

# [Mechatronic Modeling of Real-Time Wheel-Rail Contact](#)

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# Chapter 2

## Review of Wheel-Rail Contact Models

**Abstract** This chapter describes the evolution of the theories for solving the wheel-rail contact problem. The determination of the forces acting between wheel and rail is definitely the most important question for the study of the dynamic behaviour of a railway vehicle. In fact, the wheel-rail contact forces provide several fundamental functions for the vehicle: the support action for the vehicle load, the guidance action during the change of direction and the application of traction and braking actions.

### 2.1 Statement of the Problem

The determination of the forces acting between wheel and rail is definitely the most important question for the study of the dynamic behaviour of a railway vehicle. In fact, the wheel-rail contact forces provide several fundamental functions for the vehicle: the support action for the vehicle load, the guidance action during the change of direction and the application of traction and braking actions.

The wheel-rail contact problem can be formulated as a rolling contact problem between two nonlinear profiles in the presence of friction.

This is a problem of considerable complexity, both from the point of view of mathematical-analytical formulation, and from the numerical point of view. It has been studied by many authors in the past and it is still an important focus of rail research activity in order to discover more accurate and efficient formulations.

According to the methodology adopted by de Pater [6–8], it is possible to split the solution of the problem into four sub-problems:

- Geometrical Problem: Wheel-Rail profiles coupling for the identification of the location of contact points and of the geometrical parameters of interest (local curvatures, etc.);
- Normal Problem: calculation of the constraint forces acting between wheel and rail, evaluation of shape and dimension of the contact areas and the corresponding pressure distribution;

- Kinematical Problem: determination of the condition of relative motion between the wheel and rail, usually defined by the kinematic creepages;
- Tangential problem: calculation of the tangential forces generated by friction and creepages in the contact area.

Each of the listed sub-problems can be solved with different methodologies; in the literature, many alternative methods are proposed. In general, the most accurate and complex methods require higher computational times, while simplified methods give higher errors. In particular, the four sub-problems are not really independent, but most complex methods require that they must be evaluated simultaneously or with an iterative process. For example, the tangential forces can modify the contact area, therefore an iteration between the normal problem and the tangential problem can be necessary. Simplified methods usually neglect the interaction between the sub-problems.

The kinematical and the geometrical problems can be solved analytically considering stylised profiles (e.g., conical wheel on a cylindrical rail), but considering real non-linear profiles, the solution can be found only using numerical methods.

The forces acting between wheel and rail, generated by the contact constraints due to the coupling of the two profiles, are strongly influenced by the motion of the wheelset with respect to the track. On a real track, the presence of curves, gradients, cant and track irregularities produce a variation along the track of the normal forces and make their calculation more complex. Calculation of normal forces is therefore usually performed using numerical methods; analytical calculation is possible only in cases of simplified (stylised) profiles, and is not useful in cases of real vehicle simulations.

The calculation of the constraint forces can be made using three different approaches:

### 2.1.1 Rigid Contact

In this case the wheel-rail interface is expressed by a set of algebraic equations that therefore form a bilateral constraint given by the equations (for a single wheelset):

Wheelset Roll

$$\theta = f_1(y, \psi, s) \quad (2.1)$$

Wheelset Vertical

$$z = f_1(y, \psi, s) \quad (2.2)$$

where the independent variables are the lateral displacement of the wheelset  $y$ , the wheelset yaw angle  $\psi$  and the longitudinal coordinate along the track mean line “ $s$ ”. Some simplified approaches are only bi-dimensional, and the yaw angle influence is neglected. In general, Eqs. 2.1 and 2.2 can be obtained by imposing the condition:

$$\text{Min}_{y_C} d(\mathbf{q}, y_C) = 0 \quad (2.3)$$

where  $d$  is the distance between the profiles,  $y_C$  is the lateral coordinate of the contact point and  $\mathbf{q}$  is the state vector of the system. Basically, Eq. 2.3 minimises the distance between the wheel and rail profile and, in cases of regular, monotone and continuous profiles, can be achieved by imposing the tangent condition of the two profiles. In a more general case (real profiles), it can be solved numerically by finding the zeros of the distance function.

