"The model approximates the complex prototype asymptotically. The trend of the approximation is to become identical with the prototype. In the limit, model and prototype coincide. For a typical example, the best model of a cat is another cat, or indeed, the original cat proper. In other words, if the model realizes the purpose of the prototype in all its aspects, then it is possible to explore the prototype in all detail, and no model is needed".

Norbert Wiener (26.11.1894–18.3.1964)

2

Methodology of Modeling

Modeling, as a specific form of experiment, enables one to recognize the laws of Nature. Its broad development, numerous forms and many application areas have resulted in the fact that modeling has gradually become a relatively disorganized field. It follows from this that modeling methodology is actually itself a problem.

Apart from other things, the task of methodology is to find a common point of view in sorting models, which enables their classification and explains their specific position in a broader system of experimental means. Clearly, this could be a point of view from which to look at information and similarity.

2.1. Identification, Modeling and Simulation

Modeling represents one of the most general ways of representing the outer world and studying the objective laws existing in it. It is an experimental information process where another physical or abstract object, called a model, is assigned unambiguously to the examined system (original, object or work) according to certain criteria. Dynamic system modeling, with direct or indirect feedback on the examined object, is called simulation.

With modeling, diverse forms of similarity and analogy can be connected. In this sense, similarity can be understood as unambiguous mutual assignment between different systems in terms of their structure, properties and behavior. Physical similarity refers to the similarity between systems and processes, having the same physical substances,
and involves, besides geometric similarity, the similarity of parameters and values of the system state. Mathematical similarity expresses the similarity between systems and processes having identical mathematical descriptions. It is called an analogy when it is about different systems and processes. Finally, cybernetic (functional) similarity expresses the mathematical similarity in the external behavior of systems.

In accordance with the above-mentioned three kinds of similarity, modeling can be classified as physical, mathematical and cybernetic. Cybernetic modeling makes use of black box models. This black box concept, introduced by N. Wiener, is understood to be a system which does not deliver any information on its inner structure but on its external behavior only. Opposite to a black box is a white or, better, a transparent box, which represents the inner structure of a system and a process under way in it. Similarly, a grey box concept can be used, delivering partial information about the inner structure of the system. The experiment is the general superior concept to the modeling one.

Essentially, there are two ways to generate a mathematical model (Fig. 2.1). In direct identification, one proceeds from a summary of knowledge about the behavior of the examined object which has been obtained either as a result of the object identification or as a result of the development of knowledge in a corresponding field. Often, the necessary information is obtained indirectly as a result of identification

Fig. 2.1. Procedure diagram of direct and indirect identification of an object and generation of mathematical and simulation models.
of a physical model and this is called indirect identification. The *results of an experiment* are represented by a summary of knowledge expressed mostly in the form of a phenomenological description of a physical model or an examined object, because of assignment of the model to the examined object. To obtain a general mathematical model, a detailed analysis of the physical principles and of results of an experiment is necessary. This is the only way to create a simulation model and to get with it some credible information on system behavior.

### 2.1.1. Identification and Simulation

In modeling, reliable identification of the properties of the examined object is fundamentally significant. The credibility of a mathematical and simulation model and, therefore, even of modeling results, is closely connected with this. Therefore, the identification represents an important step for systems modeling and simulation on which the efficiency of the modeling depends to a large extent. *Identification* is an experimental way to determine important system and process characteristics enabling the construction of the mathematical model. The *system characteristics* include various physical properties, the structure and parameters of the system and the process. The *system diagnostic* is used to determine the state of the system and the process, especially its variations from a supposed state.

The procedure of identification of an examined dynamic system and consequential simulation process is in the block chart (Fig. 2.2).

---

**Fig. 2.2.** Identification and simulation in the process of investigation of an examined system.
Substantial information on system behavior (about an examined object, original, prototype or work) are gained by direct measurement by means of a measuring system. The identification process results in the formulation of a mathematical model. The mathematical model is transformed into a shape which enables building a simulation model by selected technical means. Based on the comparison of the examined system with a simulation model, one can assume model credibility and utilize the obtained information to improve the original.

2.1.2. Cybernetics and Modeling

The contribution of cybernetics to the development of scientific knowledge is closely connected to improving traditional modeling methods and introducing computer aided modeling. This enrichment is based on changes in our way of thinking because cybernetics is the theory of models in science without strong boundaries among physical, biological and economic processes (S. Bear). It is based on the ability to solve quantitatively higher tasks of optimization and complex dynamic systems control by making use of the theory of cybernetic similarity and analogy. By removing the boundaries among disciplines, cybernetics enables the synthesis of knowledge from diverse fields, the mutual influence of fields and, as a result, even their accelerated development. System modeling and simulation play a crucial role in this process.

