

Chapter 2

A Method for Configuration Representation of Metamorphic Mechanisms with Information of Component Variation

Wuxiang Zhang and Xilun Ding

Abstract The metamorphic mechanism has the characteristics of multi-configuration, variable constraints, and multi-function. For achieving the variation characteristics of the topological structures in different configurations and the coupling relations intuitively during the process of configuration transformation, a novel comprehensive symbolic matrix for representing the topological structures of the metamorphic mechanism is proposed. The information including variation of components, the relative joint orientations is involved in this new adjacency matrix. The variation characteristics and coupling features of the metamorphic mechanism can be obtained by the generalized operations including intersection and difference on the corresponding symbolic matrices.

Keywords Configuration representation · Component variation · Metamorphic mechanisms

2.1 Introduction

In contrast to a traditional mechanism, the metamorphic mechanism has the characteristics of multi-configuration, variable constraints, and multi-function. It forms a class of mechanisms that have the ability to change configuration sequentially from one to another as a resultant change of the number of effective links and topological structure to achieve different tasks following its working

W. Zhang (✉) · X. Ding
Robotics Institute, Beijing University of Aeronautics and Astronautics,
Beijing, P.R. China
e-mail: zhangwuxiang@me.buaa.edu.cn

X. Ding
e-mail: xlding@buaa.edu.cn

conditions and requirements. Thus a metamorphic mechanism can be considered as a mechanism set composed by multiple kinematic chains which have the ability of reconfiguration [1].

In the practical process of the transformation between these kinematic chains, there exist three basic metamorphic characteristics including variation of components, adjacent relations and properties of kinematic joints. In most cases, these three ways are occurred simultaneously. On the other hand, some basic constituent elements (component and its connectivity relationship) of these mechanisms keep unchanged to make the mechanisms in adjacent configurations have complex coupling features [2]. These two aspects are key factors affecting the study on the method for configuration synthesis of metamorphic mechanisms [3].

Therefore, for achieving the variation characteristics and the coupling relations of the topological structures in different configurations, a description approach which is appropriate to the metamorphic mechanism should be researched firstly.

Mechanism diagram and topological graph are two simple and intuitive methods for describing the structure of a mechanism, but it can not be represented in a mathematical form. Therefore, adjacency matrix method was used to describe the topological structure of the mechanism in a single configuration. Further, elementary transformation of the matrices was proposed to represent the corresponding variation of mechanisms [4–7]. Wang and Dai [1] introduced the joint symbols into the adjacency matrix for expressing the variation of kinematic joints. In this method, all the components are numbered sequentially and placed in the principal diagonal position. And the off-diagonal elements are expressed by using the joint symbols, such as revolute joint (R), prismatic joint (P) and higher joint (G), etc. representing the connectivity relationship of the corresponding rows and columns. So the matrix is symmetrical, and the number of its rows (columns) is equal to the number of equivalent components. But the joint axis information is not be expressed by using this method, so planar mechanisms and spatial mechanisms cannot be distinguished.

Because the axis orientation of the kinematic joints are time-varying, it is infeasible to express the corresponding information by taking advantage of eulorian angles relative to a fix coordinate in adjacency matrix. The relative axis orientation of the kinematic joints located at two ends of the same component is invariable while in motion so that we can utilize use it. Yang [8] introduced the concept of geometric constraint for expressing the relative position and orientation of the joint axes and generalized it into six types: parallelism, coincidence, intersection, perpendicularity and randomness. Li, Wang and Dai [9, 10] developed a topological representation matrix with information of loops, types of links and joints, and orientations of joints.

As stated previously, the components have their own unified numbers in these methods, and the variation information of these components can not be showed up with producing confusion of components and their connectivity relationships when comparing the adjacency matrices. In addition, different description sequence to the same mechanism will lead to different adjacency matrices which need to perform additional isomorphism identification.

Based on the above-mentioned research achievements and insufficiencies, a symbolic matrix which can represent the topological structures of the mechanism in all configurations with information of component variation is proposed.

2.2 The Method for Configuration Representation of Metamorphic Mechanisms

The basic characteristic of the method is to construct a symbolic adjacency matrix by introducing the information of component variation and joint orientation. The variation characteristics and coupling features of the mechanisms in adjacent configurations can be obtained by the generalized operations including intersecting and difference on the corresponding symbolic matrices.

