1

Introduction

This book provides an introduction to the topic of aberrations in optical imaging systems. Aberrations are of interest because they often degrade the image quality in optical systems. To obtain sharp images the aberrations must be corrected, balanced, minimized, or avoided. Figure 1.1 shows images of light point sources in the presence of aberrations: spherical aberration, coma, and astigmatism. These images exhibit axial, plane, and double plane symmetry.

There is a plurality of effects that create aberrations and degrade an image: for example, variations in the media where light propagates, roughness in the surfaces of an optical system, and misalignment of optical components. This book is concerned with the classical aberrations of spherical aberration, coma, astigmatism, field curvature, distortion, and higher-order forms.

The subject of aberrations is discussed using the wave theory of aberrations pioneered by H. H. Hopkins [1]. To precisely determine image quality it is necessary to have knowledge of the optical field at the exit pupil of an optical system and then perform a diffraction calculation. Wave theory provides the geometrical field that is used to determine the nature of the image formed by a given optical system. This book provides a modern presentation of the wave theory of aberrations, and also discusses sixth-order aberrations, pupil aberrations, the irradiance function, aberrations in non-axially symmetric systems, aberration fields, and polarization aberrations. These recent topics advance our knowledge about how light propagates in optical systems. The value of the theory of aberrations lies in providing insights and understanding about the behavior of light as it propagates. This knowledge is essential for the design of optical systems[1] and for interpreting correctly the results of computer ray tracing. Overall, the book provides a solid and useful foundation about aberrations in optical imaging systems.

[1] See for example M. J. Kidger, "The importance of aberration theory in understanding lens design," in C. H. F. Velzed (ed.), *Fifth International Topical Meeting on Education and Training in Optics Proc. SPIE*, Vol. 3190, SPIE, 1997, pp. 26–33.

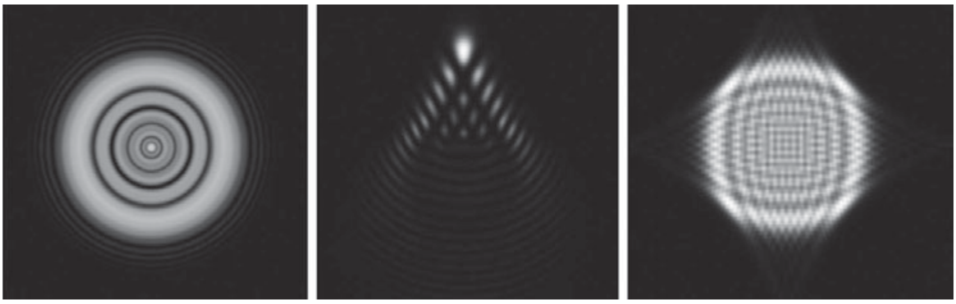

Figure 1.1 Images of light point sources in the presence of spherical aberration, coma, and astigmatism aberrations.

1.1 Optical systems and imaging aberrations

Optical systems can be divided into imaging and non-imaging. The imaging systems considered in this book produce images by reflection, refraction, or diffraction of light. Imaging systems are intended to form images of objects that are self-luminous or that redirect light. Intuitively an image should be similar to the object or be precisely defined by an ideal model. Ideal images are often defined by application dependent requirements. Because of the inherent nature of optical systems, the images formed are not perfect as optical aberrations degrade them. Aberrations can be considered as departures from an ideal behavior.

Historically the analysis of extended images has been done by first analyzing the images of point sources located within the field of view of the optical system. A point source is a non-physical entity that is useful for establishing the point from which light rays emerge. Rays are normal to the geometrical wavefront, which is defined as a surface of equal optical path measured along the rays and from the point source. In an ideal optical system the geometrical spherical wavefront originating from an object point propagates through the optical system and converges as a spherical wavefront towards an ideal real point (or appears to diverge from an ideal virtual point), as shown in Figure 1.2.

In practice, and because of the inherent geometrical shape of the optical surfaces in the system, the converging wavefront may not be spherical but deformed. We are interested in determining the wavefront deformation from a spherical shape as a function of the field of view and aperture of the optical system.