Since the beginning of the formulation of cybernetics as a discipline, it has been accompanied by the idea of constructing a computer working as a model analogous with the human brain. This idea steadily influences the development of cybernetic modeling, which represents a highly abstract modeling form. To a certain extent, the power of cybernetic modeling depends on utilizing functional analogies between various dynamic systems.

The systems theory is concerned with general questions of mathematical modeling, such as the analysis and synthesis of complicated technical, biological, economic, social, and other systems. Abstraction, modeling and analogy are fundamental tools in systems theory.

In the mathematical meaning of the word, system means a set of elements connected mutually and to their surroundings by means of interactions which create the properties enabling the fulfillment of their functions. The structure is the inner system arrangement expressed by mutual couplings and actions of components, the inner organization of the whole or the parts of which it is formed.
A formal description of a system can be in the form of $S(X, R)$, where

$$X = \{x_1, x_2, ..., x_n\}$$  \hspace{1cm} (2.1)

is a set of elements and

$$R = \{r_1, r_2, ..., r_m\}$$  \hspace{1cm} (2.2)

is a set of relations among elements where $n, m \in N$. In the simplest case, the relation can be binary $r_k(x_i, x_j)$. The simplest system consists of a pair of elements whereby the set $R$ involves a single relation.

The concept of a system can be enlarged implicitly. In general, one can say that each system element is a system itself and is denoted as a subsystem. Analogously, by connecting systems, a composed system (super-system) is created. The system branching is called a system hierarchy. To model and simulate complex systems, the hierarchical approach has not only methodological but even practical significance.

In Fig. 2.3, the system consists of two subsystems, $S_1$ and $S_2$, where mutual couplings among the elements $x_1$ up to $x_6$ and couplings among the subsystems are displayed. This can concern, e.g. a bearing-base system. Individual elements of the subsystem $S_1$ represent parts of the bearing, whereas the elements of the subsystem $S_2$ are the parts of the base.

Dynamic system means a system $S(\tau)$ in which the state values occur in space and time $\tau$. The limit state of a dynamic system is its initial or steady state. Dynamic systems can be divided into deterministic and stochastic ones, which is a point of view being applied significantly in

![Fig. 2.3. Real system diagram (a) and its abstract model (b).](image-url)
modeling as well. Further, a dynamic system can be classified as non-determined, adaptive, etc. Depending on the method of space and time expression, dynamic systems are divided into continuous or discrete ones in space and time. Dynamic systems can also be classified as linear and non-linear, non-stationary or stationary.

**Composed dynamic system** is a term used in studying the behavior of complicated sets (complexes). Every composed system can become an element (subsystem) of a more complex composed system. A subsystem forms an autonomic system part and its behavior need not correspond to the behavior of the system as a whole, but must be compatible with it by means of direct interactions or information flows. A **partial system** is a detached system part.

**Adaptive dynamic systems** are very important in simulation of controlled processes in situations where not all process parameters or properties are known at the beginning. The system parameters can be estimated only during the course of the process on the basis of information about previous behavior.

In simulating various controlled dynamic systems, the **state function** \( F(U, Fo) \) of the following compound process, but not the function \( U(M, Fo) \), appears often as a **controlled state quantity**, characterizing the fundamental process in place \( M(x, y, z) \) and the time expressed by the Fourier number \( Fo \). A **limiting condition** determines the extent of control of the boundary condition of the basic process. A simulator for starting up a turbo-set is an example, where the simulated basic thermal process is described by a mathematical model in the form of a partial 2\(^{nd}\)-order differential equation; meanwhile, the following compound thermal stress process is described by a partial 4\(^{th}\)-order differential equation. In this case, the temperature \( T \) corresponds to function \( U \), the thermoelastic potential, and the thermal stress and deformation derived from it, corresponds to the function \( F \).