The symbolic matrix A is constructed to describe the topological structures of the metamorphic mechanism completely as

$$A = \begin{pmatrix} L_1 & J_{1,2} & \cdots & J_{1,i} & \cdots & J_{1,n-1} & J_{1,n} \\ J_{1,2} & L_2 & \cdots & J_{2,i} & \cdots & J_{2,n-1} & J_{2,n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ J_{1,i} & \cdots & \cdots & L_i^K & \cdots & \cdots & J_{i,n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ J_{1,n-1} & J_{2,n-1} & \cdots & J_{i,n-1} & \cdots & L_{n-1} & J_{n-1,n} \\ J_{1,n} & J_{2,n} & \cdots & J_{i,n} & \cdots & J_{n-1,n} & L_n \end{pmatrix} \quad (2.1)$$

where the principal diagonal element L_i^K represent the component of the mechanism as well as the off-diagonal element $J_{i,j}$ represent the connectivity relationship of components L_i and L_j . The specific explanations are shown as follows.

- (1) Principal diagonal element L_i^K represent the component whose sequence number is i in the mechanism. The subscript i represents the order of the corresponding component in the matrix and the superscript K represents the driving component or the frame component as symbols D or F respectively. In general conditions, the component whose connectivity characteristic keeps unchanged during the course of configuration transformation is prior to be selected as the first principal diagonal element. Then other components should be arranged in sequence.

In the symbolic matrix, for expressing the component variation including combination and decomposition distinctly, 'direct sum' (operator ' \oplus ') is used here. The combination of more than two components can be represented as $L_i \oplus L_j \oplus \cdots$. It is different with the former representation methods. In those methods, several components fixed together was treated as one of them on the assumption of the components are coincident without considering the influence of their relative position and orientation constraints at the moment of configuration transformation. But in fact, some newly components are usually generated. The geometric

Table 2.1 Expression of the geometric constraint

Geometric constraint relations	W
Parallelism	$//R, //P, //H, \dots$
Intersection	$\widehat{R}, \widehat{RR}, \widehat{RRR}, \dots$
Coincidence	$/R, /H$
Perpendicularity	$\perp R, \perp P, \perp H, \dots$
Randomicity	$-R, -P, -H, \dots$

parameter of them will be variable with the moment of configuration transformation. So ‘direct sum’ is introduced into the matrix for expressing the component combination at a certain position. This can provide the basis for the research on sequent dimensional synthesis of metamorphic mechanisms.

(2) Off-diagonal element $J_{i,j}$ ($j = 2, \dots, nj \ 2, \dots, n$) shows the type of joint and the relative axis orientation of the joints located at two ends of the component L_i as

$$J_{i,j} = \begin{cases} U_W & \begin{array}{l} U \text{ represent the joint symbols, for example } -R \text{ (revolute joint),} \\ \text{Prismatic joint, Spherical joint, } \dots \\ \text{The subscript } W \text{ represent the relative geometric constraint relation of} \\ \text{the joint axes located at two ends of the component } L_i \\ \text{Components } L_i \text{ and } L_j \text{ are not connected} \end{array} \\ 0 & \end{cases} \tag{2.2}$$

In Eq. (2.2), the joint axis between the first component and the second component can be treated as the reference axis, and the other joint axis of the second component can be expressed as the relative orientation of the reference axis. By inference, all the joint axis of the mechanism can be represented relatively by geometric constraint relations of the two ends of the same component successively. The expressions of the geometric constraint are illustrated in Table 2.1 [8–10].

Note. For describing the intersection of more than two joint axes in the same point, the corresponding W should be expressed as $\widehat{RR} \dots$

So the matrix A has the following characteristics as follows:

- (1) The matrix is symmetrical, and the principal diagonal elements represent the sequential connected components. The quantity of its rows (columns) is equal to the quantity equivalent components.
- (2) Nonzero elements in the lower-left/upper-right part of the matrix are correspondent with the types of joints. And the subscript of these nonzero elements can represent the relative axis orientation of all joints in the mechanism for distinguishing planar mechanisms from spatial mechanisms clearly.
- (3) The quantity of the nonzero elements in the i th row (the principal diagonal element L_i is not included) N_i represents the amount of the kinematic joints connected with the component L_i . If there exist $N_i = 1$, the mechanism has at least one open chain; if all the amount of N_i are equal to 2, the mechanism is a

single closed chain; If there exist $N_i > 2$, the mechanism has multi-closed chains.

