1
Introduction

1.1 Pairing in nuclei, superconductors, liquid ^{3}He and neutrons stars

If one sweeps a magnetic field through a metallic ring (e.g. a ring made out of lead) immersed in liquid helium ($T \sim 4$ K) it induces a current which does not show any measurable decrease for a year, and a lower bound of 10^5 years for its characteristic decay time has been established using nuclear resonance to detect any slight decrease in the field produced by the circulating current (File and Mills (1963)). If a torus-shaped vessel filled with liquid helium below the critical temperature $T_c = 2.17$ K (known as He II) and packed with porous material, which provides very narrow capillary channels, is rotated around its axis of symmetry and then brought to rest, the liquid continues to flow (Reppy and Depatie (1964)), showing no reduction in the angular velocity over a twelve-hour period, and indicating that He II can flow without dissipation. Using an adiabatic cooling apparatus, Osheroff *et al.* (1972 a,b) found two anomalies in the pressure–time curve of liquid ^{3}He, when the volume was changed at a constant rate. At the critical temperature $T_c = 2.7$ mK the slope of the curve suffered a discontinuity, and at about $T_c = 1.8$ mK there was a singularity involving hysteresis (see also Osheroff (1997) and Lee (1997)). If a deformed nucleus in its ground state is set into a state of rotation by the action of a non-uniform, time-dependent Coulomb field, it displays rotational bands with a moment of inertia which is a fraction (between one-half to one-third) of the rigid moment of inertia (Belyaev (1959), Bohr and Mottelson (1975)). Rotating neutron stars (pulsars) display marked glitches, that is, sudden increases in the frequency of the emitted pulses of radiation (McKenna and Lyne (1990), McCullough *et al.* (1990), Flanagan (1990), Anderson *et al.* (1982)). All the above observations are examples of phenomena known as superconductivity and superfluidity.

From a microscopic point of view, helium atoms are structureless spherical particles interacting via a two-body potential. The attractive part of this potential, arising from weak Van der Waals-type dipole, quadrupole, etc. forces, causes helium gas to condense, at normal pressure, into a liquid at temperatures of 3.2 K and 4.2 K for ^{3}He and ^{4}He respectively.

The striking difference in the behaviour of ^{3}He and ^{4}He at even lower temperatures, in particular the fact that the critical temperature for ^{3}He to become superfluid is roughly one thousandth of the transition temperature of ^{4}He, is a consequence of the fact that ^{3}He is composed of an odd number of fermions (two protons, one neutron and two electrons), and is thus also a fermion, while ^{4}He, containing one more neutron, is a boson. Since in a Bose system single-particle states may be multiply occupied, at low temperatures this system has a tendency to condense into the lowest-energy single-particle state (Bose–Einstein condensation). It is believed that the superfluid transition in ^{4}He is a manifestation of Bose–Einstein condensation (see e.g. Leggett (1989), Pitaevskii and Stringari (2003), Pethick and Smith (2002)).

The basic feature of the Bose condensate is its phase rigidity, i.e. the fact that it is energetically favourable for the particles to condense into a single-particle state of fixed quantum-mechanical phase, such that the global gauge symmetry is spontaneously broken. For three-dimensional (3D-) systems, macroscopic flow of the condensate is (meta) stable, giving rise to the phenomenon of superfluidity (frictionless flow).

In a Fermi system, on the other hand, the Pauli exclusion principle allows only single occupation of fermion states. In the simplest approximation the fermions move independently in an average potential and occupy the lowest available single-particle states up to a Fermi energy ε_F. Fermions with energy near ε_F are, in a variety of systems, subject to a pairing residual interaction. The associated pairing correlations are important for understanding the structure of the low-lying states of nuclei, the properties of neutron stars and those of metals and of liquid helium ^{3}He at low temperatures. The relevant fermions are nucleons in nuclei, and in neutron stars, electrons in superconductors and ^{3}He atoms in liquid helium.

The pairing interaction leads to pairs of fermions bound in states coupled to integer spin (zero or one). These pairs, whose structure is different for each physical system, behave like bosons, and can at low temperatures Bose-condense, the condensate being characterized by macroscopic quantum coherence leading to the superconducting or superfluid phase. The mechanism and the consequences of this condensation in the case of nuclei is the subject of the present monograph.