### 2.1.3. Physical Similarity and Modeling

The physical similarity theory is based on similarity criteria. It is about dimensionless values replacing fully the dimensional ones in a studied phenomenon. In particular, the similarity criteria serve to state the modeling scales which state unambiguously the relations between a model and an object. The scales are determined from the equality of mutually corresponding similarity criteria for a model and an object. The determination of similarity criteria is the fundamental task in modeling, not only in simulation model creation but even for compressing and generalizing the modeling results and transferring them to other similar systems and processes. Therefore, they appear as **generalized variables**.
To determine the similarity criteria and, possibly, the dependence of criteria, similarity theory applies three methods of obtaining generalized variables:

- *dimensional analysis*,
- *analysis of a physical phenomenological model*,
- *analysis of a mathematical model*.

Sometimes, the last two methods are called the similarity analysis methods.

Because of the fact that similarity theory lowers the number of operations on a model and that of experiments and enables transferring the results to another similar problem, the efficiency of a model experiment is made higher. In Fig. 2.4, *four basic steps of modelling* are presented. By compacting the information and generalizing the variables in an experiment, the methods of similarity and modeling theory, the so-called *methods of generalized variables*, play the main role. The examined process is described by dimensional values $x_1$ to $x_N$, where $N$ equals the total number of quantities in the process. Due to the conversion of dimensional values to dimensionless ones, by means of a Pi theorem and some of the methods of generalized variables, the total number of variables is reduced. The original process, described by dimensional values, can be expressed by a functional dependence of criteria. In the second phase, the process on a model is described analogically by the functional dependence of the dimensionless criteria of the model (M). By comparing the corresponding criteria for a model and an original, the modeling scales are determined and a simulation model (3rd step) is constructed. The fourth and last step represents the

Fig. 2.4. Diagram of four fundamental modeling phases ($\pi^i$ – composed criteria, $P^i$ – simple criteria, $N$ – total number of quantities, $n$ – number of dimensionally different quantities, $r$ – number of fundamental and complementary dimensions).
simulation process itself and comparison of results with the behavior of the object.

2.1.4 Overview of the Information Theory of Modeling

Essentially, modeling is an informational process in which information about the state and behavior of an examined object is obtained by means of a model. In modeling, the information increases and its information entropy is reduced at the same time due to the increasing knowledge of the object. The extent of knowledge $R$ of an examined object can be expressed in the following form

$$ R = 1 - \frac{H}{H_{\text{max}}}, $$

where $H$ is the information entropy of the object and $H_{\text{max}}$ is its maximum value where the amount of knowledge can become $R \in (0,1)$. The impossibility of reaching the boundary values $R = 0$ and $R = 1$ is among the basic modeling theorems. These boundaries express the ideal states. It follows from the above that there must be a priori and a-posteriori information about the object. The amount of model information $I$ can be determined from the difference of uncertainty $H_1$ and residual uncertainty $H_2$

$$ I = H_1 - H_2. $$

Information entropy is described by the relation derived by K.E. Shannon

$$ H = - \sum_i p_i \log p_i, $$

where $p_i$ is the probability that the studied system is in the $i$-state.

As opposed to some other disciplines, the model information is the main matter of interest of modeling theory. Generally, it has a quantitative as well as qualitative character. When examining it, one proceeds from three fundamental points of view – syntactic, semantic, and pragmatic ones. For modeling and system simulation, the use of all three informational standpoints is characteristic. The following is an example of the application of the above-mentioned standpoints in mathematical modeling:

A: Poisson stress equation (syntactic access),
B: torsion equation (semantic access),
C: torsion equation in a turbo-alternator shaft (pragmatic access).

When comparing these three cases, the quantitatively different information about the mathematical model of shaft torsion of a turbo-alternator is obvious. The syntactic standpoint (case A) is typical for
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classic information theory and does not take into account the content of a report that can be wrong or inexact, as it is in the cited example. Nevertheless, it must be technically and grammatically entirely correct. Case B is syntactically and semantically correct and defines a complete mathematical process model, but does not include the pragmatic part and does not answer the question about the purpose of the model. Only case C involves all three informational standpoints. In addition to the process and its mathematical model, the system in which the process is proceeding is also defined.

2.1.5 Historical Development of Modeling

Looking at the long history of its development, modeling can be divided into several shorter periods defined by substantial qualitative changes in the development of model abstraction. A more detailed explanation is presented in book [2.6]. At the beginning of its development, modeling involved the similarity the primitive man observed during his conscious activities and in contact with Nature, whereas now and in the foreseeable future, there is a complicated cybernetic model in which various hierarchically organized processors, with the structure approaching that of a brain, play the main role.