- (4) The joint axis orientation between the last component and the first component in the closed kinematic chain should be represented as the geometric constraint relative to the reference axis. To the open chain, the case does not exist.

The symbolic matrix can describe the topological structure of the mechanism in a single configuration. Based on the matrix, the variation characteristics and coupling features of the metamorphic mechanism in adjacent configurations can be obtained by the corresponding operations.

2.3 Topological Variation Characteristics of the Metamorphic Mechanism

For achieving the topological variation characteristics of the metamorphic mechanism, it is necessary to apply generalized difference operation on the corresponding configuration representation matrices as

$$A_{i+1,i}^{sub} = A_{i+1} - A_i \stackrel{\text{Equivalence}}{\Leftrightarrow} A'_{i+1} - A'_i \quad (2.3)$$

where $A_{i+1,i}^{sub}$ represent the result of the generalized difference operation, A'_{i+1} and A'_i representing the increased order matrices are equivalent with the original configuration representation matrices in configurations $i + 1$ and i respectively. The orders of the two involved matrices are probably different, so it is necessary to make their orders have the same value by the operation of increasing order. The operator “-” represent the generalized difference operation for achieving the different parts. In set theory, the matrix $A_{i+1,i}^{sub}$ is the relative complement set of A'_i to A'_{i+1} . Its physical meaning is that after comparing the two topological representation matrices A_{i+1} 和 A_i , the different parts of the two matrices are extracted and reserved. The order of the result matrix is correspondent with the quantity of all related components. Its operation steps are described as follows.

Firstly, if the orders of the two matrices involved in the operation are different, the operation of increasing order should be done according to the number of all related components. Components fixed together should be resolved to several single components, and the corresponding J_{ij} in Eq. (2.1) is given as the number ‘1’. The other connectivity relations should be transferred to the corresponding components.

The operation starts with the first line and column, and the principal diagonal elements and their connectivity relationship should be compared gradually. The same elements should be assigned ‘0’ while the different elements in the minuend matrix should be reserved. Variation of the driving components and components fixed on the ground which are important factors influencing the topological structures of the metamorphic mechanism should be included. In addition, the

same joint axis may have different representations in two matrices because of component variation, so it is necessary to identify the variation in combination with the metamorphic mechanism.

2.4 Coupling Features of the Metamorphic Mechanism

There exist some basic constituent elements (component and its connectivity relations) keep unchanged to make the mechanisms in adjacent configurations have complex coupling features. For achieving these coupling features, it is necessary to apply generalized intersection operation (operator ‘ \cap ’) on the corresponding configuration representation matrices as

$$A_{i+1,i}^{int} = A_{i+1} \cap A_i \stackrel{\text{Equivalence}}{\Leftrightarrow} A'_{i+1} \cap A'_i \quad (2.4)$$

where $A_{i+1,i}^{int}$ represent the result of the generalized intersection operation whose physical meaning is that after comparing the two topological representation matrices A_{i+1} 和 A_i , the same parts of the matrix are extracted and reserved. And the different elements should be assigned ‘0’.

The operation steps are coincident with the above expressed difference operation. The first step is making the order of the involved matrices unified. Then the principal diagonal elements and their connectivity relations should be compared gradually, and the different elements should be assigned ‘0’ while the same elements should be reserved. Therefore, the nonzero elements in the result matrix manifest the coupling features.

