-Hermite” follows the *Mathematical Encyclopedia* (Hazewinkel, 2002), usage; and (d) other texts, e.g., (Morse & Feshbach, 1953, vol. 2, p. 1642; Jones, 1964, p. 86), have also used the name “Weber-Hermite”.

The WHWFs are given a physical representation as follows. The one-dimensional wave equation is:

$$-\frac{1}{2m} \frac{\partial^2 \psi}{\partial x^2} + V(x) \psi = E \psi$$

with spring potential:

$$V(x) = \frac{1}{2} k x^2 = \frac{1}{2} m \omega^2 x^2,$$

where $\omega = \sqrt{\frac{k}{m}}$ is the angular frequency, $k$ is the stiffness constant, $m$ is the mass, and $x$ is the field deflection of the oscillator. The wave equation can be written in dimensionless form by defining the independent variables $\xi = \alpha x$ and an eigenvalue, $\lambda$, and requiring:

$$\alpha^4 = mk, \quad \lambda = 2E \left( \frac{m}{k} \right)^{1/2} = \frac{2E}{\omega}.$$ 

The dimensionless form is then:

$$\frac{\partial^2 \psi}{\partial \xi^2} + (\lambda - \xi^2) \psi = 0.$$

---

**Fig. 16  Coherence Functions for SRX frequencies and target resonances.**

Target resonances: 310, 525 & 650 MHz;

DTX (on resonance) frequencies: 310, 525 & 650 MHz;

DTX (off resonance) frequencies: 290, 545 & 630 MHz.

For both (A) (on resonance), and (B) (off resonance), SRX = 310, 525 & 650 MHz.

Notice in (A) that $\gamma_{41}^2$ is minimum at 525 & 650 MHz (lack of coherence), $\gamma_{42}^2$ is minimum at 310 & 650 MHz (lack of coherence), $\gamma_{43}^2$ is minimum at 310 & 525 MHz (lack of coherence), but $\gamma_{44}^2$ is at all frequencies (maximum coherence) as expected;

Notice in (B) that $\gamma_{41}^2, \gamma_{42}^2 & \gamma_{43}^2$ are all minimum at 310, 525 & 650 MHz (lack of coherence), but $\gamma_{44}^2$ is high at all frequencies (maximum coherence) as expected.
which is also a form of Weber’s equation and permits solutions as a function of $n = \frac{E}{\beta} - \frac{1}{2}$. In order for the solutions to be quadratically integrable, it is necessary that $n$ take on integer values: $n = 0, 1, 2, \ldots$ (Morse & Feshbach, 1953, p. 1641). With normalization factors, the solutions are the Weber–Hermite or parabolic cylinder functions:

$$
\psi_n(t) = \frac{1}{\sqrt{2^n n!}} \left( \frac{\alpha}{\pi} \right)^{1/4} \exp \left( -\frac{\alpha t^2}{2} \right) H_n(t\sqrt{\alpha}),
$$

where $\alpha = m\omega$ and the $H_n$ are Hermite polynomials, and $\alpha$ is a time-frequency trade parameter/variable.

If in the case of a function, $f(x)$, an expansion of the form:

$$
f(x) = \alpha_0 \psi_0(x) + \alpha_1 \psi_1(x) + \cdots + \alpha_n \psi_n(x) + \cdots
$$

exists, and if it is legitimate to integrate term-by-term between the limits $-\infty$ and $+\infty$, then:

$$
a_n = \frac{1}{\sqrt{(2\pi)^{1/2} n!}} \int_{-\infty}^{+\infty} \psi_n(t)f(t)\,dt.
$$

The first 6 WHWFs (WHWF 0-5) are shown in the time and frequency domains in Fig. 17. It is apparent that the WHWFs increase in (temporal

![Fig. 17](image-url) The first 6 WHWFs (upper) and their Log Magnitude Spectra (lower) labeled according to an increasing $n$ ($n = 0, 1, 2, 3, 4, 5$). As $n$ increases, the temporal length and bandwidth increase, i.e., time-bandwidth product increases.
If the complex WH matrix is designated, \( W \), then, as \( W \) is a unitary matrix, \( WW^\dagger = I \), where \( W^\dagger \) is the conjugate transpose (Hermitian adjoint) of \( W \), and \( I \) is the identity matrix shown in B. The inverse of \( W \), or \( W^{-1} \), is equal to the conjugate transpose: \( W^{-1} = W^\dagger \). C and D are exploded views of the first 20 signals in A.

