Cambridge University Press
978-1-107-02139-6 - Fundamentals of Electro-Optic Systems Design: Communications, Lidar, and Imaging
Sherman Karp and Larry B. Stotts
Excerpt
More information

1 Genesis of electro-optic systems

When we decided to write this book about the design of electro-optic systems, we agreed to make it as fundamental as possible. To do this in detail would most probably make the book unwieldy. Rather, we will try to motivate all aspects of the design from fundamental principles, stating the important results, and leaving the derivations to references. We will take as our starting point the first two Laws of Thermodynamics [1]. The Three Laws of Thermodynamics are the basic foundation of our understanding of how the Universe works. Everything, no matter how large or small, is subject to the Three Laws. The Laws of Thermodynamics dictate the specifics for the movement of heat and work, both natural and man-made. The First Law of Thermodynamics is a statement of the conservation of energy – the Second Law is a statement about the nature of that conservation – and the Third Law is a statement about reaching Absolute Zero (0° K). These laws and Maxwell's equations were developed in the nineteenth century, and are the foundation upon which twentieth-century physics was founded.

Since the Laws are so obviously important, what are they?

First Law: energy can neither be created nor destroyed. It can only change forms. In any process, the total energy of the universe remains the same.

The First Law states that energy cannot be created or destroyed; rather, the amount of energy lost in a steady-state process cannot be greater than the amount of energy gained. This was a rebuff to early attempts at perpetual motion machines. This is the statement of conservation of energy for a thermodynamic system. It refers to the two ways that a closed system transfers energy to and from its surroundings – by the process of heating (or cooling) and the process of mechanical work. Whether the system is open or closed, all energy transfers must be in balance.

Second Law: the entropy of an isolated system not in equilibrium will tend to increase over time, approaching a maximum value at equilibrium.

The second law states that energy systems will always increase their entropy rather than decrease it. In other words heat can spontaneously flow from a higher-temperature region to a lower-temperature region, but not the reverse. (Heat can be made to flow from cold to hot, as in a refrigerator, but requires an external source of power, electricity.)

A way of looking at the second law for non-scientists is to look at entropy as a measure of disorder. A broken cup has less order than before being broken. Similarly solid crystals have very low entropy values, while gases which are highly disorganized have high entropy values.

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1.1 Energy

What does this have to do with optics? Well, about the same time that the thermodynamic laws were being developed, the laws of electricity were also being developed. James Clerk Maxwell unified the Laws of Gauss, Ampere and Faraday into the set of laws now referred to as Maxwell's equations [2]. These equations show that electric and magnetic fields are coupled and satisfy a wave equation,

$$\nabla^2 \phi = \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} \tag{1.1}$$

where ϕ can represent the scalar components of the electric and magnetic fields. In a vacuum the speed of light $c = 1/\sqrt{\varepsilon_0 \mu_0}$ where ε_0, μ_0 are the permittivity and permeability of free space, as derived in Maxwell's equations. Since electromagnetic fields can propagate in a vacuum, no loss occurs (unless obstacles are encountered) and the fields can propagate indefinitely since energy can be neither created nor destroyed. Thus the energy from the Sun can bring life to the planet Earth, and we can see the vast vista of stars at night, even though they are light years away. A general solution to the wave equation (in spherical coordinates) for the fields takes the form

$$\phi = \text{Constant} \times \frac{f\left(t - \frac{r}{c}\right)}{r}. \tag{1.2}$$

$t - \frac{r}{c}$ is referred to as the retarded time. That is, the waveform of the field propagates undistorted, but attenuated (spread out in space) by the factor r, when traveling the distance r, in a time r/c. Since the energy in the fields is proportional to $|\phi|^2$ [2], we see that the energy in a propagating electromagnetic field falls off as $1/|r|^2$. From this we can deduce several properties regarding the transfer of energy by electromagnetic means. For the purpose of this book we will use the following standard notation for Radiant quantities:

Radiant energy is **energy** carried from any electromagnetic field. It is denoted by Q_e. Its **SI** unit is the **joule** (J).

Radiant flux is radiant energy per unit time (also called **radiant power**); it is considered the fundamental radiometric unit. It is denoted by P_e. Its **SI** unit is the **watt** (W).

Radiant exitance, or **radiant emittance**, is radiant flux emitted from an extended source per unit **source area**. It is denoted by M_e. Its **SI** unit is: watt per square meter (W/m^2).