2.5 Relation Between Variation Characteristics and Coupling Features

Because the variation characteristics and coupling features of the metamorphic mechanism in adjacent configurations can be obtained by the generalized operations including intersecting and differentiating on the corresponding symbolic matrices, it is obvious that there exist the following relation as

$$\begin{aligned} A_{i+1,i}^{sub} \cup A_{i+1,i}^{int} &= (A_{i+1} - A_i) \cup (A_{i+1} \cap A_i) \\ &= [(A_{i+1} - A_i) \cup A_{i+1}] \cap [(A_{i+1} - A_i) \cup A_i] \\ &= A_{i+1} \cap (A_{i+1} \cup A_i) \\ &= A_{i+1} \end{aligned} \quad (2.5)$$

In Eq. (2.5), symbol “ \cup ” represent the generalized union operation. Equation (2.5) shows that the configuration representation matrix in configuration i can be

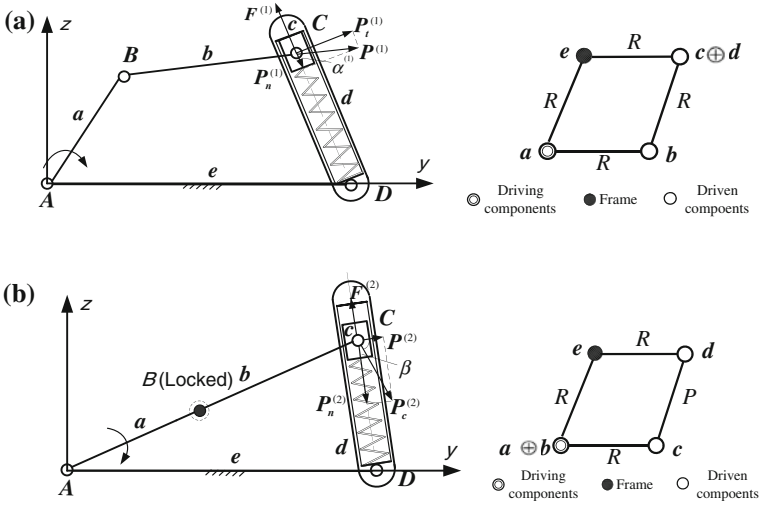


Fig. 2.1 A planar five bar force-limit metamorphic mechanism **a** Mechanism schematic in configuration 1 and its topological structure. **b** Mechanism schematic in configuration 2 and its topological structure

obtained by uniting the topological variation characteristics matrix and coupling characteristics matrix.

2.6 Case Study

2.6.1 Case 1

A planar five bar force-limit metamorphic mechanism is shown in Fig. 2.1 [3, 11]. When the mechanism is in configuration 1 as in Fig. 2.1a, slider c locates at the top end of the slot in component d under the action of spring force $F^{(1)}$ which satisfies $F^{(1)} \geq P_n^{(1)} = P^{(1)} \cos \alpha^{(1)}$. Force $P_n^{(1)}$ (the force direction is along the link d) is the component force of the transmission force $P^{(1)}$ generated by link b , $\alpha^{(1)}$ is the transmission angle. The mechanism in this configuration can be treated as a four bar mechanism, in this case, spring force $F^{(1)}$ guarantees the slider c stays static. Without such a force $F^{(1)}$, slider c inclines to move down along the slot under the action of $P_n^{(1)}$, and the mechanism leads to uncertain motion. When the mechanism is in configuration 2, see Fig. 2.1b, links a and b are fixed together by locking revolute joint B using geometric limit, and the mechanism is transformed to a slider mechanism. Under this condition, spring force $F^{(2)} < P_n^{(2)} = P_c^{(2)} \cos \beta$ makes the prismatic joint C active. $P_n^{(2)}$ is the component force of $P_c^{(2)}$ (generated

by link b , and its direction is perpendicular to link b). Angle β represents the angle between the directions of $P_n^{(2)}$ and $P_c^{(2)}$.

Applying the proposed method, the symbolic matrices of the mechanisms in its two configurations can be expressed as follows:

$$A_1 = \begin{pmatrix} e^F & R & 0 & R_{\parallel R} \\ R & a^D & R_{\parallel R} & 0 \\ 0 & R_{\parallel R} & b & R_{\parallel R} \\ R_{\parallel R} & 0 & R_{\parallel R} & c \oplus d \end{pmatrix} \quad (2.6)$$

$$A_2 = \begin{pmatrix} e^F & R & 0 & R_{\parallel R} \\ R & a^D \oplus b & R_{\parallel R} & 0 \\ 0 & R_{\parallel R} & c & P_{\parallel R} \\ R_{\parallel R} & 0 & P_{\parallel R} & d \end{pmatrix} \quad (2.7)$$

The matrices show that all the joint axes are parallel to each other, indicating that it is a planar mechanism. And components a and e are the driving component and the frame respectively.