Particular emphasis is placed on the study of quantal-size-effects (QSE). These effects are due to the fact that the nucleus is a finite many-body system where the surface plays a paramount role. In fact, the nuclear surface is not only the source of space quantization and thus of the discreteness of the single-particle levels, but also, by vibrating as a whole, of the existence of collective surface modes. Furthermore, because the length at which Cooper pairs are correlated is much larger than the nuclear dimension, the nuclear superfluid can be viewed as a zero-dimensional system. Because the number of pairs which build the condensate is small, fluctuations become very important.

1.2 Macroscopic wavefunction and phase rigidity

The central idea of the macroscopic quantum state is represented by assigning a macroscopic number of particles to a single wavefunction ($\tilde{\Psi}$) (see e.g. Anderson (1964, 1984), Mercerau (1969), Tilley and Tilley (1974), Bruus and Flensberg (2004)). These particles are assumed to have condensed into a single state. This condensation results in a macroscopic density of particles (ρ_s) sharing the same quantum phase (Φ). The resulting wavefunction is then $\bar{\Psi} = \Psi \exp(\mathrm{i}\Phi)$. In this form $\rho_s = (\bar{\Psi}^* \bar{\Psi})$ is not the usual probability of finding a particle but, owing to the macroscopic number of particles involved, is actually the effective particle density. Both Ψ and Φ may be functions of space and time and their variations will therefore determine the motion of the quantum fluid.

In what follows we shall be more interested in understanding the consequences the r-dependence of Φ has on the behaviour of the system and somewhat neglect the r-dependence of Ψ. Since, by definition, the particles are in precisely the same state and must therefore behave in an identical fashion, the equations of motion for the macrostate must also be identical to the equations of motion for any single particle in this state. Because the phase is common to so many particles, its effects do not average out on a macroscopic scale, but remain to fundamentally determine the behaviour of the system.

Changes in the wavefunction are of course determined by the Schrödinger equation. In particular, the centre of mass velocity ($\vec{V}$) can be calculated for this wavefunction from the velocity operator ($\vec{v}$) common to all the particles

$$\vec{v} = -\frac{1}{m^*}(\mathrm{i}\hbar\vec{\nabla} + e^*\vec{A})$$

where e^* and m^* are, respectively, the (effective) charge and mass of the particles and $\vec{A}$ is the vector potential. The centre of mass velocity is

$$\vec{V} = \tfrac{1}{2}\left\{\bar{\Psi}\vec{v}^{\,+}\bar{\Psi}^{\,+} + \bar{\Psi}^{\,+}\vec{v}\bar{\Psi}\right\} / \left(\bar{\Psi}^{\,+}\bar{\Psi}\right)$$

giving a current

$$\vec{J} = e^*\rho_s\vec{V} = \frac{e^*\rho_s}{m^*}\left(\hbar\vec{\nabla}\Phi - e^*\vec{A}\right). \tag{1.1}$$

By taking the curl of this equation one can derive another equation of significance, namely

$$\vec{\nabla} \times \vec{J} + \frac{\rho_s e^{*2}}{m^* c}\vec{B} = 0. \tag{1.2}$$

This is the solution found by F. London and H. London (London, 1954) of the relation

$$\frac{\partial}{\partial t}\left(\vec{\nabla}\times\vec{J}+\frac{\rho_{\mathrm{s}}e^{*2}}{m^{*}c}\vec{B}\right)=0. \tag{1.3}$$

This equation together with the Maxwell equation

$$\vec{\nabla}\times\vec{B}=\frac{4\pi}{c}\vec{J}, \tag{1.4}$$

characterizes a medium that conducts electricity without dissipation. In fact, in such circumstances, electrons under the effect of an electric field will be freely accelerated without dissipation so that their mean velocity $\vec{v}_{\mathrm{s}}$ will satisfy

$$m^{*}\frac{\mathrm{d}\vec{v}_{\mathrm{s}}}{\mathrm{d}t}=-e^{*}\vec{E}.$$

Since the current density carried by these electrons is $\vec{J}=-e^{*}v_{\mathrm{s}}\rho_{\mathrm{s}}$, the above equation can be written as

$$\frac{\mathrm{d}}{\mathrm{d}t}\vec{J}=\frac{\rho_{\mathrm{s}}e^{*2}}{m^{*}}\vec{E}. \tag{1.5}$$