Most of the model development periods can be considered finished. However, this is not true for the abstract model formulation period, which culminates not only in technical fields, but also in economics, ecology, biology, sociology, and other disciplines. This period is the longest one, which began in history with the Greek astronomer and mathematician Ptolemaios, who built the first well-known mathematical model in the form of a set of cycles and hypercycles describing precisely the movement of planets he observed. The development continued in the Middle Ages, when Leonardo da Vinci assigned an abstract model to the movement of a fish in water and that of a bird in air. The development of physical similarity and modeling theory began towards the end of the 19th century and culminated in the 20th century. More recently, there has been a period of stagnation, due to the coming of computers and initial misunderstanding of the similarity theory in computer-aided modeling, especially in data compression, its treatment and the generalization of results. The development of modeling based on the physical analogy principle culminated in the 60's of the 20th century and was followed by a period of stagnation, due to the arrival of computers in modeling. Nowadays, physical analogy modeling can serve as the illustration way of task solutions.

Computer-aided modeling is undergoing intensive development which is not yet finished. The arrival of computers, the origin and
development of cybernetics and the gradual mathematization and cybernetization of the sciences influence all significant fields of human activity. However, even today’s computer-aided models will pass their dominant position in the system of modeling means to cybernetic models. The apex of development of computer-aided models can be presumed to occur in this century if one takes into account contemporary technical progress.

Technological modeling tasks still represent an important part of solving problems. But the time increment of the number of technological tasks is relatively smaller compared to the tasks in numerous other fields. Nevertheless, in technology, in addition to the nowadays tasks in classic fields such as machinery, power and electrical engineering, and technology, these tasks also appear in recently developing fields such as magneto-hydrodynamics, tribology, plasma technology, micro- and nanotechnology, geophysics, meteorology, non-equilibrium mechanics and many others.

2.2. Classification and Properties of Models

In compliance with the character of the model system, models can be divided into abstract (formal, ideal) and physical ones (objective, of material, of realization). The way to obtain the information is different in both models. The deductive process in an abstract model corresponds to experimental processing in a physical model. A physical model is built with natural or artificial materials. A physical model used for simulation of a dynamic system is called a simulation model. An abstract model consists of a non-material representation (by an idea, symbol or graphic expression, etc.) which describes an examined system. Abstract models are objective in terms of their content, which means that they express the laws of the real world and describe it. Nevertheless, their form is subjective because the same content can be expressed in many different forms. Among abstract models are mathematical models, especially, and models expressed, e.g., by programming languages, flow charts, and the like. In modeling, the use of natural language is inappropriate because it is polymorphic and uncertain.

From the model-to-object similarity point of view, models can be divided into physical models, physical analogues, mathematical analogues and cybernetic analogues. A physical model is based on physical similarity, a physical analogue is based on mathematical–physical similarity, a mathematical analogue is based on mathematical similarity, and a cybernetic analogue is based on mathematical functional similarity. In mathematical and cybernetic analogues, the
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term *analogue* is usually replaced with *model* as a superior term. A *natural model* is a limited case of a physical model, where the model is made identical with the object. *Physical analogues* are based on mathematical and physical similarity, where various phenomena are described by analogous equations; e.g., potential electric distribution and potential flow of a perfect liquid are described by a Laplace partial differential 2nd-order equation. *Mathematical models* are based on mathematical similarity and are usually computer-aided deterministic or stochastic ones. They have dominant positions in contemporary modeling. *Cybernetic models* are based on the principle of the functional input-to-output similarity of a system represented by a *black box*. Cybernetic models are contemporary ones, but first and foremost are a matter of the future, especially in solving global problems of mankind. In models based on mathematical similarity, the process of progressive transformation of mathematical models and the development of abstraction is remarkable.

There are other important terms connected to a model. The model-building process is called *composition* (*synthesis, construction*). In Fig. 2.5, the assignment of a model to an original object is suggested as an example. It is based on observing input $X_i$ and output $Y_i$ quantities. The functional relation $Y_i = F_m(X_i)$ is the model expression of quantity $X_i$ behavior on an original. $F_m$ denotes a *model operator*. *Verification* is the process of quantitative evaluation of a model’s correctness. It is verified whether a model acceptably represents a system both from the structural and the behavioral point of view. The *credibility* (*validity*) expresses the quantitative degree of agreement of a model’s behavior with that of the examined system. A model is credible if its behavior agrees with that of the original within acceptable limits under all the conditions considered in the investigation. This is not a precise check of hypotheses but a relatively subjective evaluation. *Validity* (*valence*) describes the extent of applications in which a model can be considered credible. The *variability* expresses the possibility to adapt a model according to changes of various factors, especially in connection with