length \( \times \) bandwidth), i.e., in time-bandwidth product, as \( n \) increases (Fig. 18). In a manner similar to that of the Fourier transform, a WH transform can be constructed by matrix methods. Figure 18A shows a \( 128 \times 256 \) WH (magnitude) matrix. If the complex WH matrix is designated, \( W \), then, as \( W \) is a unitary matrix, \( WW^\dagger = I \), where \( W^\dagger \) is the conjugate transpose (Hermitian adjoint) of \( W \), and \( I \) is the identity matrix shown in 18B. The inverse of \( W \), or \( W^{-1} \), is equal to the conjugate transpose: \( W^{-1} = W^\dagger \).
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Fig. 19  
(A) A: a time domain averaged MRX from a target barrel in upright position. 
(B) B: multiple window time-frequency analysis (MWTFA) spectrum of the MRX using 8 WHWFs: WH0:WH7 to obtain 8 time-frequency spectra that were then averaged.
Using WHWFs it is possible to take a time domain signal, e.g., an MRX signal elicited from a barrel target (Fig. 19A), and calculate a multiple-window time-frequency analysis (MWTFA) spectrum (Fig. 19B). The MWTFA is a WHWF expression of Thomson’s multiple window method (Thomson, 1982), that is a time-frequency distribution estimator for a random process and is a substitute for a Fourier transform periodogram — an inappropriate estimator for short, time-limited signals. Thomson’s approach to spectral estimation of such signals is to compute several periodograms using a set of orthogonal windows that are locally concentrated in frequency and then averaged (Xu et al., 1999). Optimal windows are the PSWFs, but WHWFs are used in the following analyses, being a good approximation, and whereas the WHWFs are analytic, the PSWFs are not.

6. Treatment of Nonstationary Signals

The term “non-stationary” is used in multiple senses. Here we use it in the sense shown in Fig. 20. In the case of short, time-limited signals, accurate averaging requires that those signals be aligned in time as shown on the left of this figure. Jitter destroys alignment. If similar signals are not aligned, as shown on the right, their “average” will not be a good estimator.

As observed, the terms “stationary” and “non-stationary” are used in multiple senses. In the statistics literature, all signals shown here are

![Fig. 20](image-url) 

Collections of “stationary” and “non-stationary” signals that require averaging. The way these terms are used is described below. The stationary signals on the left can be summed and averaged as they are aligned in time. The jittered non-stationary signals on the right are not aligned and cannot be accurately averaged.
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Fig. 21 (Continued)
considered non-stationary because they are not periodic. The statistics literature usually assumes that the signal commencement, e.g., at time $t_0$, is the same for all signals. However, in the engineering literature the assumption is that the signal commencement may not be the same for all signals, and many times is not. As it is mostly the case that the distance between the transceiver and a target is not constant, radar signals should be considered non-stationary. In this engineering sense, the left column of signals is stationary, and the right column is non-stationary, and this is the case also from a receiver’s point-of-view. We shall refer to “signals subjected to jitter”, when referring to non-stationary signals in the engineering sense, and as exemplified in the right column. Time averaging of the first column of signals results in a valid temporal average. Attempts at time “averaging” of the second column of signals would result in a spurious average.

If the reception of a series of short, time-limited signals are subject to jitter and are non-stationary in the sense we have declared the use of the term, taking their time average is difficult. To circumvent this problem, and also to provide insight into the spectral characteristics of both an RX’s carrier and envelope, we introduced a new double frequency spectrum described in the next section.

7. Carrier Frequency-Envelope Frequency (CFEF) Spectra

If return signal (RX) information in the time domain can be neglected, then the following frequency-frequency or Carrier Frequency-Envelope Frequency (CFEF) spectrum can be used (Fig. 21). The CCEF is obtained

Fig. 21 A: multiple window time-frequency analysis (MWTFA) spectrum of the Barrel UP target MRX using 8 WHWFs: WH0-WH7 to obtain 8 time-frequency spectra that were then averaged. This is Fig. 19B. B: Carrier Frequency-Envelope Frequency (CFEF) spectrum. This spectrum is composed of the magnitudes of the Fourier transforms of the cuts at each frequency through the time-frequency spectrum shown in Figure A. The spectra shown indicate both the spectra of the MRX carrier frequencies as well as of the modulating envelopes of those carriers. It can be seen that although there are peaks in these spectra at approximately the carrier frequencies — by reading across to the y-axis — that indicate short, almost monocycle, bursts at the appropriate frequency, by reading down to the x-axis it can be seen that the frequencies of the envelopes modulating the carriers are also indicated. On the far left a broad band carrier signal is indicated that is modulated by a low frequency envelope. Any long vertical band indicates a broad band spike burst. In the middle, there are “islands” indicating short duration envelopes modulating “carrier frequencies”, i.e., wave packets. There are also wave packets located at approximately the same frequency on the y- and x-axes indicating wave packet envelope modulations of the “carrier frequency” components of the RX signal at the same frequency as those “carrier frequency” components — a double modulation involving both a “carrier” and envelope at the same frequency. See also Section 2.2.9, below.
by the Fourier transformation of each line in a MWTFA (Fig. 19B). The CFEF \( y \)-axis is identical to the MWFTA \( y \)-axis, but the CFEF \( x \)-axis is also a frequency axis and the influence of temporal jitter has been removed. To obtain an averaged signal, multiple CFEFs are averaged and the jitter is not an influence on the average. The average is obtained for a penalty of loss time of RX arrival information.

The CFEF method provides detailed spectral information concerning a target return. As shown in Fig. 21B, information concerning a signal’s “envelope” frequencies, as well as “carrier” frequencies is revealed.

8. Polarization

The polarization and polarization changes of RX signals elicited by differently polarized TX signals constitute important information that can be exploited in a number of ways, but will not be addressed in this book. This subject is complex and requires a treatment that itself would be book-length.

In anechoic chamber tests (Section 2.2.0, below) the RXs elicited by two different orthogonally polarized TX signals were collected, but no difference in these RX signals was detected. For the targets tested, TX-RX polarization was not a discriminating variable.