This condition allows the finding of the contact point locations which is a fundamental task in order to calculate the contact forces. The calculation of the contact points, since the profiles are usually defined using a wide number of points (200 or more) is one of the most time consuming tasks in the contact force calculation process. For this reason, the condition is usually pre-calculated for different values of the lateral and yaw coordinates by generating bi-dimensional tables. This approach is efficient and can be adopted in case the profiles are not changed or moved (due to track irregularities) along the track. The case of track irregularities can be included in a pre-calculated approach by using N-dimensional tables (usually track irregularities can be defined using 4 or 6 additional coordinates depending if those are defined with respect to the track or the rail).

The rigid approach has two important defects, the first depending on the fact that the constraint is considered as bi-lateral, and therefore allows that traction forces can act between wheel and rail preventing the lifting of the wheel even in the case of track irregularities or other physical phenomena able to realistically generate uplift.

The second problem is the fact that, in order to obtain a constraint equation from the wheel and rail profiles, it is necessary that those profiles are regular enough in order to be always tangent in a single point for each possible reciprocal position. This formulation does not allow consideration of the case of double or multiple point contact, and makes it difficult to apply it to the case of worn profiles.

Despite those limitations, the rigid contact is still one of the most commonly used approaches in commercial multi body codes; this is because the algebraic equations can be easily integrated into the differential algebraic equation solution scheme of the codes, achieving a high computational efficiency.

### 2.1.2 Elastic Contact

In this case, wheel-rail constraint in the normal direction is simulated through a single side elastic contact element. The relative motion between the profiles originates areas of possible intersection between the profiles, where a reaction force proportional to the profiles intersection is applied. Normal contact forces can be calculated as:

$$|F_{N,y_c}| = \begin{cases} 0 & \text{if } d(\mathbf{q}, y_c) \geq 0 \\ \mathbf{K}_{y_c} \cdot d(\mathbf{q}, y_c) & \text{if } d(\mathbf{q}, y_c) < 0 \end{cases} \quad (2.4)$$

This approach allows the use of worn profiles and the simulation of multiple contact points (this obviously requires the definition of a normal force for each contact point). The contact stiffness ( $\mathbf{K}_{y_c}$ ) is very high, is non-linear and depends on the contact area, therefore it should be calculated at each time step. Simplified methods use a constant stiffness that can be estimated using an average Hertzian stiffness for the nominated axle load.

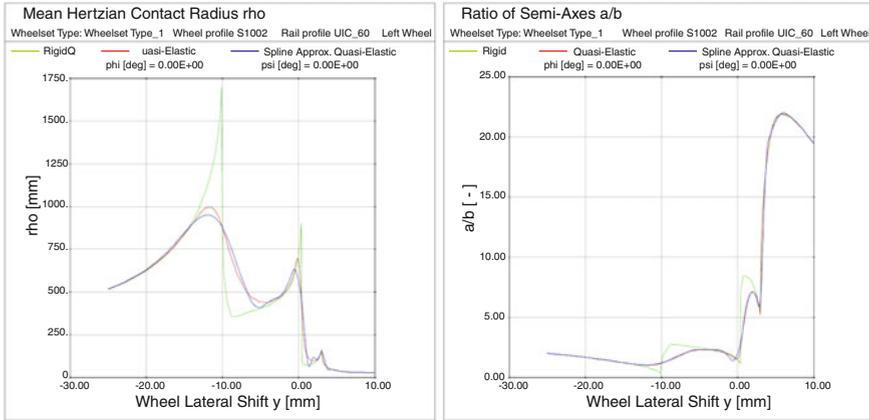
In any case, this stiffness is very high ( $10^9$  N/m) and creates computational problems related to the high frequency arising for the vertical direction. Furthermore, in cases of elastic contact, the determination of the normal load and of the contact points in cases of single/multiple contacts, does not depend only on the independent variables of the wheelset ( $y, s, \psi$ ) as shown by Eqs. 2.1 and 2.2, but depends on the dynamic behaviour of the entire vehicle. All the state variables defining the wheelset position (6 coordinate) are in this case independent, and therefore no table approximation can be adopted. In the literature, many methods have been proposed to solve the elastic contact problem; those can be basically divided into Constant stiffness, Hertzian (single contact patch), Multi-Hertzian and Non-Hertzian methods.