![Diagram of syntax and assignment of a functional model to the examined object.](image)

**Fig. 2.5.** Diagram of syntax and assignment of a functional model to the examined object.
the strategy of the simulation procedure. **Adaptability** is the capability of a model to collaborate as a subsystem with other model subsystems. **Time** and **financial demands** are important characteristics of models. Usually, the costs are expressed as dependent on required **modeling precision** or on **task complexity**. This approximate dependence is expressed in Fig. 2.6, as a chart, but cannot be fully generalized. The task complexity concept involves such things as the dimensionality and shape of the solved region, the character of system parameters and boundary and initial conditions. The accuracy is a limited quantity, especially in physical models, physical analogues, analogue computer-aided models and hybrid computer-aided models, which is connected in particular with the limited accuracy of modeling elements. Sometimes, the term **credibility**, which is related to the real object, is more suitable to be used instead of the term accuracy of solution (e.g. the numerical one).

Finally, there is a **speculative model**, which is not really a model, in fact. It is called an insufficiently credible model, which is based on an excessive amount of speculations. It occurs in generating hypotheses or in case a system has not been identified. Speculative models often originate in examining complicated social, economic, ecological, and other systems. However, they also emerge if the model is irresponsibly formulated or realized.

### 2.2.1 Creation, Development and Transformation of a Model

Continuing from the definitions of basic modeling terms, an abstract
model can be converted into a mathematical and simulation model, and various simulation models can be used.

In Fig. 2.7 the development of an abstract model in generating a simulation model is outlined. The first step is a thought model reflecting the real world object or system being examined.

Subsequent transformations of an abstract model are word models and then graphic or symbolic models, of which mathematical model is especially important. A simulation model originates with the solution's method and algorithm and further necessary transformations, depending on what means of simulation have been selected. The way back, indicated by the dotted line from a simulation model to an object in the drawing, only highlights the purposefulness of the simulation, which is directed to the knowledge of an object or system and, possibly, its improvement.

In Fig. 2.8, an unlimited set of possible objects is further divided into real objects and projected or intended (proposed) objects. All models, except the natural ones, enable the modeling of either real objects or only intended ones. However, there are a lot of objects and problems connected with them that cannot be solved nowadays. This can be due to difficulties in the realization of a physical model, or in the algorithmization of the solution, in the formulation of a mathematical or physical model, etc. The intersection of individual sets expresses the fact that many tasks can be solved with various models. The model choice depends on the aim of the solution, on the desired information, on the feasibility of the solution, and on the accessibility of modeling means, among other things. The growth of knowledge and development of natural and technical sciences, in connection with the mathematization and cybernetization of fields, has widened the limits of mathematical as well as cybernetic (functional) modeling, which is based on the similarity of the outer behavior of a system.

![Diagram](image-url)

**Fig. 2.7.** Development of an abstract model in simulation model generation.
2.2.2 Various Categorizations of Models

In Fig. 2.9, models are sorted according to several basic points of view. Depending on the character of the modeling process (A), models are divided into deterministic and stochastic ones. Deterministic models are distinguished by unambiguously determined causes and their consequences. In stochastic models, either the examined problem itself or the method of solution has a random character.

Similarity (B) of course relates to the similarity between an original and a model. The way to treat the model information (C) is among the fundamental points of view in dividing computer-aided models. Model purpose (D) concerns the function of a model as a means to obtain knowledge or to utilize it to control a process. Modeling process control (E) relates to whether the modeling process is passive from the point of view of external influence or is changing actively according to control conditions.

Fig. 2.8. Display of the sets of areas where various sorts of models are used.

Fig. 2.9. Division of models according the character of the modeling process (A), similarity between a model and an object (B), way to treat the model information (C), purpose of a model (D) and modeling process control (E).
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In Fig. 2.10, four types of stochastic models are presented. If the examined process itself is stochastic, the model will be stochastic too (a). An example on this type of model can be found in nuclear physics where only the probabilities with which the original phenomena occur are known. Other cases involve a process which can be described in a deterministic way, but some parameters or disturbing influences on the object are random. If these influences cannot be neglected, the model will be stochastic (b). A mechanical system with random excitation force is an example. If the disturbing influences on the object can be neglected, a deterministic model (c) can be built. The resulting quantity is random due to the action of disturbing influences \( S \) on the model. In evaluating the results, the proceedings, like those for stochastic models, must be used even if the problem and the model are deterministic. A special group of stochastic models (d) is represented by those models which are used to solve deterministic problems with probability methods. These stochastic models and methods are discussed in Chapter 9.