The Fourier transform of this equation gives the ordinary AC conductivity for an electron gas of density ρ_{s} in the Drude model, when the relaxation time τ becomes infinitely large, that is,

$$\vec{J}=\sigma_{\mathrm{s}}(\omega)E(\omega)$$

where

$$\sigma_{\mathrm{s}}(\omega)=\lim_{\tau\to\infty}\sigma(\omega)$$

is the frequency dependent (or AC) conductivity

$$\sigma(\omega)=\frac{\sigma_0}{1-\mathrm{i}\omega\tau},$$

the zero-frequency conductivity being

$$\sigma_0=\frac{\rho_{\mathrm{s}}e^{*2}\tau}{m^{*}}.$$

Substituting equation (1.5) into Faraday's induction law

$$\nabla\times\vec{E}=-\frac{1}{c}\frac{\partial\vec{B}}{\partial t},$$

one finds equation (1.3). In other words, $\vec{\nabla}\times\vec{J}+\frac{\rho_{\mathrm{s}}e^{*2}}{m^{*}c}\vec{B}=C$ characterizes a non-dissipative electric medium. The more restrictive London equation, which specifically characterizes superconductors and distinguishes them from mere perfect conductors, requires in addition $C = 0$.

The reason for replacing (1.3) by (1.2) is that the latter equation leads directly to essential experimental facts, by forbidding currents or magnetic fields internal to the superconductor except within a layer of thickness $\Lambda = \left(\frac{m^* c^2}{4\pi \rho_s e^{*2}}\right)^{1/2} \approx 42 \left(\frac{r_s}{a_0}\right)^{3/2} \left(\frac{\rho}{\rho_s}\right)^{1/2}$ (London penetration depth) of the surface, $r_0 = a_B r_s$ being the Wigner-Seit cell radius of the system under consideration, defining the density ρ $(r_0 = (4\pi\rho/3)^{-1/3})$. In fact, equations (1.2) and (1.4) imply

$$\nabla^2 \vec{B} = \frac{4\pi\rho_s e^{*2}}{m^* c^2} \vec{B},$$

$$\nabla^2 \vec{J} = \frac{4\pi\rho_s e^{*2}}{m^* c^2} \vec{J},$$

where the relation $\vec{\nabla} \times (\vec{\nabla} \times) = \vec{\nabla}(\vec{\nabla} \cdot) - \nabla^2$ was used. Assuming a semi-infinite superconductor occupying the half space $x > 0$,

$$B(x) = B(0) e^{-x/\Lambda},$$

and

$$J(x) = J(0) e^{-x/\Lambda}.$$

Thus, the London equation implies the Meissner effect, along with a specific picture of the surface currents that screen out the applied field. These currents occur within a surface layer of thickness $10^2 - 10^3$ Å. Within this same surface layer the field drops continuously to zero, predictions which are confirmed, among other things, by the fact that the field penetration is not complete in superconducting films as thin as or thinner than the penetration depth Λ.

Let us now return to equation (1.1). This relation can be obtained by minimizing the free energy of the system with respect to the phase Φ. In other words, subject to a phase gradient, the system minimizes its energy by carrying a current even in thermodynamical equilibrium, and such a current is always dissipationless. This is true both for charged systems (like, e.g., metals where $e^* = 2e$ and $m^* = 2m_e$), as well as for neutral systems (like, e.g., He II, where $e^* = 0$ and $m^* = m_4$).

Of course there is an energy cost for the system to carry the current, but as long as this cost is smaller than the alternative which is to go out of the superfluid or superconducting state, the current carrying state is chosen. The critical current is reached when the energies are equal (and equal to the value of the gap, see Sections 1.4 and 1.5 and Figs. 1.6 and 1.7), and then the superfluid or superconductor goes into the normal state (see equations (1.17) and (1.21), respectively).

Within this context, it should be noted that the appearance of the excitation gap is not the reason for the superfluidity or superconductivity itself, but a consequence of the spontaneous symmetry breaking of gauge invariance. In fact, gapless superconductors do exist (in this connection see Sections 5.3 and 6.2.1).

1.3 Broken symmetry and collective modes

In many phase transitions, such as that to the ferromagnetic state, or from the normal to the superconducting state, or again from a spherical to a deformed nucleus, the ground state of the low-temperature phase has a lower symmetry than the Hamiltonian used to describe the system. The situation is one of broken symmetry. In cases where the symmetry group that is broken is continuous (e.g. the rotation group), a new collective mode appears, whose frequency, in the absence of long-range forces, goes to zero in the long wavelength limit (Anderson Goldstone Nambu (AGN) mode (see Chapter 4)). For the ferromagnet, the elementary excitations required by Goldstone's theorem (Goldstone, 1961) are Bloch's spin waves (magnons), in which the magnetization precesses about its direction in the ground state (see Figs. 1.1 and 1.2).