More generally a goal of the theory of wavefront aberrations is to determine analytically the geometrical optical field $G(\vec{H}, \vec{\rho})$ at the exit pupil of an optical system as specified by

$$G(\vec{H}, \vec{\rho}) = \sqrt{I_0 \cdot I(\vec{H}, \vec{\rho})} \cdot \exp\left\{-i\frac{2\pi}{\lambda}(S(\vec{H}, \vec{\rho}) - W(\vec{H}, \vec{\rho}))\right\}, \tag{1.1}$$

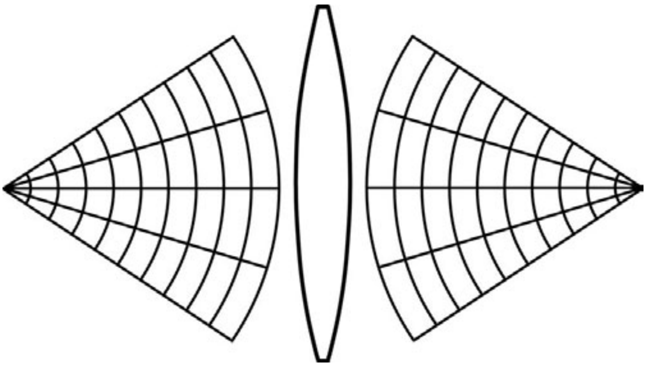

Figure 1.2 Representation of ideal propagation of rays and wavefronts in a lens system from an on-axis object point to an image point.

where $i = \sqrt{-1}$, λ is the wavelength of light, n is the index of refraction of the image space, $\vec{H}$ is the field vector, $\vec{\rho}$ is the aperture vector, I_0 is the irradiance in W/m^2 at the pupil and field center $\vec{H} = 0$, $\vec{\rho} = 0$.

The function $I(\vec{H}, \vec{\rho})$ is dimensionless and it conveys the changes in irradiance at the exit pupil plane. The function $S(\vec{H}, \vec{\rho})$ is called the sphere function and it gives the optical path length from the pupil plane to a sphere that has its center at an ideal image point. The aberration function $W(\vec{H}, \vec{\rho})$ gives the optical path difference (OPD) between the sphere function $S(\vec{H}, \vec{\rho})$ and the wavefront. In the absence of wavefront aberrations the function $W(\vec{H}, \vec{\rho})$ is zero and the sphere function $S(\vec{H}, \vec{\rho})$ gives the optical path of a spherical wavefront from the pupil plane.

To determine the nature of an image according to wave theory of light, a diffraction calculation is performed which requires knowledge of the optical field. Near the image the geometrical and physical fields are substantially different. However, the phase and amplitude of the physical field at the exit pupil of an optical system can be estimated as equal to the phase and amplitude of the geometrical field. Then the nature of the image can be determined using scalar or vector diffraction theory.

For the case of a beam of light that is focused by an aplanatic system, the diffraction calculated image for monochromatic light is the Airy pattern[2] as represented in Figure 1.3.

In 1835 G. B. Airy [2] calculated the position of the minima and maxima in the system of rings. This provided further support for the wave theory of light propagation when it was compared with the actual measured values.

[2] A detailed description of the Airy pattern is given by M. Born and E. Wolf, "The three dimensional light distribution near focus," Section 8.8 in *Principles of Optics*, 6th edition, Cambridge University Press, 1980. See also B. Richards and E. Wolf, "Electromagnetic diffraction in optical systems II. Structure of the image field in an aplanatic system," *Proc. R. Soc. Lond.* A, **253**:1274(1959), 358–379.

