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CHAPTER **ONE**

Methods in Algebra

Mathematics is the study of structure, pursued using a highly refined form of language in which every word has an exact meaning, and in which the logic is expressed with complete precision. As the structures and the logic of their explanation become more complicated, the language describing them in turn becomes more specialised, and requires systematic study for the meaning to be understood. The symbols and methods of *algebra* are one aspect of that special language, and fluency in algebra is essential for work in all the various topics of the course.

STUDY NOTES: Several topics in this chapter will probably be quite new — the four cubic identities of Section 1E, solving a set of three simultaneous equations in three variables in Section 1G, and the language of sets in Section 1J. The rest of the chapter is a concise review of algebraic work which would normally have been carefully studied in previous years, and needs will therefore vary as to the amount of work required on these exercises.

1 A Terms, Factors and Indices

A *pronumeral* is a symbol that stands for a number. The pronumeral may stand for a known number, or for an unknown number, or it may be a *variable*, standing for any one of a whole set of possible numbers. Pronumerals, being numbers, can therefore be subjected to all the operations that are possible with numbers, such as addition, subtraction, multiplication and division (except by zero).

Like and Unlike Terms: An *algebraic expression* is an expression such as

$$x^2 + 2x + 3x^2 - 4x - 3,$$

in which pronumerals and numbers and operations are combined. The five *terms* in the above expression are x^2, $2x$, $3x^2$, $-4x$ and -3. The two *like terms* x^2 and $3x^2$ can be combined to give $4x^2$, and the like terms $2x$ and $-4x$ can be combined to give $-2x$. This results in three *unlike terms* $4x^2$, $-2x$ and -3, which cannot be combined.

WORKED EXERCISE: $x^2 + 2x + 3x^2 - 4x - 3 = 4x^2 - 2x - 3$

Multiplying Terms: To simplify a product like $3xy \times (-6x^2y) \times \frac{1}{2}y$, it is best to work systematically through the signs, the numerals, and the pronumerals.

WORKED EXERCISE: (a) $4ab \times 7bc = 28ab^2c$ (b) $3xy \times (-6x^2y) \times \frac{1}{2}y = -9x^3y^3$

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Index Laws: Here are the standard laws for dealing with indices (see Chapter Six for more detail).

1 INDEX LAWS:

$$a^x\,a^y = a^{x+y} \qquad (ab)^x = a^x\,b^x$$
$$\frac{a^x}{a^y} = a^{x-y} \qquad \left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$$
$$(a^x)^n = a^{xn}$$

WORKED EXERCISE:

(a) $3x^4 \times 4x^3 = 12x^7$

(b) $(48x^7y^3) \div (16x^5y^3) = 3x^2$

(c) $(3a^4)^3 = 27a^{12}$

(d) $(-5x^2)^3 \times (2xy)^4 = -125x^6 \times 16x^4y^4$
$= -2000x^{10}y^4$

(e) $\dfrac{(6x^4y)^2}{3(x^2y^3)^3} = \dfrac{36x^8y^2}{3x^6y^9}$
$= \dfrac{12x^2}{y^7}$

Exercise 1A

1. Simplify:
(a) $3x - 2y + 5x + 6y$
(b) $2a^2 + 7a - 5a^2 - 3a$
(c) $9x^2 - 7x + 4 - 14x^2 - 5x - 7$
(d) $3a - 4b - 2c + 4a + 2b - c + 2a - b - 2c$

2. Find the sum of:
(a) $x + y + z,\ 2x + 3y - 2z$ and $3x - 4y + z$
(b) $2a - 3b + c,\ 15a - 21b - 8c$ and $24b + 7c + 3a$
(c) $5ab + bc - 3ca,\ ab - bc + ca$ and $-ab + 2ca + bc$
(d) $x^3 - 3x^2y + 3xy^2,\ -2x^2y - xy^2 - y^3$ and $x^3 + 4y^3$

3. Subtract:
(a) x from $3x$
(b) $-x$ from $3x$
(c) $2a$ from $-4a$
(d) $-b$ from $-5b$

4. From:
(a) $7x^2 - 5x + 6$ take $5x^2 - 3x + 2$
(b) $4a - 8b + c$ take $a - 3b + 5c$
(c) $3a + b - c - d$ take $6a - b + c - 3d$
(d) $ab - bc - cd$ take $-ab + bc - 3cd$