### 2.1.3 Quasi-Elastic Contact

This model has been developed [9–11] in order to allow the simulation using simple algebraic equations of worn profiles and situations that would produce multiple contact points. The result has been obtained by an opportune regularisation of the function defining the profile distance, no longer expressed as a point by point function, but as an averaged function. This has been made by Shupp, Weidemann and Mauer [9] by averaging the distance function in the area of the possible contact, by using the following formulation instead of Eq. 2.3:

$$\varepsilon \cdot \ln \left( \frac{\int_{y_{c,\min}}^{y_{c,\max}} \exp\left(\frac{d(\mathbf{q}, y_c)}{\varepsilon}\right) ds}{\int_{y_{c,\min}}^{y_{c,\max}} ds} \right) = 0 \text{ with } \varepsilon > 0 \quad (2.5)$$

where the distance function is weighted on the entire contact area ( $y_{c,\max}, y_{c,\min}$ ). This method is implemented in the Sim pack simulation package (see an example in Fig. 2.1) and allows an important reduction of calculation times compared to the elastic contact method. The method still has the problem of preventing the wheel uplift.



**Fig. 2.1** Differences between the rigid and quasi-elastic contact model in case of new profiles (S1002/UIC60), obtained using Simpack

## 2.2 Tangential Problem: Evolution of Theories

The resolution of the tangential problem has been however the question that has focused the attention of researchers, leading to define different contact theories. Tangential forces arise due the relative motion between wheel and rail. From early studies it was observed that the behaviour of a wheelset running on the rail could not be considered as a “pure rolling” motion. In fact, the evidence shows the motion is characterised by a “slow” sliding phenomena occurring at the contact. This phenomena of small sliding has been described as a pseudo sliding (or pseudo glissement) or micro-creepage or simply creepage; the forces arising from this motion are therefore indicated as creep forces.

Although from the physical- mathematical point of view, the problem can be considered as solved with the theory developed by Kalker [12] in 1967, research is still relevant to develop algorithms computationally more efficient than those proposed by Kalker.

The first models of contact formulated by Kliegens [13] and others were limited to a simplified geometric solution of the problem, identifying a rigid contact point between wheel and rail, and assuming a relative motion of pure rolling governed by Coulomb’s law for necessary dynamic assessments.

The first experiments performed on steam locomotives by Carter in the 1930’s to study issues related to traction (even using roller rigs), showed that the motion of the vehicle was not of pure rolling either in stationary conditions; this means that the peripheral speed of the wheel was not equal to the speed of the vehicle, but it was lower for the trailing axles and higher for traction axles. By increasing the traction torque applied to the axle, it was observed that, instead of a sudden transition from a condition of perfect adhesion to a full slide condition that could be assumed on the basis of a Coulomb friction model, the sliding condition

gradually increased depending on the applied torque: this was the first observation of creepage.

Considering a mono dimensional model referring only to the longitudinal direction, according to Carter's theory, the relative motion condition can be expressed by defining the (longitudinal) kinematic creepage as:

$$\xi = \frac{V_0 - \omega \cdot r_0}{V_0} \quad (2.6)$$

where  $V_0$  is the longitudinal velocity of the vehicle,  $\omega$  is the angular velocity of the wheel and  $r_0$  is the rolling radius.

Increasing the traction force  $F$ , the experimental results show that in a first phase of the traction force is directly proportional to the creepage; the law can be expressed as:

$$F = c \cdot \xi \quad (2.7)$$

In this area, the phenomenon is governed by an elastic deformation of the bodies and  $c$  is an appropriate constant depending on the geometry and the normal load.

When the force approaches the saturation of the friction force, the trend becomes non-linear; at this stage, in a portion of the contact area, a loss of adhesion (localised slip) is generated. Further increasing the traction force causes the proportion of sliding to increase until it reaches the limit set by Coulomb's law, when sliding occurs over the entire area:

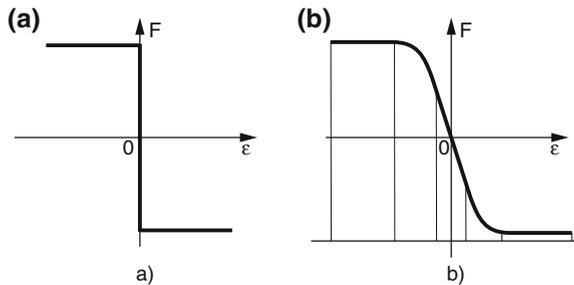
$$F = \mu \cdot N \quad (2.8)$$

Qualitative force behaviour is shown in Fig. 2.2 and compared with Coulomb's friction law.