Irradiance is radiant flux incident on a surface unit area. It is denoted by E_e. Its **SI** unit is: **watt per unit area** (W/m^2).

Radiant intensity is radiant flux emitted from a point source per unit **solid angle**. It is denoted by I_e. Its **SI** unit is: watt per **steradian** (W/sr).

Radiance is radiant flux emitted from an extended source per unit **solid angle** and per unit **projected source area**. It is denoted by L_e. Its **SI** unit is: watt per steradian and square meter (W/(sr m^2)).

Spectral in front of any of these quantities implies the same **SI** unit per unit **wavelength**.

Thus spectral radiant energy is the energy radiated per unit wavelength. These noise sources generally are not described analytically, and depending upon the instrument used fall into one of the categories described above.

1.2 The range equation

We will try to show how these terms are used, with a few examples.

Example 1.1 Suppose a point source radiated (transmitted) an amount of radiant power, $P_t = Q_e$ joules/second, equally into all directions (4π steradians or omnidirectional). Then the radiant intensity would be equal to

$$I_t = \frac{P_t}{4\pi} \tag{1.3}$$

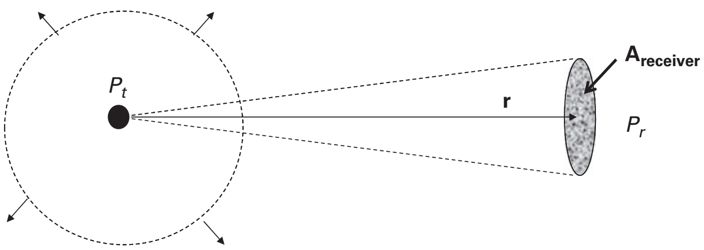
A receiver with an area A_r located a distance r from the point source would subtend the solid angle $A_r/|r|^2$ to the source. Therefore such a receiver could capture an amount of radiant power (power) equal to Figure 1.1,

$$P_r = \frac{P_t A_r}{4\pi|r|^2} \tag{1.4}$$

which conforms to the $1/|r|^2$ requirement of a propagating field. Suppose that a second identical source could direct all its energy toward a receiver, uniformly into a solid angle $\Omega < 4\pi$. Such a source would then transmit $4\pi/\Omega$ times more power to the receiver, without the receiver knowing that it wasn't a source with $(4\pi/\Omega)P_t$ watts. For this reason $4\pi/\Omega$ is called the gain (G_t – over isotropic) of the second system, and we write the received power as

$$P_r = \frac{P_t A_r G_t}{4\pi|r|^2} \tag{1.5}$$

This is referred to as the frequency-independent form of the range equation, and defines the transfer of power in a radiating system.



$$P_r = \frac{P_t}{4\pi} \cdot \frac{A_r}{|r|^2}$$

Figure 1.1 Range equation in geometric form.

Example 1.2 How can we calculate the total power emitted by the Sun? The Sun is located a distance $R_{sun} = 92 \times 10^6$ miles from the Earth. The irradiance of the Sun at the top of the Earth's atmosphere (solar constant), I_{sun}, is approximately 1500 w/m^2. The Earth has a cross section, A_{earth}, of $\pi(4000)^2 = 50.3 \times 10^6$ square miles, or a cross section of 1.3×10^{14} m^2. Thus the Earth intercepts $I_{sun}A_{earth}$ watts from the Sun, which is approximately 1.96×10^{17} watts. The solid angle subtended from the Earth to the Sun is $\Omega_{sun-earth} = A_{earth}/R_{sun}^2$ steradians, so the Earth intercepts the percentage $\Omega_{sun-earth}/4\pi$ of the Sun's emission, or only 5.81×10^{-7} per cent of the energy radiated by the sun. This means that the sun emits a total of $\left(4\pi/\Omega_{sun-earth}\right)I_{sun}A_{earth} = 3.37 \times 10^{25}$ watts.