5. Subtract:
(a) $x^3 - x^2 + x + 1$ from $x^3 + x^2 - x + 1$
(b) $3xy^2 - 3x^2y + x^3 - y^3$ from $x^3 + 3x^2y + 3xy^2 + y^3$
(c) $b^3 + c^3 - 2abc$ from $a^3 + b^3 - 3abc$
(d) $x^4 + 5 + x - 3x^3$ from $5x^4 - 8x^3 - 2x^2 + 7$

6. Multiply:
(a) $5a$ by 2
(b) $6x$ by -3
(c) $-3a$ by a
(d) $-2a^2$ by $-3ab$
(e) $4x^2$ by $-2x^3$
(f) $-3p^2q$ by $2pq^3$

7. Simplify:
(a) $2a^2b^4 \times 3a^3b^2$
(b) $-6ab^5 \times 4a^3b^3$
(c) $(-3a^3)^2$
(d) $(-2a^4b)^3$

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8. If $a = -2$, find the value of: (a) $3a^2 - a + 4$ (b) $a^4 + 3a^3 + 2a^2 - a$

9. If $x = 2$ and $y = -3$, find the value of: (a) $8x^2 - y^3$ (b) $x^2 - 3xy + 2y^2$

10. Simplify: (a) $\dfrac{5x}{x}$ (b) $\dfrac{-7x^3}{x}$ (c) $\dfrac{-12a^2b}{-ab}$ (d) $\dfrac{-27x^6y^7z^2}{9x^3y^3z}$

11. Divide:

(a) $-2x$ by x
(b) $3x^3$ by x^2
(c) x^3y^2 by x^2y
(d) a^6x^3 by $-a^2x^3$
(e) $14a^5b^4$ by $-2a^4b$
(f) $-50a^2b^5c^8$ by $-10ab^3c^2$

DEVELOPMENT

12. Simplify: (a) $\dfrac{3a \times 3a \times 3a}{3a + 3a + 3a}$ (b) $\dfrac{3c \times 4c^2 \times 5c^3}{3c^2 + 4c^2 + 5c^2}$ (c) $\dfrac{ab^2 \times 2b^2c^3 \times 3c^3a^4}{a^3b^3 + 2a^3b^3 + 3a^3b^3}$

13. Simplify: (a) $\dfrac{(-2x^2)^3}{-4x}$ (b) $\dfrac{(3xy^3)^3}{3x^2y^4}$ (c) $\dfrac{(-ab)^3 \times (-ab^2)^2}{-a^5b^3}$ (d) $\dfrac{(-2a^3b^2)^2 \times 16a^7b}{(2a^2b)^5}$

14. What must be added to $4x^3 - 3x^2 + 2$ to give $3x^3 + 7x - 6$?

15. Take the sum of $2a - 3b - 4c$ and $-4a + 7b - 5c$ from the sum of $4c - 2b$ and $5b - 2a - 2c$.

16. If $X = 2b + 3c - 5d$ and $Y = 4d - 7c - b$, take $X - Y$ from $X + Y$.

17. Divide the product of $(-3x^7y^5)^4$ and $(-2xy^6)^3$ by $(-6x^3y^8)^2$.

EXTENSION

18. For what values of x is it true that: (a) $x \times x \le x + x$? (b) $x \times x \times x \le x + x + x$?

1 B Expanding Brackets

The laws of arithmetic tell us that $a(x + y) = ax + ay$, whatever the values of a, x and y. This enables expressions with brackets to be *expanded*, meaning that they can be written in a form without brackets.

WORKED EXERCISE:

(a) $3x(x - 2xy) = 3x^2 - 6x^2y$

(b) $a^2(a - b) - b^2(b - a)$
$= a^3 - a^2b - b^3 + ab^2$

(c) $(4x - 2)(4x - 3)$
$= 4x(4x - 3) - 2(4x - 3)$
$= 16x^2 - 12x - 8x + 6$
$= 16x^2 - 20x + 6$

Special Quadratic Identities: These three identities are so important that they need to be memorised rather than worked out each time.