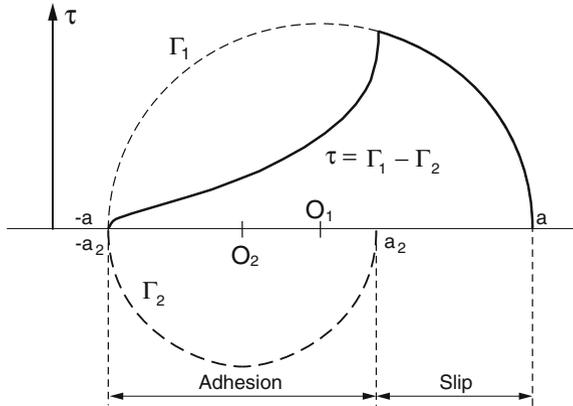
Carter has developed a model of contact [14], considering elastic bodies and therefore finite contact areas and assuming that the contact area is divided into a portion where sliding occurs and another where there is adhesion with varying proportions depending on the applied traction force.

The model proposed to calculate the size of the contact areas on the basis of the Hertz theory for the case of contact between a cylinder and a plane. The total

**Fig. 2.2** **a** Coulomb friction force, and **b** wheel-rail friction force



**Fig. 2.3** Tangential stress behaviour according to Carter's theory



tangential forces exchanged between wheel and rail are calculated by integrating the tangential stress on the entire contact area. To calculate the distribution of tangential efforts, Carter introduced the hypothesis of an elastic half-space and obtained an analytical solution.

In summary, the shear stress  $\tau$  can be calculated as the difference between two circles, the first with a diameter equal to the size of the contact area ( $\Gamma_1$  shown in Fig. 2.3) and the second with a radius  $a_2$  ( $\Gamma_2$ ), which varies depending on the creepage  $\zeta$ .

Carter assumed that the position of the lower circle represented the area of adhesion, tangent to the bigger circle at the leading edge, on the side of the area that is “coming” into contact because of the rolling of the wheel.

The main limitations of the Carter's theory are that it is a mono-dimensional theory that is not suitable for the study of lateral dynamics of vehicles, and secondly that the value of the coefficient of proportionality in the elastic region has been incorrectly calculated using the formula:

$$c = A \cdot \sqrt{r \cdot N} \tag{2.9}$$

The first tri-dimensional contact model was proposed by Johnson, who in 1958 published two works considering a sphere running on a plane. The resulting model was tri-dimensional because a sphere in contact with a plane has 3 degrees of freedom: the longitudinal displacement in the direction of movement, the lateral displacement (laying on the plane and normal to the direction of movement), and the spin rotation defined around the axis originated from the contact point and normal to the plane. For each of these degrees of freedom it is possible to define a specific relative slide between the two bodies, and therefore to define the corresponding kinematic creepages. Johnson has, in the first work [15], analysed the influence of spin, and in the second [16], the effects of lateral and longitudinal creepages. The model had the limitation of considering only circular contact areas, therefore, with the cooperation of Vermeulen in 1964, he extended the theory to

the case of elliptical contact areas in the presence of lateral and longitudinal creepages [17].

The pressure distribution in the model of Vermeulen and Johnson was such as to predict that the adhesion area was elliptical, as for the contact area, and tangential to the contact area at a single point corresponding to the leading edge.

Johnson himself observed that the assumption of an elliptical adhesion area, tangential to the contact area at a single point could not be correct due to the presence in the area of a transition of sliding- adhesion-sliding.

In 1963 Haines and Ollerton [18] developed a method to obtain a more accurate distribution for tangential stresses for the case of pure longitudinal creepage; this method, the strip theory, allowed them to obtain the traction force by integrating the tangential stress using strips parallel to the creepage, starting from the leading edge (where new particles of the wheel enter into the contact area) and assuming slip at the trailing edge ( $\tau = \mu p$ ). The integration was complex from the mathematical point of view, and this method was only fully implemented later by Kalker [19], who also extended its application to a more general case.