In the majority of cases, models have been comprehended hitherto as a means of gaining new knowledge. Essentially, this is a passive role in which a model acts neither on an examined process nor on an object directly. Nowadays, the significance of models acting actively on a process or object increases. They are applied especially where the necessary information about process behavior cannot be obtained by direct measurement.

The technical point of view for the sorting of simulation models is the *means used for simulation*. All models, deterministic and stochastic,
analogue, numerical and hybrid, can be divided into computer-aided, built on programmable universal simulation means (namely computers), single-purpose programmable computer-aided models and simulators, and single-purpose non-programmable models and simulators.

Expressions of space and time in a model represent fundamental aspects of the division of models and simulation methods. It is displayed in Tab. 2.1. Among many other standpoints applied in distinguishing simulation models, the one for model similarity preservation is significant. In compliance with this point of view, simulation models can be sorted into complete, incomplete and approximate. Complete models ensure complete model similarity in space and time. Incomplete models (partial, local) are distinguished by partial similarity, e.g. in space and time. In practice, incomplete modeling often occurs when the similarity is preserved only in certain places (O, A, B, C) of a space or time system of coordinates, as can be seen in Fig. 2.11. In general, the function $U(X, \tau)$ is not similar on a model and an object. Modeling can be carried out when the investigation is limited to certain places (Fig. 2.11a) where the similarity conditions are fulfilled. Another example of incomplete modeling is that with similarity in partial sections OA and OA' of the characteristic, see Fig. 2.11b. This case is frequent, e.g. when the elastic stress of bodies is modeled. In approximate models the similarity criteria are usually not preserved. The modeling error depends on the approximate expression of the parameters and on its total influence on the course of the process as well.

**Tab. 2.1 Sorting of models with respect to space and time**

<table>
<thead>
<tr>
<th>Models</th>
<th>Unsteady Continuous time</th>
<th>Unsteady Discrete time</th>
<th>Steady</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuous space</td>
<td>CSCT</td>
<td>CSDT</td>
<td>CS</td>
</tr>
<tr>
<td>Discrete space</td>
<td>DSCT</td>
<td>DSDT</td>
<td>DS</td>
</tr>
</tbody>
</table>

**Fig. 2.11.** Example of incomplete similarity in modeling – local (a) and initial (b) similarity.
2.2.3. Sorting Models According to Degree of Abstraction

The basic point of view applied in sorting models is the level of abstraction from a real object (original, work). The amount of information the model preserves compared to the object is decisive. According to the degree of abstraction, simulation models can be divided as follows.

**Zero-abstraction-degree model**
The model is the original under working conditions directly. The amount of information is one hundred percent.

**First-abstraction-degree model (Fig. 2.12a)**
The physical process is fully preserved. Some modeling scales are changed. Essentially, it is the object (original, prototype, machine) under various testing conditions. This is a natural physical model.

**Second-abstraction-degree model (Fig. 2.12b)**
The physical process is fully or partially preserved in the inner system structure in which another substance can act. For example, in liquid flow modeling the aggressive substance can be replaced by another non-aggressive one, etc. The laws of physical similarity are decisive. Some scales are not unit ones. The mathematical description is identical with that of the object but for the solution it need not be known. This is a physical model.

**Third-abstraction-degree model (Fig. 2.12c)**
The original physical process is replaced by an analogous one. The analogy between object and model inner structures is preserved. The physical similarity laws are fully or partially valid. This is based on the similarity of mathematical models of both analogue processes. The model is a physical analogue.

**Fourth-abstraction-degree model**
The similarity between the inner structures of the object and the model is not preserved. A system with distributed parameters is modeled by a system with concentrated parameters. The similarity exists in nodal points of the model only. The parameters distributed in the whole volume of an object cell are concentrated into one node in the model. The mathematical and, to a certain extent, even the physical similarity are preserved. The most widespread computer-aided models, working on the principle of numerical methods, and various discrete analogue models are among this group. If the concentration of model parameters must be emphasized, the model is called a passive network analogue.
Fifth-abstraction-degree model
The original physical process in a system with distributed parameters is modeled by means of an active network analog. The mathematical process similarity is decisive at points where the inputs and outputs of elements of a mathematical model meet. The fundamental differential equation is converted to a set of ordinary differential or algebraic equations. It represents a mathematical model.