Superconductors furnish an example of a system in which the excitations required by the symmetry-breaking process have a finite frequency in the

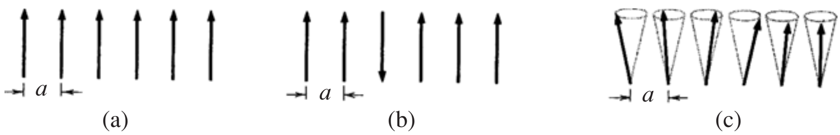

Figure 1.1. (a) Classical picture of the ground state of a simple ferromagnet; all spins are parallel. (b) A possible excitation; one spin is reversed. (c) The low-lying elementary excitations are spin waves. The ends of the spin vectors precess on the surfaces of cones, with successive spins advanced in phase by a constant angle (after C. Kittel (1968)). From *Introduction to Solid State Physics*, 7th edition, by Charles Kittel, Copyright 1995 John Wiley & Sons Inc. Reprinted with permission of John Wiley & Sons Inc.

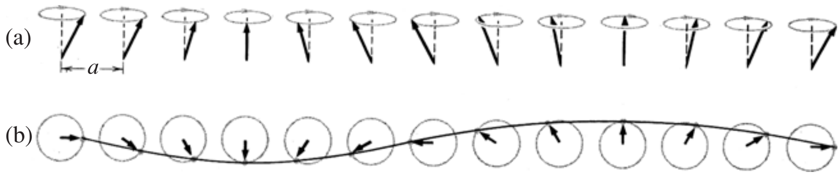

Figure 1.2. A spin wave on a line of spins. (a) The spins viewed in perspective. (b) Spins viewed from above, showing one wavelength. The wave is drawn through the ends of the spin vectors (after Kittel (1968)). From *Introduction to Solid State Physics*, 7th edition, by Charles Kittel, Copyright 1995 John Wiley & Sons Inc. Reprinted with permission of John Wiley & Sons Inc.

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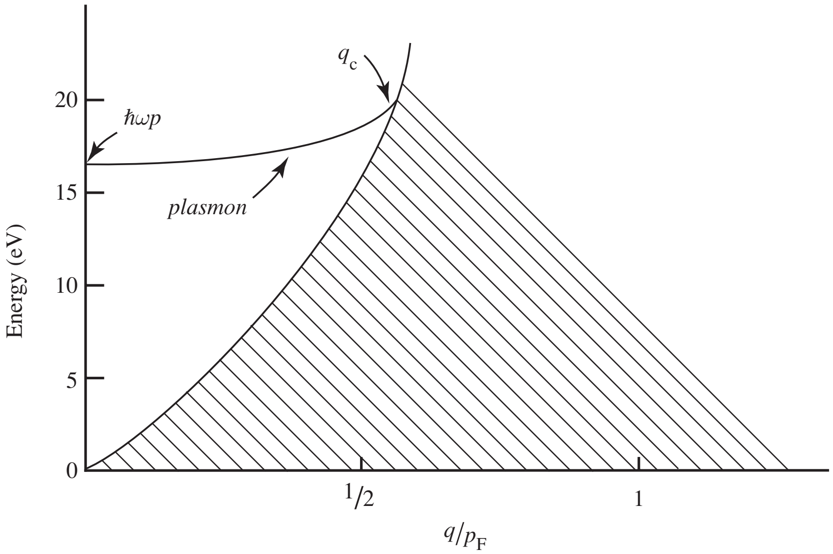

Figure 1.3. Excitation spectrum of density fluctuations in a quantum plasma with the density of Al, as calculated in the random phase approximation. Plasmons are essentially undamped (see also Section 8.3.4) for wavevectors less than q_c, and are strongly damped (Landau damping) beyond q_c by the single particle–hole excitations, whose energies lie within the hatched region (after Pines (1963)).

long wavelength limit (because of the infinite range of the Coulomb force): the corresponding Goldstone mode is the familiar plasma oscillation (see Fig. 1.3).

For a neutral fermion superfluid, on the other hand, the collective mode is the zero-sound mode proposed by Anderson (1958) and Bogoliubov (1958a), which has a vanishing frequency at long wavelengths (see Section 4.3.1).