2
SQUARE OF A SUM: $(A + B)^2 = A^2 + 2AB + B^2$
SQUARE OF A DIFFERENCE: $(A - B)^2 = A^2 - 2AB + B^2$
DIFFERENCE OF SQUARES: $(A + B)(A - B) = A^2 - B^2$

WORKED EXERCISE:

(a) $(4x + 5y)^2 = 16x^2 + 40xy + 25y^2$ (square of a sum)

(b) $\left(t - \dfrac{1}{t}\right)^2 = t^2 - 2 + \dfrac{1}{t^2}$ (square of a difference)

(c) $(x^2 + 3y)(x^2 - 3y) = x^4 - 9y^2$ (difference of squares)

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Exercise 1B

1. Expand:
(a) $4(a+2b)$
(b) $x(x-7)$
(c) $-3(x-2y)$
(d) $-a(a+4)$
(e) $5(a+3b-2c)$
(f) $-3(2x-3y+5z)$
(g) $-2x(x^3-2x^2-3x+1)$
(h) $3xy(2x^2y-5x^3)$
(i) $-2a^2b(a^2b^3-2a^3b)$

2. Expand and simplify:
(a) $3(x-2)-2(x-5)$
(b) $-7(2a-3b+c)-6(-a+4b-2c)$
(c) $x^2(x^3-5x^2+6x-1)-2x(x^4+10x^3-2x^2-7x+3)$
(d) $-2x^3y(3x^2y^4-4xy^5+5y^7)-3xy^2(x^2y^6+2x^4y^3-2x^3y^4)$

3. Expand and simplify:
(a) $(x+2)(x+3)$
(b) $(2a+3)(a+5)$
(c) $(x-4)(x+2)$
(d) $(2b-7)(b-3)$
(e) $(3x+8)(4x-5)$
(f) $(6-7x)(5-6x)$

4. (a) By expanding $(A+B)(A+B)$, prove the special expansion $(A+B)^2=A^2+2AB+B^2$.
(b) Similarly, prove the special expansions:
(i) $(A-B)^2=A^2-2AB+B^2$
(ii) $(A-B)(A+B)=A^2-B^2$

5. Expand, using the special expansions:
(a) $(x-y)^2$
(b) $(a+3)^2$
(c) $(n-5)^2$
(d) $(c-2)(c+2)$
(e) $(2a+1)^2$
(f) $(3p-2)^2$
(g) $(3x+4y)(3x-4y)$
(h) $(4y-5x)^2$

6. Multiply:
(a) $a-2b$ by $a+2b$
(b) $2-5x$ by $5+4x$
(c) $4x+7$ by itself
(d) x^2+3y by x^2-4y
(e) $a+b-c$ by $a-b$
(f) $9x^2-3x+1$ by $3x+1$

7. Expand and simplify:
(a) $\left(t+\frac{1}{t}\right)^2$
(b) $\left(t-\frac{1}{t}\right)^2$
(c) $\left(t+\frac{1}{t}\right)\left(t-\frac{1}{t}\right)$

DEVELOPMENT

8. (a) Subtract $a(b+c-a)$ from the sum of $b(c+a-b)$ and $c(a+b-c)$.
(b) Subtract the sum of $2x^2-3(x-1)$ and $2x+3(x^2-2)$ from the sum of $5x^2-(x-2)$ and $x^2-2(x+1)$.

9. Simplify: (a) $14-\big(10-(3x-7)-8x\big)$ (b) $4\Big(a-2(b-c)-\big(a-(b-2)\big)\Big)$

10. Use the special expansions to find the value of: (a) 102^2 (b) 999^2 (c) 203×197

11. Expand and simplify:
(a) $(a-b)(a+b)-a(a-2b)$
(b) $(x+2)^2-(x+1)^2$
(c) $(a-3)^2-(a-3)(a+3)$
(d) $(p+q)^2-(p-q)^2$
(e) $(2x+3)(x-1)-(x-2)(x+1)$
(f) $3(a-4)(a-2)-2(a-3)(a-5)$

12. If $X=x-a$ and $Y=2x+a$, find the product of $Y-X$ and $X+3Y$ in terms of x and a.

13. Expand and simplify:
(a) $(x-2)^3$
(b) $(x+y+z)^2-2(xy+yz+zx)$
(c) $(x+y-z)(x-y+z)$
(d) $(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

14. Prove the identities:

(a) $(a+b+c)(ab+bc+ca)-abc=(a+b)(b+c)(c+a)$

(b) $(ax+by)^2+(ay-bx)^2+c^2(x^2+y^2)=(x^2+y^2)(a^2+b^2+c^2)$

EXTENSION

15. If $2x=a+b+c$, show that $(x-a)^2+(x-b)^2+(x-c)^2+x^2=a^2+b^2+c^2$.

16. If $(a+b)^2+(b+c)^2+(c+d)^2=4(ab+bc+cd)$, prove that $a=b=c=d$.

1 C Factorisation

Factorisation is the reverse process of expanding brackets, and will be needed on a routine basis throughout the course. The various methods of factorisation are listed systematically, but in every situation common factors should always be taken out first.