Sixth-abstraction-degree model
The process in a model is similar to that of an object in terms of external behavior only – functional similarity. Neither physical nor mathematical similarities in the inner structure are considered. It is a functional model of a black box. In Fig. 2.14, a functional system model is represented by a black box with the function $f_1(T)$ at the input and the function $f_2(T)$ at the output. The function $F$ which is modeled depends, for example, on dimensions, physical material properties and external conditions.

Seventh-abstraction-degree model
This is a functional model of two black boxes. The similarity in the inner
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structure: is not considered. Only the mathematical similarity in external mutual influence oil sub-systems A and B is considered. An example of a two-element functional model is in Fig. 2.14b, displaying the action of sub-system A on sub-system B.

**Eighth-abstraction-degree model**
This is a *functional model with many black boxes* and mutual influences among them. The mathematical similarity in external behavior of individual elements of a model set is decisive. Figure 2.14c shows a multi-element functional model of mutual influences among individual sub-systems.

**Ninth-abstraction-degree model**
This is a *two-set hierarchic functional model* in which there are inner mutual couplings in each of both sets, with resulting action of one black box (element) set on the second set.

**Tenth-abstraction-degree model**
These are the most complicated *functional multi-set models, organized in a hierarchy*. Such models are characteristically used for complicated biological systems especially.

**Relation between model abstraction and similarity**

Figure 2.15 shows a relationship between similarity and the degree of abstraction. To previously defined concepts of similarity and abstraction...
one can add that mathematical similarity concerns either an inner process (up to 5th degree of abstraction) or external system behavior (6th and higher degree of abstraction). According to this, all models can be divided into *models of inner and external behavior*. The inner behavior models can preserve similarity either at all points of a continuous model (up to 3rd degree of abstraction) or at selected nodal points only (models of 4th and 5th degree of abstraction).

### 2.2.4 Model Simplification and Credibility

*Simplification* is a phenomenon accompanying modeling. The more complicated the investigated problem is, the more simplification is expressed. Because there is no absolute model-original conformity even from the information standpoint, one of the fundamental questions in modeling is how to find the optimal relation between model simplification and credibility. This is valid for building both an abstract mathematical model and a physical simulation model.

There is no general rule to find this relationship. The character of the task to be solved, the aim of the solution and good knowledge of the problem’s physical parameters are most significant. The designer’s ability is very important to distinguish and sort out essential things from non-essential ones and to take the simplification to a limit where the model remains credible and the solution is low-cost but adequately accurate. Usually, simplification concerns the character of the process, dimensionality and shape of the modeled zone (region), physical parameters, variability of boundary and initial conditions, inner sources or troughs.

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**Fig. 2.15.** Sorting models according to similarity and the degree of abstraction.
In achieving model credibility, the problem of its simplification arises especially if complicated technical and even non-technical systems, e.g. biological ones, are modeled. Figure 2.16 shows an example of human-body model simplification to parts in the shape of a multilayer plate or cylinder expressing the anatomical structure of the body. However, this simplification can be made especially if medium-integral quantities, e.g. blood temperature in a body, are being investigated. A practical case of thermal process modeling of the human body exposed to a thermal shock is discussed in Chapter 8.

The above-mentioned example of the simplification of a model of the human body, as a complex biological system, to a simple plate is an extreme case which has many analogies, even in other areas. On the other hand, there are a lot of cases in which the zone shape cannot be simplified. The fact that each mathematical and simulation model always has errors does not belittle the significance and results of modeling. In simulation, diverse differential diagrams of an approximate mathematical model can be defined very precisely, but one must consider whether higher modeling claims are satisfactorily counterbalanced by more precise results.

2.2.5. Tasks and Algorithmization of Solutions

Input analysis of the problem and determination of the task character are fundamentals requirements for success of modeling.

Fig. 2.16. Example of model simplification of a complex biological system – the human body.
Problem analysis

The input analysis in which a complicated task is divided into partial physical tasks is an important part of every problem solution. Depending on the character of partial tasks, theoretical or experimental solutions or, possibly, both of them, are chosen. Figure 2.17 shows a procedure diagram for problem analysis, method selection and solution. The theoretical problem analysis, aimed at building physical and mathematical descriptions, is the most difficult part. The choice of the method, for which a deterministic or stochastic approach can be applied, is a further step. As will be described in further chapters, each of these approaches can give a completely different significance to the concept of the task complexity. Logical, analytical, numerical, analogous, and hybrid methods are available. After the method has been chosen, a modeling solution follows in which specific properties of numerical, analogue, and hybrid methods can be utilized.