1.3 Characterization of noise

First we will relate all the radiant quantities described earlier. We start first with the most elemental of the quantities, the spectral radiance, $N_e(r,\theta,\lambda)$. To obtain the radiance, we would perform the integration, which for narrowband filters can be approximated, as follows:

$$L_e(r,\Omega) = \int_{\lambda_1}^{\lambda_2} N_e(r,\Omega,\lambda)d\lambda \approx N_e(r,\Omega,\lambda)\Delta\lambda \tag{1.6}$$

From $L_e(r, \Omega)$ we can go in one of two directions. To obtain the radiant emittance we perform the integral

$$M_e(r) = \int_0^{\Omega_0} L_e(r,\Omega)d\Omega \approx L_e(r,\Omega)\Omega_0 = N_e(r,\Omega,\lambda)\Omega_0\Delta\lambda \tag{1.7}$$

while to get the radiant intensity, we perform the integral

$$I_e(\Omega) = \int_A L_e(r,\Omega)dS \approx L_e(r,\Omega)A = N_e(r,\Omega,\lambda)A\Delta\lambda \tag{1.8}$$

Finally, to obtain the power, we perform either one of two integrals:

$$P_e = \int_A M_e(r)dS \approx M_e(r)A = L_e(r,\Omega)A\Omega_0 = N_e(r,\Omega,\lambda)A\Omega_0\Delta\lambda \tag{1.9}$$

or

$$P_e = \int_0^{\Omega_0} I_e(\Omega)d\Omega \approx I_e(\Omega)\Omega_0 = L_e(r,\Omega)A\Omega_0 = N_e(r,\Omega,\lambda)A\Omega_0\Delta\lambda \tag{1.10}$$

We have defined I_e to be from a point source, but by allowing a variation with angle it becomes more general. A point source would have no variation with angle. We have also used Ω to denote (θ, ϕ) and, $d\Omega$ the Jacobian. The Sun, which has a diameter of 863 000 miles, only subtends 0.5 degrees to Earth, and for many cases appears like a point source, but can project different values in different directions. We have used the description of frequency and bandwidth in customary terms of wavelength and differential wavelength, because these sources were first studied by astronomers, and that is how they were described. We also have the relationship between frequency and wavelength of $f = c/\lambda$ with $df = |(c/\lambda^2)|d\lambda$.

Example 1.3 Let us assume that a source radiates a spectral radiance $N_\lambda(w/m^2 - sr - BW)$. Suppose that the receiver has a field of view (FOV – the solid angle that it views) equal to Ω_{rec}, a collection area A_{rec}, and a filter bandwidth $\Delta\lambda$. If R is the range from the source to the receiver, then integrating over the source area yields (1) $\Omega_{rec}R^2$ if the source fills the FOV, or (2) A_{source} if it doesn't. The receiver subtends the solid angle A_{rec}/R^2 to the source. Therefore the total power collected by the receiver becomes

$$N_\lambda \Omega_{rec} R^2 \frac{A_{rec}}{R^2} \Delta\lambda = N_\lambda \Omega_{rec} A_{rec} \Delta\lambda \tag{1.11}$$

if the source fills the receiver FOV. This would be in cases like imaging systems, where the FOV of each detector pixel fills the cell in the image that is being viewed. On the other hand in a detection system, such as a system searching for a hot target, the FOV might be larger than the target and

$$N_\lambda \frac{A_{source}}{R^2} A_{rec} \Delta\lambda \tag{1.12}$$

would most likely apply.

1.4 Black-body radiation

Black-body or "pure-temperature" radiation was the name given to radiation emanating from a system in thermal equilibrium. This is the fundamental source of primary (suns, stars, plasmas, etc.) and secondary (moons, planets, atmosphere, etc.) electromagnetic noise, and was the exciting area of research in thermodynamics. In addition, the outcome proved fundamental to the development of the quantum theory.

It was known that in a sealed chamber at a given temperature, the spectral distribution of radiation is independent of the material out of which the chamber is made. In 1884, Boltzmann [3] showed that the total irradiance energy in this distribution varied as the fourth power of the temperature. The constant of proportionality, $\delta = 0.567 \times 10^{-8}$ J–m^{-2} deg^{-4} s^{-1}, is known as the Stefan-Boltzmann constant,

$$\text{Irradiance} \sim \delta T_0^4 \tag{1.13}$$

and the law as the Stefan-Boltzmann law. In 1893 Wien [4] showed that this functional dependence must be of the form

$$\psi_\lambda \sim T_0^5 f(\lambda T_0) = \frac{1}{\lambda^5} F(\lambda T_0) \tag{1.14}$$

where T_0 is the temperature in degrees Kelvin. He assumed a form

$$\psi_\lambda \sim \lambda^{-5} e^{k/\lambda T_0} \tag{1.15}$$

called Wien's law. He also developed Wien's displacement law, which states that if the product λT_0 is held constant, ψ_λ / T_0^5 is the same at all temperatures.