3 METHODS OF FACTORISATION:

- HIGHEST COMMON FACTOR: Always try this first.
- DIFFERENCE OF SQUARES: This involves two terms.
- QUADRATICS: This involves three terms.
- GROUPING: This involves four or more terms.

Factoring should continue until each factor is *irreducible*, meaning that it cannot be factored further.

Factoring by Highest Common Factor and Difference of Squares: In every situation, look for any common factors of all the terms, and then take out the highest common factor.

WORKED EXERCISE: Factor: (a) $18a^2b^4-30b^3$ (b) $80x^4-5y^4$

SOLUTION:

(a) The highest common factor of $18a^2b^4$ and $30b^3$ is $6b^3$,
so $18a^2b^4-30b^3=6b^3(3a^2b-5)$.

(b) $80x^4-5y^4=5(16x^4-y^4)$ (highest common factor)
$=5(4x^2-y^2)(4x^2+y^2)$ (difference of squares)
$=5(2x-y)(2x+y)(4x^2+y^2)$ (difference of squares again)

Factoring Monic Quadratics: A quadratic is called *monic* if the coefficient of x^2 is 1. Suppose that we want to factor a monic quadratic expression like $x^2-13x+36$. We look for two numbers whose sum is -13 (the coefficient of x) and whose product is 36 (the constant).

WORKED EXERCISE: Factor: (a) $x^2-13x+36$ (b) $a^2+12ac-28c^2$

SOLUTION:

(a) The numbers with sum -13
and product 36 are -9 and -4,
so $x^2-13x+36$
$=(x-9)(x-4)$.

(b) The numbers with sum 12
and product -28 are 14 and -2,
so $a^2+12ac-28c^2$
$=(a+14c)(a-2c)$.

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Factoring Non-monic Quadratics: In a *non-monic* quadratic like $2x^2 + 11x + 12$, where the coefficient of x^2 is not 1, we look for two numbers whose sum is 11 (the coefficient of x), and whose product is 24 (the product of the constant term and the coefficient of x^2).

WORKED EXERCISE: Factor: (a) $2x^2 + 11x + 12$ (b) $6s^2 - 11st - 10t^2$

SOLUTION:

(a) The numbers with sum 11 and product 24 are 8 and 3,
so $2x^2 + 11x + 12$
$= (2x^2 + 8x) + (3x + 12)$
$= 2x(x + 4) + 3(x + 4)$
$= (2x + 3)(x + 4)$.

(b) The numbers with sum -11 and product -60 are -15 and 4,
so $6s^2 - 11st - 10t^2$
$= (6s^2 - 15st) + (4st - 10t^2)$
$= 3s(2s - 5t) + 2t(2s - 5t)$
$= (3s + 2t)(2s - 5t)$.

Factoring by Grouping: When there are four or more terms, it is sometimes possible to split the expression into groups, factor each group in turn, and then factor the whole expression by taking out a common factor or by some other method.

WORKED EXERCISE: Factor: (a) $12xy - 9x - 16y + 12$ (b) $s^2 - t^2 + s - t$

SOLUTION:

(a) $12xy - 9x - 16y + 12 = 3x(4y - 3) - 4(4y - 3)$
$= (3x - 4)(4y - 3)$

(b) $s^2 - t^2 + s - t = (s + t)(s - t) + (s - t)$
$= (s - t)(s + t + 1)$

Exercise 1C

1. Write as a product of two factors:

(a) $ax - ay$
(b) $x^2 + 3x$
(c) $3a^2 - 6ab$
(d) $12x^2 + 18x$
(e) $6a^3 + 2a^4 + 4a^5$
(f) $7x^3y - 14x^2y^2 + 21xy^2$