At the Delft University, Kalker performed his studies using limitations as low as possible, considering an elliptical contact area with the simultaneous presence of creepage  $\xi$  (longitudinal),  $\eta$  (lateral) and  $\phi$  (spin). During his activity he developed several theories and algorithms that can be considered as the knowledge-base of the modern wheel-rail contact theories. Equation 2.6 gives a simplified formulation of the longitudinal creepage for a wheelset whose motion is not affected by lateral movements and yaw rotations. In order to study a more general case, Kalker considered a different formulation for the three creepages (also in agreement with definitions given previously by Johnson), given by a ratio between the relative velocity in the relevant direction and the vehicle reference velocity  $V_0$ .

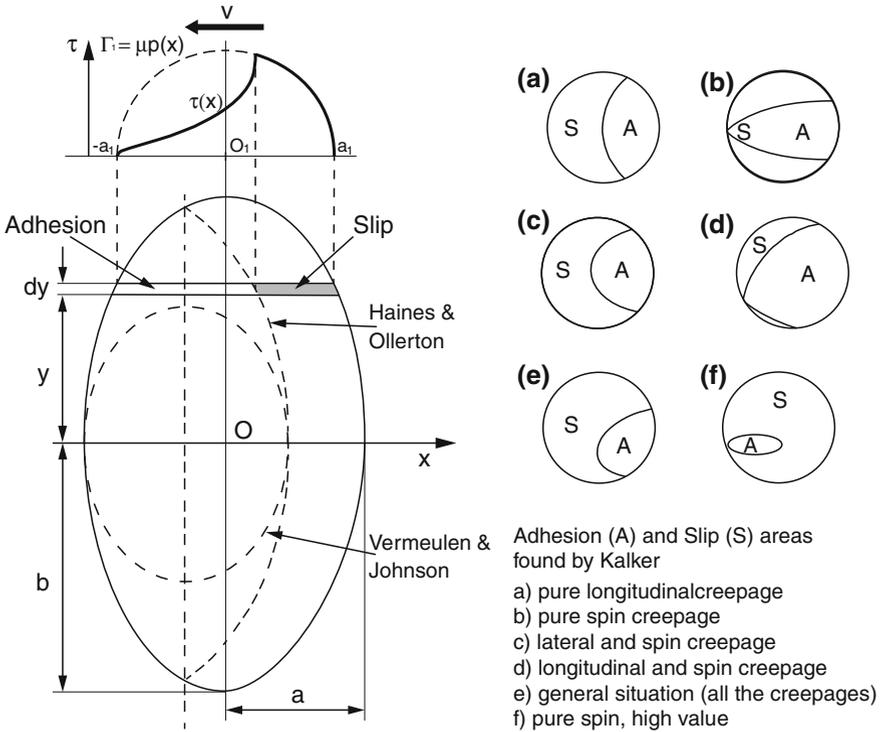
Figure 2.4 shows a comparison of the different theories and the behaviour of the slip/adhesion region for different combination of the creepages.

The creepages can be calculated as follows:

$$\xi = \frac{v_x}{V_0} \quad \eta = \frac{v_y}{V_0} \quad \phi = \frac{\omega_\phi}{V_0} \quad (2.10)$$

where  $v_x$ ,  $v_y$  are the relative velocities between wheel and rail in the longitudinal and lateral directions and  $\omega_\phi$  is the relative angular velocity around the normal direction at the contact point.

In his Ph.D. thesis [12], starting from the Hertz theory, assuming that the contact area could be considered as an elastic half-space and that a condition of steady state rolling existed, Kalker has described an exact analytical method to calculate the contact forces in the linear portion of the force-creepage curve; this is known as the Kalker linear-theory. In matrix form, the forces exchanged between wheel and rail can be expressed as function of the creepage as follows:



**Fig. 2.4** Comparison between the tangential stress distributions obtained by Haines and Ollerton, Vermeulen and Johnson and Kalker for different creepage combinations

$$\begin{Bmatrix} F_x \\ F_y \\ M_z \end{Bmatrix} = G \cdot a \cdot b \cdot \begin{bmatrix} C_{11} & 0 & 0 \\ 0 & C_{22} & \sqrt{a \cdot b} \cdot C_{23} \\ 0 & -\sqrt{a \cdot b} \cdot C_{23} & a \cdot b \cdot C_{33} \end{bmatrix} \cdot \begin{Bmatrix} \xi \\ \eta \\ \phi \end{Bmatrix} \quad (2.11)$$

where  $G$  is the tangential elasticity modulus and the  $C_{ij}$  coefficients are known as Kalker's coefficients and can be calculated as functions of the  $a/b$  ratio and of the Poisson's module ( $\nu$ ).