In 1900, Rayleigh [5] made a suggestion which was based on a technique used successfully in statistical mechanics. This was followed up with a calculation made by Jeans, resulting in the Rayleigh-Jeans formula. The idea relied on the equipartition of energy, and went as follows: one can expand the electromagnetic field contained in an enclosure into orthogonal and independent modes (degrees of freedom). By then associating the total energy kT_0 with each degree of freedom (used in the ideal gas law) an expression for the radiation can be obtained. Jeans calculated the number of modes per cycle to be D_{BB}, where

$$D_{BB} = \frac{8\pi f^2}{c^3} \tag{1.16}$$

yielding the expression for the energy to be

$$E = kT_0 D_{BB} = \frac{8\pi f^2 k T_0}{c^3} . \tag{1.17}$$

Unfortunately, this predicted an infinite energy instead of the fourth-power dependence on T_0. The two expressions E and ψ_λ, are plotted in Figure 1.2, along with precise

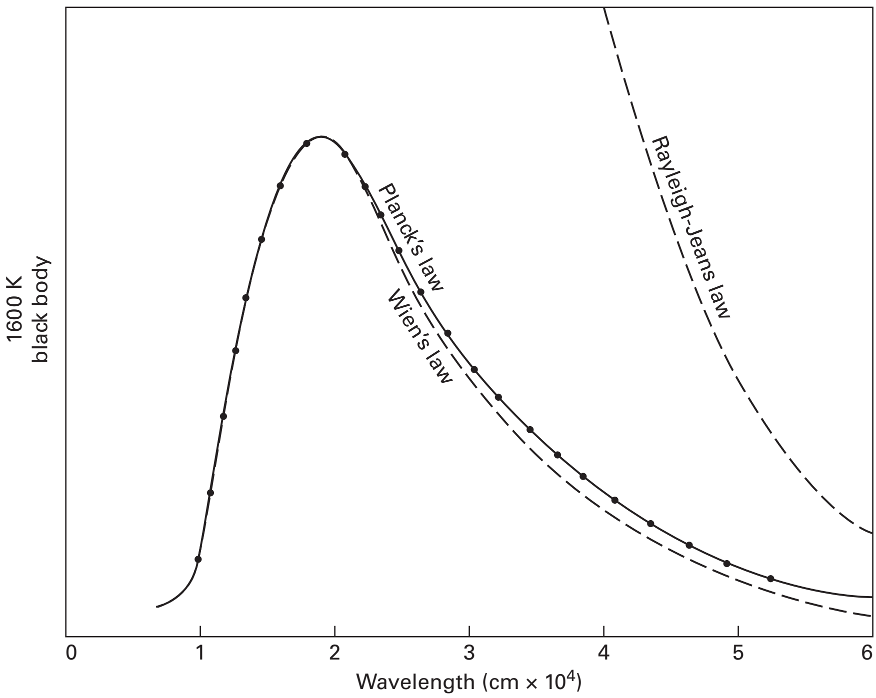


Figure 1.2 Black-body radiation, Wien's law, and the Rayleigh-Jeans approximation. The experimental curve is fit exactly by Planck's law.

measurements taken by Lummer and Pringsheim [6]. Neither was correct, but Wien's law was asymptotically correct at high frequencies, and the Rayleigh-Jeans law was asymptotically correct at low frequencies.

This was the picture as seen by Planck around 1900. Plank's derivation of the correct equation for the black-body law came through his understanding of thermodynamics. He curve-fitted the second derivative of entropy with respect to energy to yield the correct two asymptotic forms, and worked them back to obtain the energy equation [7]. The most commonly used derivation (suggested by Einstein) multiplies the mode density D_{BB} by the quantum mechanical equivalent of kT_0, which is

$$\frac{hf}{e^{hf/kT_0} - 1} \tag{1.18}$$

Thus we see that the equation for black-body radiation can be derived to be

$$\begin{aligned} \psi_\lambda &= \frac{8\pi f^2}{c^3} \frac{hf}{e^{hf/kT_0} - 1} \\ &= \frac{8\pi hc}{(\lambda)^5} \frac{1}{e^{hc/k\lambda T_0} - 1} \end{aligned} \tag{1.19}$$

which fits the energy density curves exactly.