2. Factor by grouping in pairs:

(a) $ax - ay + bx - by$
(b) $a^2 + ab + ac + bc$
(c) $x^2 - 3x - xy + 3y$
(d) $2ax - bx - 2ay + by$
(e) $ab + ac - b - c$
(f) $2x^3 - 6x^2 - ax + 3a$

3. Factor each difference of squares:

(a) $x^2 - 9$
(b) $1 - a^2$
(c) $4x^2 - y^2$
(d) $25x^2 - 16$
(e) $1 - 49k^2$
(f) $81a^2b^2 - 64$

4. Factor each of these quadratic expressions:

(a) $x^2 + 8x + 15$
(b) $x^2 - 4x + 3$
(c) $a^2 + 2a - 8$
(d) $y^2 - 3y - 28$
(e) $c^2 - 12c + 27$
(f) $p^2 + 9p - 36$
(g) $u^2 - 16u - 80$
(h) $x^2 - 20x + 51$
(i) $t^2 + 23t - 50$
(j) $x^2 - 9x - 90$
(k) $x^2 - 5xy + 6y^2$
(l) $x^2 + 6xy + 8y^2$
(m) $a^2 - ab - 6b^2$
(n) $p^2 + 3pq - 40q^2$
(o) $c^2 - 24cd + 143d^2$

5. Write each quadratic expression as a product of two factors:

(a) $2x^2 + 5x + 2$
(b) $3x^2 + 8x + 4$
(c) $6x^2 - 11x + 3$
(d) $3x^2 + 14x - 5$
(e) $9x^2 - 6x - 8$
(f) $6x^2 - 7x - 3$
(g) $6x^2 - 5x + 1$
(h) $3x^2 + 13x - 30$
(i) $12x^2 - 7x - 12$
(j) $12x^2 + 31x - 15$
(k) $24x^2 - 50x + 25$
(l) $2x^2 + xy - y^2$
(m) $4a^2 - 8ab + 3b^2$
(n) $6p^2 + 5pq - 4q^2$
(o) $18u^2 - 19uv - 12v^2$

6. Write each expression as a product of three factors:

(a) $3a^2 - 12$
(b) $x^4 - y^4$
(c) $x^3 - x$
(d) $5x^2 - 5x - 30$
(e) $25y - y^3$
(f) $16 - a^4$
(g) $4x^2 + 14x - 30$
(h) $x^3 - 8x^2 + 7x$
(i) $x^4 - 3x^2 - 4$
(j) $ax^2 - a - 2x^2 + 2$
(k) $16m^3 - mn^2$
(l) $ax^2 - a^2x - 20a^3$

DEVELOPMENT

7. Factor as fully as possible:

(a) $72 + x - x^2$
(b) $(a - b)^2 - c^2$
(c) $a^3 - 10a^2b + 24ab^2$
(d) $a^2 - b^2 - a + b$
(e) $x^4 - 256$
(f) $4p^2 - (q + r)^2$
(g) $6x^4 - x^3 - 2x^2$
(h) $a^2 - bc - b + a^2c$
(i) $9x^2 + 36x - 45$
(j) $4x^4 - 37x^2 + 9$
(k) $x^2y^2 - 13xy - 48$
(l) $x(x - y)^2 - xz^2$
(m) $20 - 9x - 20x^2$
(n) $4x^3 - 12x^2 - x + 3$
(o) $12x^2 - 8xy - 15y^2$
(p) $x^2 + 2ax + a^2 - b^2$
(q) $9x^2 - 18x - 315$
(r) $x^4 - x^2 - 2x - 1$
(s) $10x^3 - 13x^2y - 9xy^2$
(t) $x^2 + 4xy + 4y^2 - a^2 + 2ab - b^2$
(u) $(x + y)^2 - (x - y)^2$

EXTENSION

8. Factor fully:

(a) $a^2 + b(b + 1)a + b^3$
(b) $a(b + c - d) - c(a - b + d)$
(c) $(a^2 - b^2)^2 - (a - b)^4$
(d) $4x^4 - 2x^3y - 3xy^3 - 9y^4$
(e) $(x^2 + xy)^2 - (xy + y^2)^2$
(f) $(a^2 - b^2 - c^2)^2 - 4b^2c^2$
(g) $(ax + by)^2 + (ay - bx)^2 + c^2(x^2 + y^2)$
(h) $x^2 + (a - b)xy - aby^2$
(i) $a^4 + a^2b^2 + b^4$
(j) $a^4 + 4b^4$

1 D Algebraic Fractions

An *algebraic fraction* is a fraction containing pronumerals. They are manipulated in the same way as arithmetic fractions, and factorisation plays a major role.