By applying the generalized difference operation, the topological structure variation characteristics can be achieved as

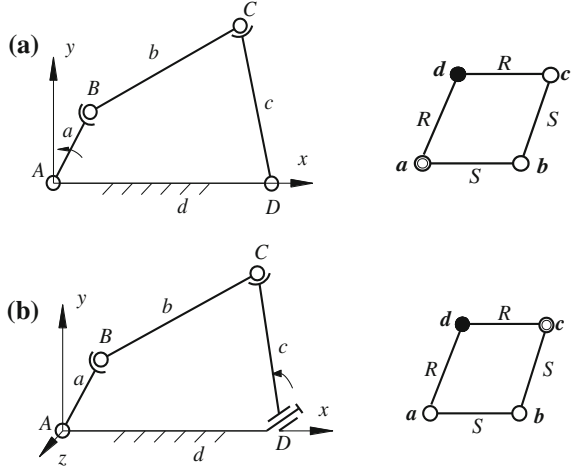
$$\begin{aligned} A_{2,1}^{sub} &= A_2 - A_1 \stackrel{\text{Equivalence}}{\Leftrightarrow} A_2' - A_1' \\ &= \begin{bmatrix} e^F & R & 0 & 0 & R_{\parallel R} \\ R & a^D & 1 & 0 & 0 \\ 0 & 1 & b & R_{\parallel R} & 0 \\ 0 & 0 & R_{\parallel R} & c & P_{\parallel R} \\ R_{\parallel R} & 0 & 0 & P_{\parallel R} & d \end{bmatrix} - \begin{bmatrix} e^F & R & 0 & 0 & R_{\parallel R} \\ R & a^D & R_{\parallel R} & 0 & 0 \\ 0 & R_{\parallel R} & b & R_{\parallel R} & 0 \\ 0 & 0 & R_{\parallel R} & c & 1 \\ R_{\parallel R} & 0 & 0 & 1 & d \end{bmatrix} \\ &= \begin{bmatrix} e^F & 0 & 0 & 0 & 0 \\ 0 & a^D & 1 & 0 & 0 \\ 0 & 1 & b & 0 & 0 \\ 0 & 0 & 0 & c & P_{\parallel R} \\ 0 & 0 & 0 & P_{\parallel R} & d \end{bmatrix} \end{aligned} \quad (2.8)$$

According to Eq. (2.8), the element indicating the connectivity relation of components a and b is '1', it means the two components are fixed together to generate a new link $a \oplus b$. Component d is separated from the component $c \oplus d$ and their connectivity joint with component c is changed to a prismatic joint.

The coupling features of the mechanisms in two configurations can be obtained as

$$A_{2,1}^{int} = A_2 \cap A_1 \stackrel{\text{Equivalence}}{\Leftrightarrow} \begin{bmatrix} e^F & R & 0 & 0 & R_{\parallel R} \\ R & a^D & 0 & 0 & 0 \\ 0 & 0 & b & R_{\parallel R} & 0 \\ 0 & 0 & R_{\parallel R} & c & 0 \\ R_{\parallel R} & 0 & 0 & 0 & d \end{bmatrix} \quad (2.9)$$

Fig. 2.2 A planar-spatial four bar metamorphic mechanism **a** Mechanism schematic in configuration 1 and its topological structure **b** Mechanism schematic in configuration 2 and its topological structure



by applying the generalized intersection operation. According to Eq. (2.9), the connectivity relationship between component e and the related components such as links a and d keep unchanged.