This is the energy per unit of wavelength (bandwidth) contained in a sealed container. When we consider the radiation from an isotropic black body, we must multiply the energy by the speed of light, c, divided by 4π, for isotropy, and divide by two for each polarization. This yields

$$\psi_\lambda \frac{c}{4\pi} \frac{1}{2} = \frac{8\pi f^2}{c^3} \frac{hf}{e^{hf/kT_0} - 1} \frac{c}{4\pi} \frac{1}{2} = \frac{f^2}{c^2} \frac{hf}{e^{hf/kT_0} - 1} \tag{1.20}$$

which has the dimensions of energy per unit bandwidth per unit solid angle per unit area. As such it is a spectral radiance, N_{BB}. Reflecting back on example 1.3, the Sun appears as a 5778° black body. If the receiver field of view is greater than 0.5 degrees we can treat example 1.2 as before (example 1.3b). If the field of view is less than 0.5 degrees we are only looking at a small cell in the Sun, which in turn fills the field of view of the pixel viewing it (example 1.3a). When an instrument measures the solar irradiance, E_{sun}, its field of view is larger than that subtended by the Sun and hence measures

$$E_{sun} = N_\lambda \frac{A_{sun}}{R^2}, \tag{1.21}$$

which has dimensions of joules/μm-m^2 as given in example 1.2.

The quantum mechanical equipartition energy has the two asymptotic forms. At the low energy (low frequency), $hf < kT_0$, we have

$$\frac{hf}{e^{hf/kT_0}-1} \approx \frac{hf}{1+\frac{hf}{kT_0}-1} = kT_0 \qquad (1.22)$$

the classic value. At the high end we have $hf\exp(-hf/kT_0)$, which approaches Wien's law.

What Planck also observed about black-body radiation was that this described a sum of harmonic oscillators,

$$\frac{8\pi f^2}{c^3}\frac{hf}{e^{hf/kT_0}-1} = \frac{8\pi f^2}{c^3}\frac{hf}{e^{hf/kT_0}\left(1-e^{-hf/kT_0}\right)} = \frac{8\pi hf^3}{c^3 e^{hf/kT_0}}\sum_{n=0}^{\infty} e^{-nhf/kT_0} \qquad (1.23)$$

which was the first quantum hypothesis.

We have expanded on the derivation of black-body radiation because it is so important in understanding the composition of background noise. This will become clearer as we see the relevance of modes, mode number and energy per mode.

We show several representative curves of astronomical radiation in Figures 1.3–1.10. In Figure 1.3, we show the approximate black-body radiation of the Sun outside the atmosphere, and also modified by the absorption bands of the atmosphere [8]. In Figure 1.4, we show this in logarithmic form as a function of the zenith angle. Notice that the units are different, but the results are the same [8]. In Figure 1.5, we show the sky radiance during the day, which

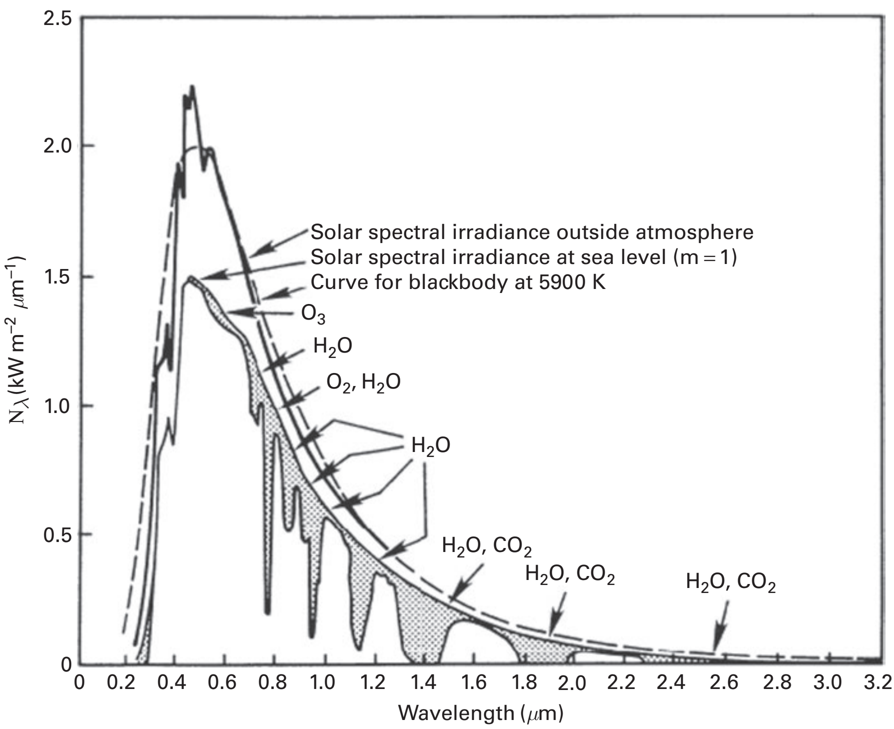


Figure 1.3 Spectral distribution curves related to the Sun. The shaded areas indicated absorpion at sea level due to the atmospheric constituents shown.