Classification of tasks

In modeling, the tasks can be divided essentially into four basic groups, which differ in their characters and also the method of solution. There are often required direct tasks, and indirect tasks, identification and optimization tasks as well.

Figure 2.18 shows a diagram for classification of tasks in modeling. The task always involves a fundamental equation or a set of them and further conditions for an unambiguous solution, among which are
boundary and initial conditions, and further restrictive and optimization conditions. In solving stationary tasks, a boundary condition is sufficient to obtain an unambiguous solution, whereas for non-stationary processes the initial condition must also be considered. In optimization, restrictive and optimization conditions are joined to the above-mentioned ones.

**Direct tasks**

In solving direct tasks, it is essential to determine the inner system response to external stimulation (boundary condition) and initial system state (initial condition), with given system parameters (zone shape, physical parameters, inner sources, movement of a boundary or a source) and known mathematical description. The solution results in a statement of a dependent variable as a function of place and time. Direct tasks can be sorted according various characteristics, e.g. into *steady* and *unsteady*, *linear* and *non-linear*, those with *simple* or *composed boundary conditions*, those with *movement of a boundary, phase change*, with *inner sources or sinks* of energy, and the like.

![Task classification in modeling](image)

**Fig. 2.18.** Task classification in modeling.
The majority of direct tasks in the last century can be explained by the fact that the focus of problems was in analysis of results for investigated processes rather than in optimal control of these processes.

**Indirect tasks**

Indirect tasks are opposite to direct ones and are therefore called reversed or, sometimes, return tasks. The corresponding external influences, initial state or system parameters are determined for the known-in-advance or requested inner behavior of the system, expressed by a known dependent variable in a system. There are many indirect tasks which can be put into three basic groups. For a boundary indirect task, a boundary condition is sought with the known dependent variable values in the system, known system parameters, initial conditions and a mathematical description. For an initial indirect task, an initial condition is sought with given boundary conditions, system parameters, dependent variable values and a mathematical description. For a parametric indirect task, a certain system parameter (zone shape, physical parameters, inner sources, boundary or source motion) is sought with the given boundary and initial conditions, known dependent variable values and a mathematical description. A parametric task is often called an inner or inverse task. The solution of strongly non-linear parametric tasks is very complicated.

**Identification tasks**

Identification tasks are solved to state or, more often, to increase the accuracy of a system model, i.e. of a fundamental equation or a set of equations, and of equations corresponding to the initial conditions or system parameter distribution. Essentially, an input-to-output transmission of an investigated system is being sought. In solving, the system responses to external stimulations are being sought without knowing the inner system behavior, with the system represented in such a case by a black box for which the input-to-output transmission is being solved. In the case of linear systems, one can pass, by means of a reverse transformation, from a measured transmission to a fundamental mathematical model or to making it more precise. Sometimes, the identifications tasks are called inductive ones.

**Optimization tasks**

The above-mentioned direct, indirect and identification tasks deal with system behavior and a model of it. The optimization tasks are directed
to optimize the system behavior in accordance with a given criterion, i.e. to control the process being underway therein, in order to fulfil the conditions for the optimization of the process with the given restrictive conditions (limiting the model variability during its optimization) and optimization conditions (expressing optimal values of a system dependent variable). Because the system behavior can be affected mainly by regulating the boundary or initial conditions or system parameters, the optimization tasks are closely related to indirect tasks. The restrictive and optimization conditions are given by the character of the task to be solved.

The optimization tasks can be divided into four groups. A boundary optimization task is most frequent. With the given initial condition and system parameters, the boundary condition distribution in time and space is being sought so that the dependent variable may fulfil the restrictive and optimization conditions in addition to a fundamental equation. Similarly, for an initial optimization task, the initial condition distribution is being sought, while for a parametric optimization task some system parameter distribution in time and space is being sought. A mixed optimization task is a combination of these tasks.

The solving optimization tasks involves a qualitative change in the approach to various technical and even non-technical task solutions. Indirect tasks as well as optimization ones can be solved as a large number of alternatives (approximations) of direct tasks too. One then speaks about an alternative (approximation) indirect or optimization task. Although the alternative task solution is simpler, it is usually much more time-consuming to solve the large number of alternative tasks than the indirect or optimization task.