An example of AGN boson in a neutral system is provided by the fourth sound in superfluid ^{3}He, which corresponds to the oscillatory motion of the superfluid phase in a confined geometry (superleak) where the normal fluid is clamped. For example, assume a porous medium. In it, the normal-fluid fraction (see equation (1.12)) is clamped by the scattering of quasiparticles with the surface of the very narrow channels. The superfluid fraction is barely affected by the confining walls, provided that the channel diameter is greater than the coherence length $\xi(T)$ (equation (1.32)), and thus may move freely. The oscillatory motion of the superfluid phase in such a confined geometry is called fourth sound (see Vollhardt and Wölfle (1990)). In the case of atomic nuclei, the very occurrence of collective rotational degrees of freedom may be said to originate in a breaking of rotational invariance, which introduces a 'deformation' that makes it possible to specify an orientation of the system (Bohr and Mottelson, 1975). Rotation (see Fig. 1.4) represents the collective mode associated with such a spontaneous

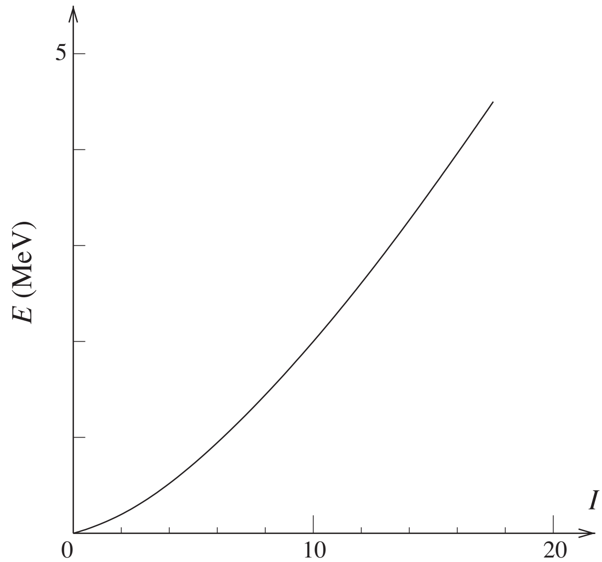

Figure 1.4. Schematic representation of the (discrete) energy levels of the (ground state) rotational band of a quadrupole deformed atomic nucleus as a function of the angular momentum I ($E = (\hbar^2/2\mathcal{I})I(I+1)$, where $\mathcal{I}$ is the moment of inertia).

symmetry breaking (AGN boson). The full degrees of freedom associated with rotations in three-dimensional space come into play if the deformation completely breaks the rotational symmetry, thus permitting a unique specification of the orientation. If the deformation is invariant with respect to a subgroup of rotations, the corresponding elements are part of the intrinsic degree of freedom, and the collective rotational modes of excitation are correspondingly reduced, disappearing entirely in the limit of spherical symmetry.

1.4 Superfluid ^{4}He (He II)

^{4}He becomes liquid under its own vapour pressure at 4.21 K. The liquid phase at this temperature, helium I, behaves like a normal liquid, but at 2.17 K it shows a further phase transition – to helium II. Helium II is a most peculiar liquid: it shows superfluidity, i.e. a lack of viscosity when flowing through a narrow slit or capillary. At 2.17 K the specific heat shows a very strong pronounced peak, resembling the Greek letter λ, whence Ehrenfest suggested the name λ-point for the transition point (see Fig. 1.5).

The theory developed by Landau (Landau (1941, 1947)) was constructed upon the basic idea that the equilibrium properties of liquid helium below the λ-point could be expressed in terms of the energy spectrum of the elementary

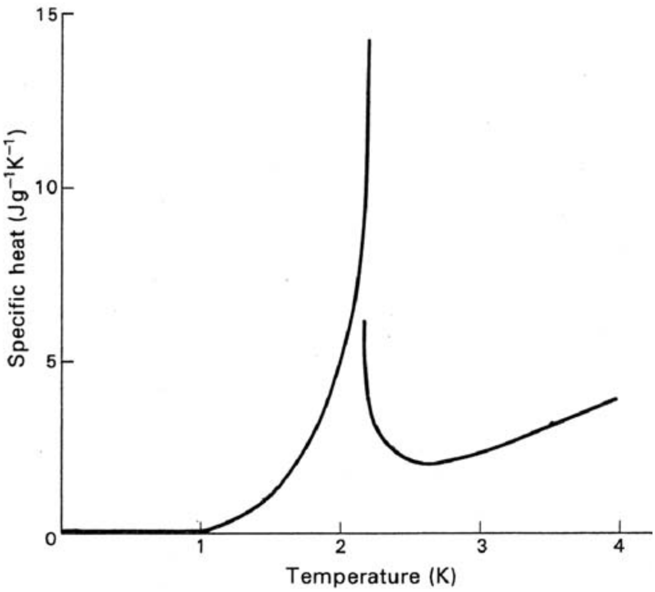

Figure 1.5. Specific heat of ^{4}He (after Atkins (1959)).