2.6.2 Case 2

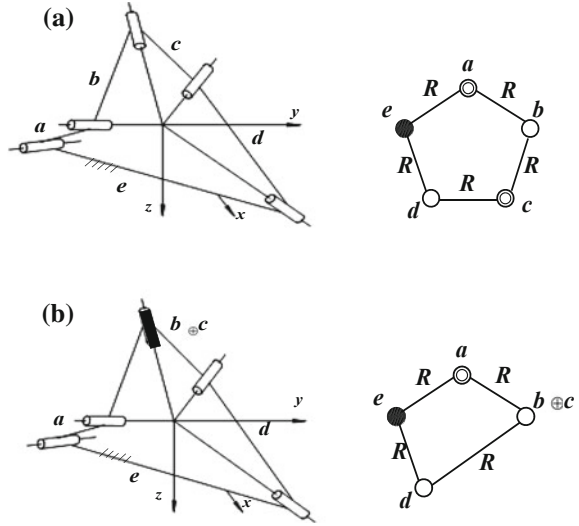
A planar-spatial four bar metamorphic mechanism is shown in Fig. 2.2. When the mechanism is in configuration 1 as in Fig. 2.2a, it can be treated as a planar RSSR four bar mechanism. Links a and d are the driving component and frame respectively. At the moment of configuration transformation, the axis of kinematic joint between components c and d is changed to make the mechanism transformed to a spatial mechanism as shown in Fig. 2.2b as well as the driving component becomes link a . The special metamorphic mechanism transforms its configuration by changing joint orientation [10].

The topological structures in these two configurations can be expressed in matrix form as follows:

$$A_1 = \begin{pmatrix} d^F & R & 0 & R_{\parallel R} \\ R & a^D & S_{-R} & 0 \\ 0 & S_{-R} & b & S_{-s} \\ R_{\parallel R} & 0 & S_{-s} & c \end{pmatrix} \quad (2.10)$$

$$A_2 = \begin{pmatrix} d^F & R & 0 & R_{\perp R} \\ R & a & S_{-R} & 0 \\ 0 & S_{-R} & b & S_{-s} \\ R_{\perp R} & 0 & S_{-s} & c^D \end{pmatrix} \quad (2.11)$$

Fig. 2.3 A spherical 5R–4R metamorphic mechanism
a Mechanism schematic in configuration 1 and its topological structure.
b Mechanism schematic in configuration 2 and its topological structure



The topological structure variation characteristics and coupling features can be achieved as

$$A_{2,1}^{sub} = A_2 - A_1 = \begin{pmatrix} d^F & 0 & 0 & R_{\perp R} \\ 0 & a^D & 0 & 0 \\ 0 & 0 & b & 0 \\ R_{\perp R} & 0 & 0 & c^D \end{pmatrix} \quad (2.12)$$

$$A_{2,1}^{int} = A_2 \cap A_1 = \begin{pmatrix} d^F & R & 0 & 0 \\ R & a^D & S_{-R} & 0 \\ 0 & S_{-R} & b & S_{-S} \\ 0 & 0 & S_{-S} & c^D \end{pmatrix} \quad (2.13)$$

Equations (2.12) and (2.13) show that the driving components becomes link c and the axis geometric constraint relationship between two ends of component d are changed from parallelism to perpendicularity as shown in Fig. 2.2.

2.6.3 Case 3

A spherical 5R–4R metamorphic mechanism and its topological structures are shown in Fig. 2.3. All the components are connected by revolute joints whose joint axes intersect in the origin of coordinates. When the mechanism is transformed to configuration 2, components b and c are fixed together to realize configuration transformation.

The topological structures of the mechanisms in two configurations can be expressed in matrix form as follows:

$$A_1 = \begin{pmatrix} e^F & R & 0 & 0 & R_{\widehat{R}} \\ R & a^D & R_{\widehat{R}} & 0 & 0 \\ 0 & R_{\widehat{R}} & b^R & R_{\widehat{RR}} & 0 \\ 0 & 0^R & R_{\widehat{RR}} & c & R_{\widehat{RRR}} \\ R_{\widehat{R}} & 0 & 0 & R_{\widehat{RRR}} & d \end{pmatrix} \quad (2.14)$$

$$A_2 = \begin{pmatrix} d^F & R & 0 & R_{\widehat{R}} \\ R & a^D & R_{\widehat{R}} & 0 \\ 0 & R_{\widehat{R}} & b \oplus c & R_{\widehat{RR}} \\ R_{\widehat{R}} & 0 & R_{\widehat{RR}} & d \end{pmatrix} \quad (2.15)$$