Cambridge University Press
978-1-107-00633-1 - Introduction to Aberrations in Optical Imaging Systems
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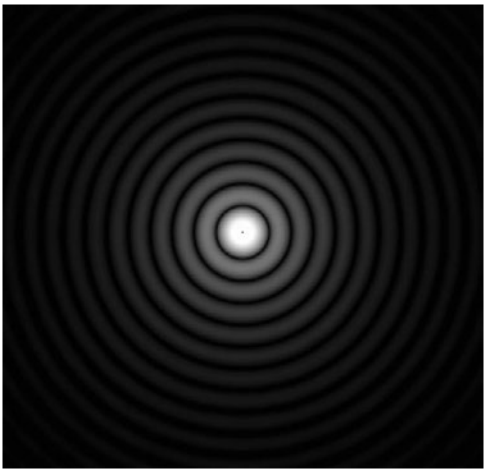

Figure 1.3 Enhanced representation of the Airy pattern.

1.2 Historical highlights

The deviation of rays of light from focusing in a point is called ray aberration. The presence of image defects due to aberrations in optical instruments such as telescopes and microscopes was known following their invention. Optical surfaces are naturally made spherical in form since in the grinding and polishing process the spherical form allows two surfaces to be in uniform contact independently of their relative position. The ray aberration from mirrors or lenses with spherical surfaces was called spherical aberration and the aberration resulting from the different refractivity (light dispersion) of a transparent substance was called chromatic aberration. Spherical aberration has axial symmetry and this makes its basic understanding simple. The phenomenon of spherical aberration in spherical mirrors was pointed out by Roger Bacon in the thirteenth century [3] and described by J. Kepler in his *Dioptrique* (1611) and by R. Descartes in his *Dioptrique* (1637) [4].

The chromatic change of focus had also been recognized, and as early as 1616 the use of a concave mirror in place of a glass objective had been proposed by N. Zucchi [4]. By the end of the eighteenth century a variety of refracting telescopes with achromatic objective lenses were being sold in England by the Dollonds [4]. The main effort in the understanding of aberrations was given to the correction of spherical aberration and the chromatic change of focus. Owing to the small field of view of telescopes and microscopes there was little attention given to off-axis aberrations. The understanding of off-axis aberrations was not as simple as that of spherical aberration. However, by the beginning of the nineteenth century the comet-like flare in the image of stars and in the images of microscopes had already been noticed. The development of the telescope, the microscope, the camera

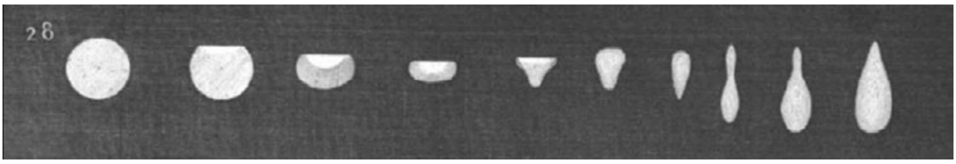

Figure 1.4 Thomas Young's sketch of the images produced by oblique rays passing through a lens, and at different distances from the lens (through focus).

obscura, and the study of the defects of the eye led to the basic understanding of optical aberrations.

The discovery of astigmatism, and especially the understanding of coma aberration, required intelligent inquiry. Initially there was no distinction between spherical aberration, coma, and astigmatism; they were considered part of the same phenomenon of spherical aberration and ray caustics. After the papers by T. Young [5] and G. Airy [6] the aberration of astigmatism was defined and understood [7]. The formulas for the object and image conjugates upon oblique refraction are due to T. Young [8] [9]. These formulas are often known as Coddington's equations [10]. The term astigmatism (not stigmatic, or not pointy) was introduced by G. Airy [11]. T. Young's sketch [5] of the aberrated images produced by oblique rays passing through a lens suggests the basic understanding, at that time, of an off-axis aberrated image; see Figure 1.4.