Addition and Subtraction of Algebraic Fractions: A common denominator is required, but finding the lowest common denominator can involve factoring all the denominators.

4 ADDITION AND SUBTRACTION OF ALGEBRAIC FRACTIONS: First factor all denominators. Then work with the lowest common denominator.

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WORKED EXERCISE:

(a) $\dfrac{1}{x-4} - \dfrac{1}{x}$
$= \dfrac{x-(x-4)}{x(x-4)}$
$= \dfrac{4}{x(x-4)}$

(b) $\dfrac{2}{x^2-x} - \dfrac{5}{x^2-1} = \dfrac{2}{x(x-1)} - \dfrac{5}{(x-1)(x+1)}$
$= \dfrac{2(x+1)-5x}{x(x-1)(x+1)}$
$= \dfrac{2-3x}{x(x-1)(x+1)}$

Multiplication and Division of Algebraic Fractions: The key step here is to factor all numerators and denominators completely before cancelling factors.

5 MULTIPLICATION AND DIVISION OF ALGEBRAIC FRACTIONS: First factor all numerators and denominators completely. Then cancel common factors.

To divide by an algebraic fraction, multiply by its reciprocal in the usual way.

WORKED EXERCISE:

(a) $\dfrac{2a}{9-a^2} \times \dfrac{a-3}{a^3+a} = \dfrac{2a}{(3-a)(3+a)} \times \dfrac{a-3}{a(a^2+1)}$
$= -\dfrac{2}{(a+3)(a^2+1)}$

(b) $\dfrac{6abc}{ab+bc} \div \dfrac{6ac}{a^2+2ac+c^2} = \dfrac{6abc}{b(a+c)} \times \dfrac{(a+c)^2}{6ac}$
$= a+c$

Simplifying Compound Fractions: A *compound fraction* is a fraction in which either the numerator or the denominator is itself a fraction.

6 SIMPLIFYING COMPOUND FRACTIONS: Multiply top and bottom by something that will clear fractions from numerator and denominator together.

WORKED EXERCISE:

(a) $\dfrac{\frac{1}{2}-\frac{1}{3}}{\frac{1}{4}+\frac{1}{6}} = \dfrac{\frac{1}{2}-\frac{1}{3}}{\frac{1}{4}+\frac{1}{6}} \times \dfrac{12}{12}$
$= \dfrac{6-4}{3+2}$
$= \frac{2}{5}$

(b) $\dfrac{\dfrac{1}{t}+\dfrac{1}{t+1}}{\dfrac{1}{t}-\dfrac{1}{t+1}} = \dfrac{\dfrac{1}{t}+\dfrac{1}{t+1}}{\dfrac{1}{t}-\dfrac{1}{t+1}} \times \dfrac{t(t+1)}{t(t+1)}$
$= \dfrac{(t+1)+t}{(t+1)-t}$
$= 2t+1$

Exercise 1D

1. Simplify:

(a) $\dfrac{x}{2x}$ (c) $\dfrac{3x^2}{9xy}$ (e) $\dfrac{12xy^2z}{15x^2yz^2}$

(b) $\dfrac{a}{a^2}$ (d) $\dfrac{12ab}{4a^2b}$ (f) $\dfrac{uvw^2}{u^3v^2w}$

2. Simplify:

(a) $\frac{x}{3} \times \frac{3}{x}$
(b) $\frac{a}{4} \div \frac{a}{2}$
(c) $x \times \frac{3}{x^2}$
(d) $\frac{a^2}{2b} \times \frac{b^2}{a^2}$
(e) $\frac{3x^2}{4y^2} \times \frac{2y}{x}$
(f) $\frac{x^2}{3ay^3} \div \frac{x^2}{3ay^3}$
(g) $\frac{5}{a} \div 10$
(h) $\frac{2ab}{3c} \times \frac{c^2}{ab^2}$
(i) $\frac{8a^3b}{5} \div \frac{4ab}{15}$
(j) $\frac{2a}{3b} \times \frac{5c^2}{2a^2b} \times \frac{3b^2}{2c}$
(k) $\frac{12x^2yz}{8xy^3} \times \frac{24xy^2}{36yz^2}$
(l) $\frac{3a^2b}{4b^3c} \times \frac{2c^2}{8a^3} \div \frac{6ac}{16b^2}$