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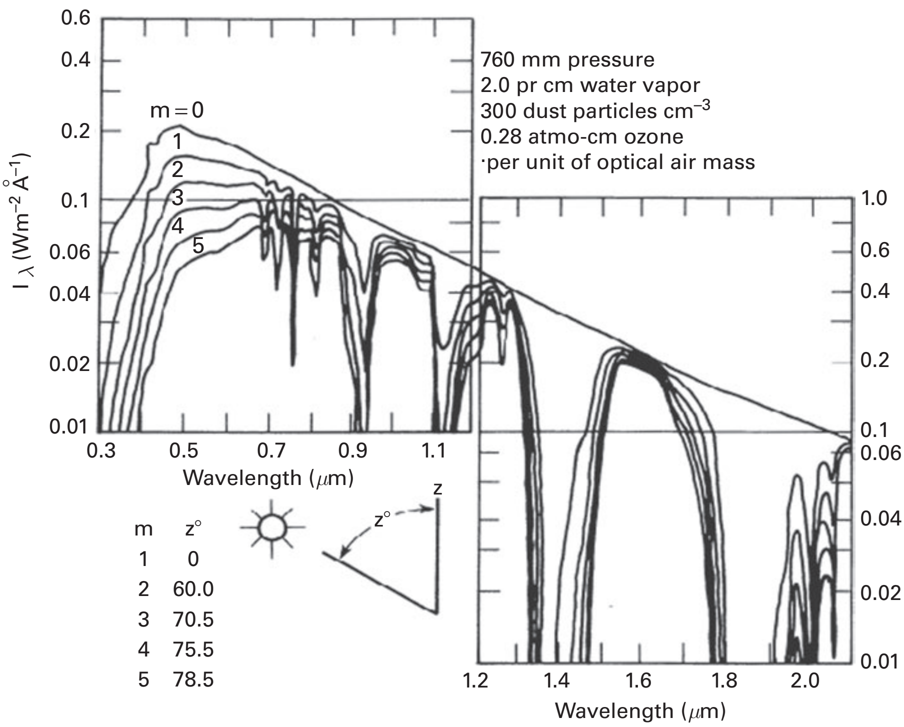

Figure 1.4 Solar spectral irradiance curves at sea level for various optical air masses. The value of the solar constant used in this calculation was 1322 W/m^2 [23].

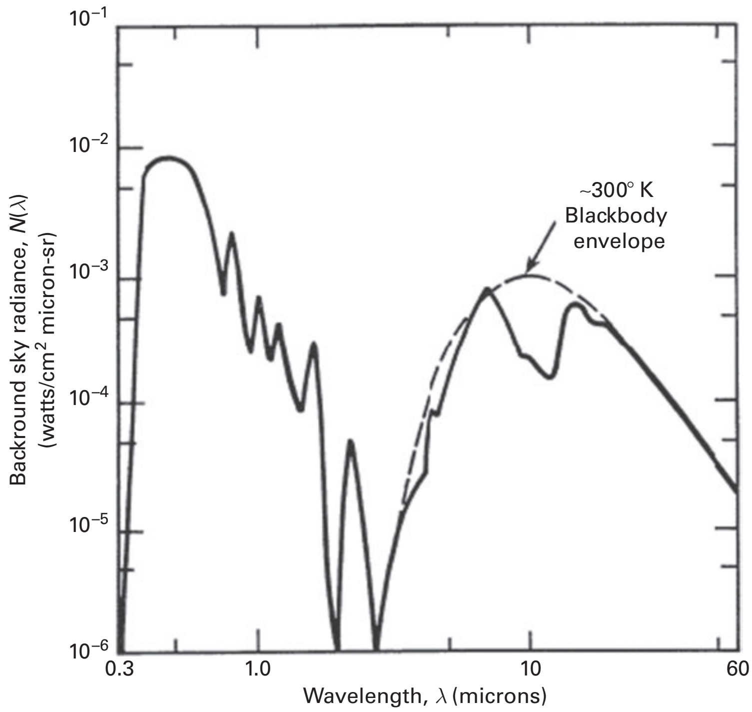

Figure 1.5 Diffuse component of typical background radiance from sea level; zenith angle, 45°; excellent visibility [23].