By applying the generalized difference and intersection, topological structure variation characteristics of the adjacent matrices can be obtained as

$$\begin{aligned} A_{2,1}^{sub} &= A_2 - A_1 \stackrel{\text{Equivalence}}{\Leftrightarrow} A'_2 \cap A'_1 \\ &= \begin{pmatrix} e^F & R & 0 & 0 & R_{\widehat{R}} \\ R & a^D & R_{\widehat{R}} & 0 & 0 \\ 0 & R_{\widehat{R}} & b & 1 & 0 \\ 0 & 0^R & 1 & c & R_{\widehat{RR}} \\ R_{\widehat{R}} & 0 & 0 & R_{\widehat{RR}} & d \end{pmatrix} - \begin{pmatrix} e^F & R & 0 & 0 & R_{\widehat{R}} \\ R & a^D & R_{\widehat{R}} & 0 & 0 \\ 0 & R_{\widehat{R}} & b & R_{\widehat{RR}} & 0 \\ 0 & 0^R & R_{\widehat{RR}} & c & R_{\widehat{RRR}} \\ R_{\widehat{R}} & 0 & 0 & R_{\widehat{RRR}} & d \end{pmatrix} \\ &= \begin{pmatrix} e^F & 0 & 0 & 0 & 0 \\ 0 & a^D & 0 & 0 & 0 \\ 0 & 0 & b & 1 & 0 \\ 0 & 0 & 1 & c & 0 \\ 0 & 0 & 0 & 0 & d \end{pmatrix} \end{aligned} \quad (2.16)$$

$$A_{2,1}^{int} = A_2 \cap A_1 = \begin{pmatrix} e^F & R & 0 & 0 & R_{\widehat{R}} \\ R & a^D & R_{\widehat{R}} & 0 & 0 \\ 0 & R_{\widehat{R}} & b^R & 0 & 0 \\ 0 & 0^R & 0 & c & R_{\widehat{RR}} \\ R_{\widehat{R}} & 0 & 0 & R_{\widehat{RR}} & d \end{pmatrix} \quad (2.17)$$

As shown in Eqs. (2.16) and (2.17), components b and c are fixed together to produce a new component $b \oplus c$.

2.7 Conclusion

A novel comprehensive symbolic matrix for representing the topological structures of the metamorphic mechanism is proposed. The information including variation of components, relative orientation of the kinematic joints are involved in this new

adjacency matrix. The variation characteristics and coupling features of the metamorphic mechanism in adjacent configurations can be obtained by the generalized operations including intersection and difference on the corresponding symbolic matrices.

Acknowledgments The authors are thankful for the fundamental support of the Natural Science Foundation of China (NSFC) under grant number 51125020&51105013 and the Research Fund for the Doctoral Program of Higher Education of China (RFDP) under grant number 200800060009.

References

1. Wang DL, Dai JS (2007) Theoretical foundation of metamorphic mechanism and its synthesis. *Chin Mech Eng* 43(8):32–42
2. National natural science foundation of China (2010) Development strategy for mechanical engineering. Science Press, Beijing
3. Zhang WX, Ding XL, Dai JS (2011) Morphological synthesis of metamorphic mechanisms based on constraint variation. *Proc IME C J Mech Eng Sci* 225(1):2997–3010
4. Dai JS, Ding XL, Wang DL (2005) Topological changes and the corresponding matrix operations of a spatial metamorphic mechanism. *Chin Mech Eng* 41(8):30–35
5. Dai JS, Rees JJ (2005) Matrix representation of topological changes in metamorphic mechanisms. *ASME Trans J Mech Des* 127(4):610–619
6. Wu YR, Jin GG, Li DF et al (2007) Adjacent matrix method describing the structure changing of metamorphic mechanisms. *Chin Mech Eng* 43(7):23–26
7. Lan ZH, Du R (2008) Representation of topological changes in metamorphic mechanisms with matrices of the same dimension. *J Mech Des* 130:074501(1–4)
8. Yang TL (2004) Topology structure design of robot mechanisms. China Machine Press, Beijing
9. Li SJ, Wang DL, Dai JS (2009) Topology of kinematic chains with loops and orientation of joints axes. *Chin Mech Eng* 25(6):34–40
10. Li SJ, Dai SJ (2010) Configuration transformation matrix of metamorphic mechanisms and joint-orientation change metamorphic method. *Chin Mech Eng* 21(14):1698–1703
11. Li SJ, Dai JS (2010) Structure of metamorphic mechanisms based on augmented assur groups. *Chin Mech Eng* 46(13):22–41