To obtain the distribution of the tangential tension and therefore the behaviours of the creep forces in the non-linear part of the curve, Kalker developed a first theory specifically for the case of elliptical contact areas.

This theory, which is still nowadays the reference for the tangential force calculation, is based on the Hertz theory to determine the contact areas. Therefore it is based on the same assumptions made by Hertz which were:

- Bodies directly pressed in contact;
- Bodies in contact are elastic and isotopic;
- Non-conformal contact: the contact area is small with respect to the radii of curvature of the bodies;

- Absence of friction.

The last assumption obviously cannot be met in the wheel-rail cases studied due to the presence of the tangential forces. Therefore Kalker has considered two additional assumptions to ensure that the contact area and the normal stress distribution cannot be modified by the tangential stresses:

- Elastic half-space: if the contact area is small with respect to the curvatures of the bodies, it is possible to solve the problem as bi-dimensional by approximating the contact surfaces using a plane;
- Quasi-identity: the elastic properties of the two half-spaces of the bodies in contact are considered identical in which case the tangential stress does not affect the normal stress distribution.

Using the Hertz theory, it is possible to evaluate the contact area  $C$  once the curvatures of the profiles around the contact point are known; this area can be described by the equation:

$$C = \left\{ (x, y) \left| \left( \frac{x}{a} \right)^2 + \left( \frac{y}{b} \right)^2 \leq 1 \right. \right\} \quad (2.12)$$

The normal pressure distribution is found to be semi-elliptical in the contact area, and is given by:

$$p_z(x, y) = G \cdot f_{00} \cdot \sqrt{1 - \left( \frac{x}{a} \right)^2 - \left( \frac{y}{b} \right)^2} \quad (2.13)$$

The reference system has an origin at  $O(x, y)$  in the centre of the contact ellipse, which has semi-axes  $a$  and  $b$ ,  $G$  being the tangential elasticity modulus and  $f_{00}$  an appropriate coefficient obtained using Hertz's theory. Therefore in each point of the contact area, the maximal tangential tension is shown to be:

$$|p_\tau| \leq \mu \cdot p_z \quad (2.14)$$

By introducing the local slip  $s_\tau$ :

$$\begin{aligned} p_\tau(x, y) &= -\frac{\mu \cdot p_z \cdot s_\tau}{|s_\tau|} & \text{con } |s_\tau| \neq 0 \\ p_\tau(x, y) &= 0 & \text{con } |s_\tau| = 0 \end{aligned} \quad (2.15)$$

Starting from the kinematic creepages, it is possible to define a displacement field  $u_\tau(x, y)$  on the contact area, and under the assumption of the elastic half space, it is possible to use the laws of elasticity to relate the stresses ( $p_\tau$ ) to the deformations ( $u_\tau$ ).

Note that only under the assumption of quasi-identity does the tangential stress not affect the normal stress distribution given by Eq. 2.13 and the shape of the contact area. In this case, according to what is known as the ‘‘Johnson process’’, it is possible to evaluate the normal stress according to the Hertz theory (not

considering the tangential stresses) and, depending on the normal stress (that remain unchanged during the iteration process), evaluate the tangential stresses.

The two fields of tangential stress and deformation, and therefore the areas of adhesion and slip, have been found by Kalker solving a constrained optimisation problem (the constraints are given by Eqs. 2.14 and 2.15) and implemented in the DUVROL algorithm [19] and later improved with the CONTACT [20] algorithm.

The determination of a field of stress as a solution of an optimisation problem has been found by Kalker on the basis of the variational theory developed by several authors [21–23], and according to this approach it is possible to obtain the stress tangential distribution by minimising the elastic energy potential or maximising the complementary energy over the contact area (the latter being the strategy chosen by Kalker). The existence of a solution has been proven by Fichera [22] for a field of normal stresses and by Oden [23] for a field of tangential stresses.