W. H. Wollaston's [12] [13] discovery in 1804 of periscopic lenses and their application to spectacles and to the camera obscura furthered the understanding of aberrations. It was clear now that the aperture stop location played an important role in controlling off-axis aberration. G. B. Airy [14] [15] in 1827 explained spherical aberration (in a broad sense) as producing three effects: image distortion, field curvature, and lack of image sharpness. He carried out an analytic study of periscopic lenses applied to the camera obscura and discovered that there is a trade-off between astigmatism and field curvature. The understanding of this trade-off turned out to be critical in the development of the photographic lens. In his analytical study G. B. Airy developed equations for the curvature of the astigmatic curves. H. Coddington [16] further developed Airy's treatment and by 1829 obtained general formulas for the curvature of astigmatic surfaces, including Petzval's field curvature. H. Coddington was aware of the works of T. Young, G. B. Airy, and J. Herschel [17] [18] and credits them in his treatise preface. H. Coddington adds the following:

> Having been fortunate enough to discover a general method which enabled me to extend to all analogous cases investigations similar to one of those which had furnished the most important results, the symmetry, and beauty of the results . . .

In paragraph 142 of his treatise H. Coddington writes

If we pass on to the equations belonging to a combination of lenses, we find in like manner, that the locus of the secondary focal line has for its radius of curvature

$$\frac{1}{\sum\left\{\frac{V}{f}+\frac{1}{\mu f}\right\}},$$

that of the primary focal line,

$$\frac{1}{\sum\left\{3\frac{V}{f}+\frac{1}{\mu f}\right\}},$$

and that of the circle of least confusion

$$\frac{1}{\sum\left\{2\frac{V}{f}+\frac{1}{\mu f}\right\}};$$

whence we conclude that if

$$\sum\left\{\frac{V}{f}\right\}=0,$$

there will be a distinct image with a radius of curvature,

$$1\div\sum\left\{\frac{1}{\mu f}\right\}.$$

If the lenses be all of the same refracting substance, this radius of curvature of the image takes the form

$$\frac{\mu}{\sum\frac{1}{f}},$$

and is remarkable as being independent of the manner in which the several lenses are disposed, their distances from each other, or in fact anything but their absolute focal lengths.

It is consequently impossible to have a distinct image formed on a plane surface by any combination of convex lenses, since f being for each a positive quantity,

$$\sum\frac{1}{\mu f}$$

can never become infinity; but if

$$\sum\left\{2\frac{V}{f}+\frac{1}{\mu f}\right\}=0,$$

the locus of the circle of least confusion will be plane, and a tolerably distinct image will be formed in that plane.

In these expressions μ stands for the index of refraction, $f = (\mu - 1)(\frac{1}{r_1} - \frac{1}{r_2})$, V refers to astigmatism, and r_1 and r_2 are the surface radii of curvature of a given lens.

Clearly, H. Coddington understood astigmatism and field curvature aberration, as well as the trade-off between them. H. Coddington appears to have priority in the credit for the famous Petzval sum; this view is also expressed by G. C. Steward.[3] The formula is credited to J. Petzval after his paper [19] in which no derivation is provided. S. F. Ray [20] has noted, "No doubt he [J. Petzval] was influenced by the work of Airy and Lister as described." Furthermore, the concept of aberration expansions and aberration sums was well-known after H. Coddington's treatise. At that time there was in England a golden period with strong contributors to optics and optical instrument makers.

The term aplanatic (without error) is attributed to J. Herschel [18]; it has been used differently to indicate absence of spherical aberration, or absence of spherical aberration and chromatic aberration, and currently it is used to indicate absence of spherical aberration and coma.[4] The aberration of coma was not so obvious to discern or to fully understand. Because of the schematic figures of ray caustics, coma was conceived as a caustic asymmetry. The modern geometrical representation of coma probably started with the detailed study by H. D. Taylor [21] published in 1906. J. J. Lister [22] in 1830 used the term coma to describe the comet-like flare in images and showed how by proper arrangement of the doublets in a microscope objective the coma could be reduced or corrected. Credit is given to J. Fraunhofer [23] [24] in designing achromatic doublets with uniform correction for spherical aberration over the field of view.