3. Write as a single fraction:

(a) $\frac{x}{2} + \frac{x}{5}$
(b) $\frac{a}{3} - \frac{a}{6}$
(c) $\frac{x}{8} - \frac{y}{12}$
(d) $\frac{2a}{3} + \frac{3a}{2}$
(e) $\frac{7b}{10} - \frac{19b}{30}$
(f) $\frac{xy}{30} - \frac{xy}{18}$
(g) $\frac{1}{x} + \frac{1}{2x}$
(h) $\frac{3}{4x} + \frac{4}{3x}$
(i) $\frac{1}{a} - \frac{1}{b}$
(j) $x + \frac{1}{x}$
(k) $a + \frac{b}{a}$
(l) $\frac{1}{2x} - \frac{1}{x^2}$

4. Simplify:

(a) $\frac{x+1}{2} + \frac{x+2}{3}$
(b) $\frac{2x-1}{5} - \frac{x+3}{2}$
(c) $\frac{2x+1}{3} - \frac{x-5}{6} + \frac{x+4}{4}$
(d) $\frac{3x-7}{5} + \frac{4x+3}{2} - \frac{2x-5}{10}$
(e) $\frac{x-5}{3x} - \frac{x-3}{5x}$
(f) $\frac{1}{x} - \frac{1}{x+1}$
(g) $\frac{1}{x+1} - \frac{1}{x+1}$
(h) $\frac{2}{x-3} + \frac{3}{x-2}$
(i) $\frac{2}{x+3} - \frac{2}{x-2}$
(j) $\frac{x}{x+y} + \frac{y}{x-y}$
(k) $\frac{a}{x+a} - \frac{b}{x+b}$
(l) $\frac{x}{x-1} - \frac{x}{x+1}$

5. Factor where possible and then simplify:

(a) $\frac{a}{ax+ay}$
(b) $\frac{3a^2-6ab}{2a^2b-4ab^2}$
(c) $\frac{x^2+2x}{x^2-4}$
(d) $\frac{a^2-9}{a^2+a-12}$
(e) $\frac{x^2+2xy+y^2}{x^2-y^2}$
(f) $\frac{x^2+10x+25}{x^2+9x+20}$
(g) $\frac{ac+ad+bc+bd}{a^2+ab}$
(h) $\frac{y^2-8y+15}{2y^2-5y-3}$
(i) $\frac{9ax+6bx-6ay-4by}{9x^2-4y^2}$

6. Simplify:

(a) $\frac{3x+3}{2x} \times \frac{x^2}{x^2-1}$
(b) $\frac{a^2+a-2}{a+2} \times \frac{a^2-3a}{a^2-4a+3}$
(c) $\frac{c^2+5c+6}{c^2-16} \div \frac{c+3}{c-4}$
(d) $\frac{x^2-x-20}{x^2-25} \times \frac{x^2-x-2}{x^2+2x-8} \div \frac{x+1}{x^2+5x}$
(e) $\frac{ax+bx-2a-2b}{3x^2-5x-2} \times \frac{9x^2-1}{a^2+2ab+b^2}$
(f) $\frac{2x^2+x-15}{x^2+3x-28} \div \frac{x^2+6x+9}{x^2-4x} \div \frac{6x^2-15x}{x^2-49}$

7. Simplify:

(a) $\frac{1}{x^2+x} + \frac{1}{x^2-x}$
(b) $\frac{1}{x^2-4} + \frac{1}{x^2-4x+4}$
(c) $\frac{1}{x-y} + \frac{2x-y}{x^2-y^2}$
(d) $\frac{3}{x^2+2x-8} - \frac{2}{x^2+x-6}$
(e) $\frac{x}{a^2-b^2} - \frac{x}{a^2+ab}$
(f) $\frac{1}{x^2-4x+3} + \frac{1}{x^2-5x+6} - \frac{1}{x^2-3x+2}$