excitations possible in helium, namely phonons and rotons. Landau considers the quantization of liquids and reaches the conclusion that there are states possible in the liquid for which

$$\text{curl}\,\vec{v} = 0, \tag{1.6}$$

where $\vec{v}$ is the velocity of the liquid. Note that this relation is obtained from equation (1.1) for $e^* = 0$ (neutral system). Such states correspond to potential flow, as would be the case in classical hydrodynamics, because, just as there is no continuous transition in quantum mechanics between states with zero angular momentum and with non-vanishing angular momentum, in the same way there may be no continuous transition between states with curl $\vec{v} = 0$ and those with curl $\vec{v} \neq 0$. Consequently, one concludes that there will be an energy gap Δ between the lowest energy level corresponding to potential flow and the lowest energy level of vortex motion (curl $\vec{v} \neq 0$). In order that the liquid be superfluid, it is necessary that the vortex motions start at a higher energy than the potential flow motions.

The spectrum of helium II can thus be seen as a superposition of two continuous spectra: one corresponding to potential flow and one corresponding to vortex motion. The potential flow part of the spectrum corresponds to longitudinal waves, i.e. sound waves. The elementary excitations are thus phonons, the energy

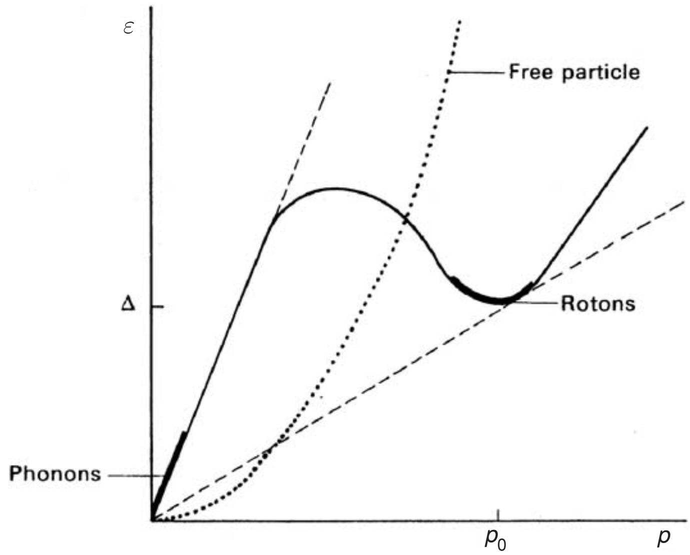

Figure 1.6. Phonon–roton spectrum suggested by Landau. Broken lines indicate superfluid critical velocities. Dotted line shows free-particle parabola for comparison.

spectrum of which is known to be (Fig. 1.6)

$$\varepsilon_{\mathrm{ph}} = c_{\mathrm{s}} p,$$

where p is the momentum of the excitation while c_{s} is the sound velocity.

The elementary excitations of the vortex motion were called rotons by Tamm. The roton spectrum is given by

$$\varepsilon_{\mathrm{r}} = \Delta + \frac{(p - p_0)^2}{2\mu}, \tag{1.7}$$

where Δ is the energy gap mentioned above while μ is the inertia of the rotons.

It should be emphasized that the above two equations (see also Fig. 1.6) give the energy of the excitation spectrum of the elementary excitations of the helium II and not the energy spectrum of the single helium atoms

$$\varepsilon_{\mathrm{sp}} = \frac{p^2}{2m_4}.$$

Note that given the dispersion relation shown in Fig. 1.6 it is difficult to speak strictly of rotons and phonons as qualitatively different types of excitations. It could be more correct to speak simply of the long wave (small p) and short wave (p in the neighbourhood of p_0) excitations. In any case, there is an essential difference between phonons and rotons. Phonons can have zero energy in the long wavelength limit and thus qualify as AGN modes (Anderson (1952, 1963), Nambu (1959, 1960)), while rotons have always an energy $\geq \Delta$ and can thus