The primary aberrations were theoretically established after the series of papers "Theory of Systems of Rays" by W. R. Hamilton that appeared in the *Transactions of the Royal Irish Academy* [25] [26] [27] [28]. The summary [29] of Hamilton's research of 1833 refers to the function $T = T^{(0)} + T^{(2)} + T^{(4)}$ as describing the properties of instruments of revolution, where

$$T^{(2)} = P_1(\alpha^2 + \beta^2) + P_2(\alpha\alpha' + \beta\beta') + P_3(\alpha'^2 + \beta'^2)$$

and

$$\begin{aligned} T^{(4)} = {} & Q_1(\alpha^2 + \beta^2)^2 + Q_2(\alpha^2 + \beta^2)(\alpha\alpha' + \beta\beta') \\ & + Q_3(\alpha^2 + \beta^2)(\alpha'^2 + \beta'^2) + Q_4(\alpha\alpha' + \beta\beta')^2 \\ & + Q_5(\alpha\alpha' + \beta\beta')(\alpha'^2 + \beta'^2) + Q_6(\alpha'^2 + \beta'^2)^2. \end{aligned}$$

3 G. C. Steward, "The geometrical aberrations," Chapter 3 in *The Symmetrical Optical System*, Cambridge University Press, 1928.

4 More generally the term aplanatic can be understood to signify absence of spherical aberration and linear phase variations as a function of the field of view.

The second-order coefficients P_1, P_2, P_3 are identified as defining the focal lengths, the magnifying powers, and the chromatic aberrations. The fourth-order coefficients Q_1, Q_2, Q_3, Q_4, Q_5, Q_6 are identified as defining the spherical aberrations (in Airy's sense). The variables $(\alpha^2 + \beta^2)$, $(\alpha\alpha' + \beta\beta')$, and $(\alpha'^2 + \beta'^2)$ are the rotational invariants. The concept of independent aberrations by order was then clearly established.

In connection with the precise definition of focal length, C. F. Gauss [30] in 1836 developed a first-order theory where the concepts of principal points and planes are introduced. C. F. Gauss's theory is congruent with the theory of collinear transformation and therefore it provides a consistent theoretical model for describing and calculating the basic imaging properties of axially symmetric lens systems.

The calculation of aberration coefficients, except perhaps for coma and distortion, in a system of lenses was understood to a significant extent by the middle of the nineteenth century. However, the systematic calculation of the primary aberrations in a system of optical surfaces became feasible when L. Seidel [23] in 1856 extended C. F. Gauss's treatment to third-order of approximation and provided specific formulas. O. Lummer [31] provides an interesting review of Seidel's work.

J. Petzval designed his famous portrait lens in 1839 [32] [33] . It is said that the manuscripts of his theoretical work on aberrations were stolen by thieves, burned in a house fire, or destroyed. In part the merit of Petzval was to understand the state-of-the-art in optical design, and become the world's optical design expert of his time. As a young scientist and newcomer to the field of optics, and to be successful, he needed to understand the correction of chromatic aberrations, spherical aberration including higher orders, coma and its correction by compensation, the trade-off between astigmatism and field curvature, light vignetting, fabrication issues, and had to have the ability to perform ray tracing. The design of the Petzval lens was quite a feat. Apparently Petzval did not have an optics shop available to him to help the design process by making some test lenses. It is remarkable that after he handed over the lens prescription to P. W. F. Voigtländer for the lens to be made, Petzval's portrait lens was found to work!

In 1905 K. Schwarzschild [34] published his investigations on geometrical optics where he used W. R. Hamilton's characteristic function (or H. Bruns's [35] eikonal) to develop an aberration theory to fifth-order of approximation. Later and independently G. C. Steward [36], M. Herzberger [37], and H. A. Buchdahl [38] [39] further developed W. R. Hamilton's ideas and calculated higher-order transverse ray aberration coefficients. J. Focke [40] provides a review of higher-order aberration theory, and recent advances in the subject have been contributed by C. H. F. Velzel and J. L. F. Meijere [41], and by F. Bociort, T. B. Anderson, and L. H. Beckmann [42].

Since the advent of digital computers and precise manufacturing techniques there have been a plurality of new topics such as aberrations in gradient-index optics [43] [44] [45], aberrations in diffractive optical elements [46] [47] [48], aberrations in multiple aperture systems [49], polarization aberrations [50], and random aberrations [51] [52]. Overall the field of optical aberrations is of technological importance and continues to grow with new exciting developments.

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