8. Simplify:

(a) $\dfrac{b-a}{a-b}$

(b) $\dfrac{v^2-u^2}{u-v}$

(c) $\dfrac{x^2-5x+6}{2-x}$

(d) $\dfrac{1}{a-b}-\dfrac{1}{b-a}$

(e) $\dfrac{m}{m-n}+\dfrac{n}{n-m}$

(f) $\dfrac{x-y}{y^2+xy-2x^2}$

DEVELOPMENT

9. Study the worked exercise on compound fractions and then simplify:

(a) $\dfrac{1-\frac{1}{2}}{1+\frac{1}{2}}$

(b) $\dfrac{2+\frac{1}{3}}{5-\frac{2}{3}}$

(c) $\dfrac{\frac{1}{2}-\frac{1}{5}}{1+\frac{1}{10}}$

(d) $\dfrac{\frac{17}{20}-\frac{3}{4}}{\frac{4}{5}-\frac{3}{10}}$

(e) $\dfrac{\frac{1}{x}}{1+\frac{2}{x}}$

(f) $\dfrac{t-\frac{1}{t}}{t+\frac{1}{t}}$

(g) $\dfrac{1}{\frac{1}{b}+\frac{1}{a}}$

(h) $\dfrac{\frac{x}{y}+\frac{y}{x}}{\frac{x}{y}-\frac{y}{x}}$

(i) $\dfrac{1-\frac{1}{x+1}}{\frac{1}{x}+\frac{1}{x+1}}$

(j) $\dfrac{\frac{3}{x+2}-\frac{2}{x+1}}{\frac{5}{x+2}-\frac{4}{x+1}}$

10. If $x=\dfrac{1}{\lambda}$ and $y=\dfrac{1}{1-x}$ and $z=\dfrac{y}{y-1}$, show that $z=\lambda$.

11. Simplify:

(a) $\dfrac{x^4-y^4}{x^2-2xy+y^2}\div\dfrac{x^2+y^2}{x-y}$

(b) $\dfrac{8x^2+14x+3}{8x^2-10x+3}\times\dfrac{12x^2-6x}{4x^2+5x+1}\div\dfrac{18x^2-6x}{4x^2+x-3}$

(c) $\dfrac{(a-b)^2-c^2}{ab-b^2-bc}\times\dfrac{c}{a^2+ab-ac}\div\dfrac{ac-bc+c^2}{a^2-(b-c)^2}$

(d) $\dfrac{x-y}{x}+\dfrac{x^3+y^3}{xy^2}-\dfrac{x^2+y^2}{x^2}$

(e) $\dfrac{x+4}{x-4}-\dfrac{x-4}{x+4}$

(f) $\dfrac{4y}{x^2+2xy}-\dfrac{3x}{xy+2y^2}+\dfrac{3x-2y}{xy}$

(g) $\dfrac{8x}{x^2+5x+6}-\dfrac{5x}{x^2+3x+2}-\dfrac{3x}{x^2+4x+3}$

(h) $\dfrac{1}{x-1}+\dfrac{2}{x+1}-\dfrac{3x-2}{x^2-1}-\dfrac{1}{x^2+2x+1}$

12. (a) Expand $\left(x+\dfrac{1}{x}\right)^2$.

(b) Suppose that $x+\dfrac{1}{x}=3$. Use part (a) to evaluate $x^2+\dfrac{1}{x^2}$ without attempting to find the value of x.

EXTENSION

13. Simplify these algebraic fractions:

(a) $\dfrac{1}{(a-b)(a-c)}+\dfrac{1}{(b-c)(b-a)}+\dfrac{1}{(c-a)(c-b)}$

(b) $\left(1+\dfrac{45}{x-8}-\dfrac{26}{x-6}\right)\left(3-\dfrac{65}{x+7}+\dfrac{8}{x-2}\right)$

(c) $\left(2-\dfrac{3n}{m}+\dfrac{9n^2-2m^2}{m^2+2mn}\right)\div\left(\dfrac{1}{m}-\dfrac{1}{m-2n-\dfrac{4n^2}{m+n}}\right)$

(d) $\dfrac{1}{x+\dfrac{1}{x+2}}\times\dfrac{1}{x+\dfrac{1}{x-2}}\div\dfrac{x-\dfrac{4}{x}}{x^2-2+\dfrac{1}{x^2}}$