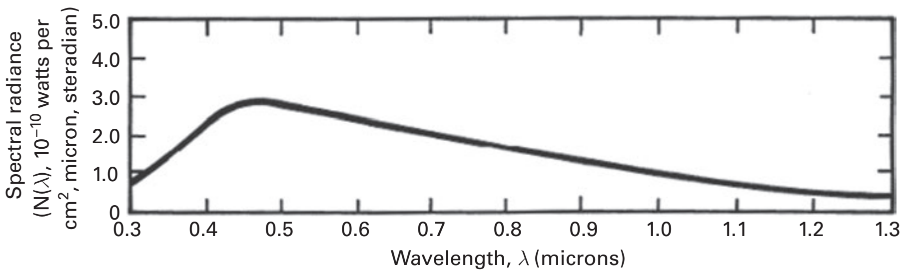

Figure 1.6 Nighttime sky radiance from zenith due to zodiacal light, galactic light, and scattered starlight [23]. (Courtesy of National Bureau of Standards.)

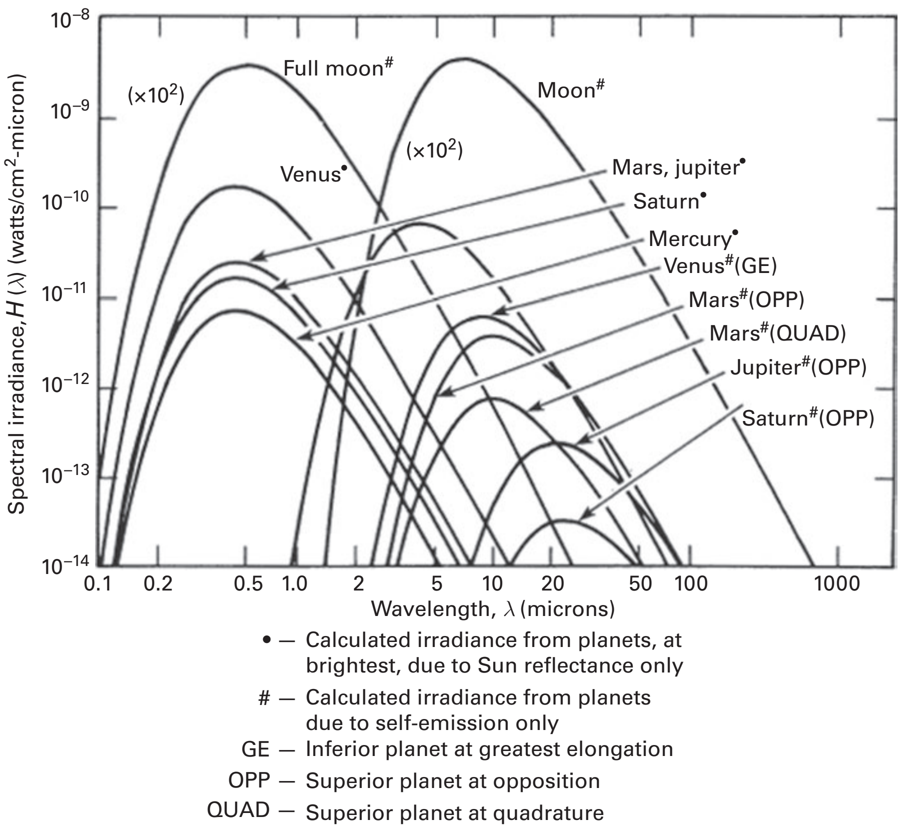

Figure 1.7 Calculated planetary and lunar spectral irradiance outside the terrestrial atmosphere [23].

consists of the re-radiation of the solar energy plus the re-radiation of the Earth's emission at approximately 300° Kelvin [9]. In Figure 1.6, we see the sky radiance at night [10]. In Figure 1.7, we show the irradiance from the planets, Moon and Venus [11]. In Figure 1.8, we see their spectral albedo (spectral albedo is defined as the ratio of the total reflected energy