Kalker in the DUVROL algorithm has imposed for the tangential stress distribution a polynomial form, and the solution of the optimisation problem requires finding the coefficient of the polynomial; this process however, solved numerically, requires a high computational effort. In order to be able to use a contact algorithm within a numerical code to simulate an entire vehicle, a faster solution is needed. For this reason, Kalker himself has developed his *simplified theory*, and then implemented it using a faster algorithm called FASTSIM [3] which is faster by a factor of thousands than the more accurate codes.

The simplified theory is based on a relaxation of the elastic relations between surface deformations and tangential stresses, which is given by a single equivalent flexibility parameter instead of three parameters used in the exact theory. The solution is obtained by numerical strip integration over the contact area which is a dimensionalised and discretised over a small number of elements (usually  $20 \times 20$ ).

Results obtained using FASTSIM, according to the calculation performed by Kalker [20], can give errors up to 25 % on the tangential forces; in any case, the code requires an iterative process and is still highly time consuming to ensure a good calculation for complex vehicle models on a long track.

Many authors have therefore proposed to use interpolation methods that, in general, can be defined as a non-linear function of several variables, where the function can be defined either by tables or specific formulas:

$$\begin{Bmatrix} F_X \\ F_Y \\ M_Z \end{Bmatrix} = f(\xi, \eta, \varphi, a/b, N, \mu) \quad (2.16)$$

Those methods are indeed more efficient than FASTSIM because they do not require any iterative cycles; furthermore, the interpolation method can be applied with more accurate codes (e.g., CONTACT) or experimental results.

An interpolation method that had a wide usage in commercial codes due to its efficiency (computation times are approximately 10 % of the time required using

FASTSIM) and accuracy (the method has been obtained by interpolating the results of the CONTACT code) is the Polach's method [4, 24]. Polach's method also allows the study of non steady state conditions [5] by using a friction coefficient which varies with the vehicle velocity, while FASTSIM has been developed to operate in a steady state condition (constant velocity).

### 2.3 Non-Hertzian Contact Methods

The assumption of elliptical contact areas is found to be correct using simplified profiles (conical wheel and cylindrical rails), but in cases of real profiles it can be inaccurate.

In particular, the elliptical areas are related to non conformal contact between the profiles; this condition can be unrealistic near the flange contact, especially considering worn profiles. In fact, for worn profiles, the radius of curvature of the bodies are approximately of the same dimension as the contact area, as shown in Fig. 2.5.

The problem of non conformal contact has been studied by many authors (Johnson [25], Kalker [1, 20, 26], Novell and others [28], Knothe [29]) and two possible solutions have been found. The simpler solution consists of considering equivalent Hertzian areas instead of the non-Hertzian areas; the equivalent areas are usually pre-calculated. This approach achieves good computational times, and values of the creep forces are in good agreement with the results from more complex methods.

This approach can be inaccurate when studying complex phenomena related to the contact area, such as wear prediction, where it is necessary to know the stress distribution over the contact areas. A problem can be also found when studying profiles which are variable along the track (switch simulation for example [30]), because it is not possible to pre-calculate the equivalent elliptical areas, and this make the method inefficient.

The other approach consists of developing more realistic non-elliptical contact areas that can be found by intersecting the profiles considering the local elasticity and, in more complex methods, the deformation caused by all the stresses (including the tangential stresses).

The problem has also been studied by Kalker, who has improved his theory based on an assumption of quasi-identity is not applicable leading to non-Hertzian contact areas, and this more general approach has been implemented in the CONTACT algorithm [20]. In the case where the quasi-identity assumption is not applicable, a normal stress distribution cannot be defined a priori for the tangential force calculation; to solve this problem, Kalker has used the Pangiotopoulos [31, 32] iterative process that can be described as follows:

Step 1:  $i = 0$ ;  $p_x^i = p_y^i = 0$ .

Step 2: calculation of  $p_z^i$  with  $p_x^i, p_y^i$  as tangential stresses.

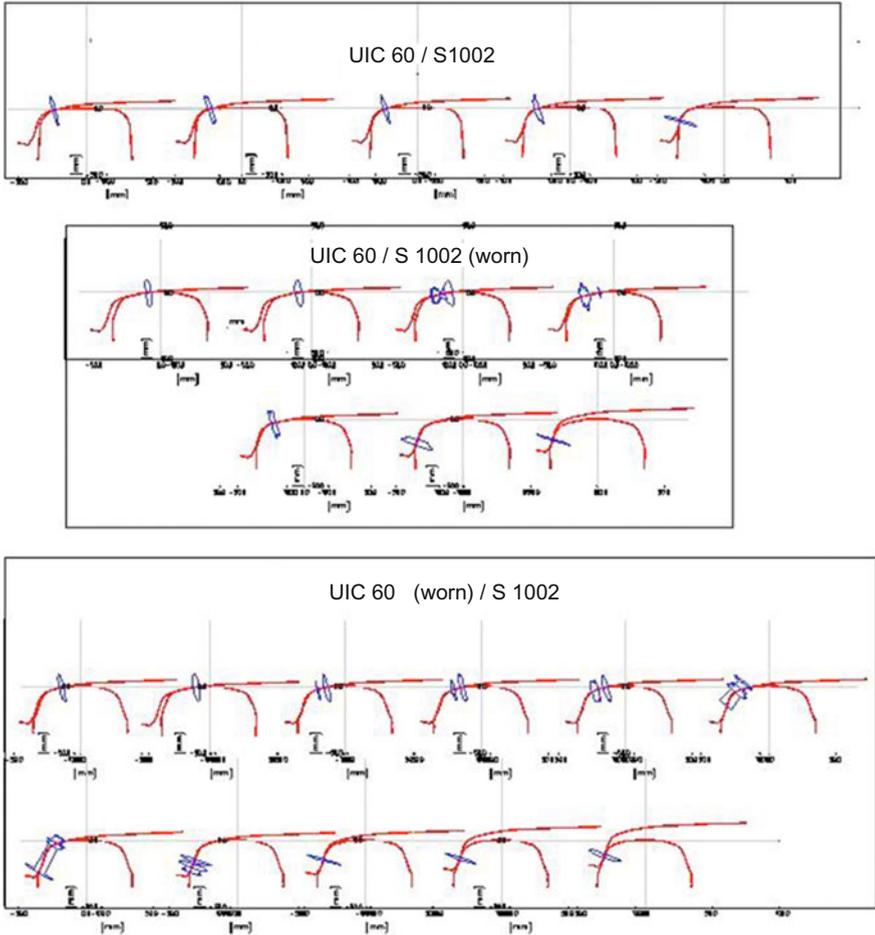


Fig. 2.5 Examples of non-conformal contact and non-Hertzian contact areas

Step 3: calculation of  $p_x^{i+1}, p_y^{i+1}$  with  $p_z^i$  as normal stress and  $u(x, y)$  as the assigned displacement field using the optimisation algorithm.

Step 4: iteration of steps 2 and 3 until convergence of  $p_x^{i+1}, p_y^{i+1}$  to  $p_x^i, p_y^i$  with the desired error.

It is evident that the process includes a double iteration to calculate both the normal and tangential stress distribution, therefore this leads to a huge computational complexity.

Kalker has also studied, together with other authors (Kalker-Piotrowsky [26], Li-Kalker [27]) an extension of his theories to the case of conformal contact.

An interesting approach for studying the problem of non conformal contact is given by the use of the Multi-Hertzian approach [2, 26, 33, 34]. Conformal contact

results in wide contact areas, that cannot be described by a single value of the curvature of the profiles. The Multi-Hertzian approach consists of the representation of a non-elliptical contact area with the superposition of several elliptical areas, the  $a/b$  ratio of each ellipse being obtained from the local value of the curvature, and the size of each ellipse is given by the portion of normal load acting on it. The distribution of the normal load can be found by considering a Hertzian stiffness acting on each area that gives a parallel system of stiffness where the load is distributed.

The method requires non-trivial corrections to account for the fact that the multiple Hertzian ellipses are not disjointed but often superimposed for large portions. Another complexity is related to the determination of the Hertzian stiffness on each area, which is dependent on the normal load acting on the considered ellipse.

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