3-D Structural Geology

A Practical Guide to Quantitative Surface and Subsurface Map Interpretation

Bearbeitet von
Richard H Groshong

Neuausgabe 2008. Buch. xvi, 400 S.
ISBN 978 3 540 31054 9
Format (B x L): 0 x 0 cm
Gewicht: 831 g

Chapter 2

Location and Attitude

2.1 Introduction

This chapter covers the basic building blocks required to construct 3-D models of geological structures. Included here are methods for locating points in three dimensions on a map or in a well, for determining the attitude of a plane and the orientation of a line, and for representing planes with structure contours. Both graphical and analytical solutions for finding and displaying lines and planes are presented. Many of the analytical techniques are based on vector geometry, a topic treated separately in Chap. 12. The graphical techniques utilize stereograms and tangent diagrams.

2.2 Location

The locations of data points recorded on maps and well logs must be converted to a single, internally consistent coordinate system in order to be used in the interpretive calculations given in this book. The positions of points in three dimensions will be described in terms of a right-handed Cartesian coordinate system with $+x$ = east, $+y$ = north, and $+z$ = up. Dimensions will be given in feet and kilofeet or meters and kilometers, depending on the units of the original source of the data. Parts of a foot will be expressed as a decimal fraction. Unit conversions are a common source of error which are largely avoided by retaining the original units of the map or well log. The relationships between locations on a topographic map or well log and the *xyz* coordinate system are given next.

2.2.1 Map Coordinate Systems, Scale, Accuracy

The true locations of points on or in the earth are given in the spherical coordinate system of latitude, longitude, and the position along an earth radius. Maps are converted from the spherical coordinate system to a plane Cartesian coordinate system by projection. Some distortion of lengths or angles or both is inherent in every projection technique, the amount of which depends on the type of projection and the scale of the map (Greenhood 1964; Robinson and Sale 1969; Bolstad 2002) but is not significant at the scale of normal field mapping.

In many regions, maps are based on the transverse Mercator or polar stereographic projections and contain a superimposed rectangular grid called the Universal Transverse Mercator (UTM) or the Universal Stereographic Projection (USP) grid (Robinson

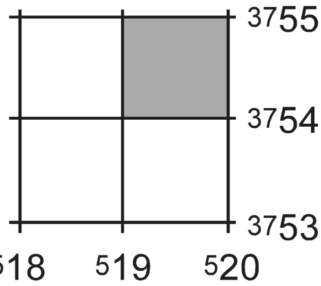


Fig. 2.1.
UTM grid (NAD-27) in north-central Alabama as given on a USGS topographic map. Each square block is 1 000 m on a side. Block 1 954 is *shaded*

and Sale 1969; Snyder 1987). The UTM grid is used at lower latitudes and the USP grid is used in the polar regions. The UTM grid divides the earth into $6 \times 8°$ quadrilaterals that are identified by reference numbers and letters. All UTM coordinates are with respect to a survey datum that should be specified. In North America, for example, UTMs are referenced to either the North American Datum of 1927 (NAD-27) or the datum of 1983 (NAD-83) (Bolstad 2002).

UTM coordinates along the margin of a United States Geological Survey (USGS) map are given at intervals of 1 000 m (Fig. 2.1). On a topographic quadrangle map, the first digit or digits of the UTM coordinates are shown as a superscript and the last three zeros are usually omitted. Locations within the grid are given as the coordinates of the southwest corner of a block within the grid system. The x value is called the easting, and the y value is called the northing. For example, the lower left coordinate in Fig. 2.1, $^{5}18$, is 518 000 m east of the origin. Any block can be subdivided into tenths in both the x and y directions, adding one significant digit to the coordinates of the sub-block. Sub-blocks may be similarly subdivided. UTM coordinates are commonly written as a single number, easting first, then the northing, with the superscript and the trailing zeros omitted. For example, a grid reference of 196 542 in the map of Fig. 2.1 represents a block 100 m on a side with its southwest corner at $x = 196$, $y = 542$. A grid reference of 19 605 420 represents a block 10 m on a side with its southwest corner at $x = 1\,960$, $y = 5\,420$. This coordinate system is internally consistent over large areas and is convenient for maintaining large databases. The United Kingdom uses a similar metric system called the National Grid (Maltman 1990).

The surface locations of wells are commonly recorded according to the coordinates given on a cadastral map, which is a map for officially recording property boundaries, land ownership, political subdivisions, etc. In much of the United States the cadastral system is based on the Land Office grid of Townships and Ranges (Fig. 2.2). The grid is aligned with latitude at base lines and with longitude at guide meridians. Every 24 miles the grid is readjusted to maintain the 6 mile dimensions of the blocks. An individual township is located according to its east-west coordinate (Township) and its north-south coordinate (Range). The township that has been subdivided (Fig. 2.2) is T.2N., R.3E. A township is subdivided into 36 sections, each 1 mile on a side and numbered from 1 to 36 as shown in Fig. 2.2. For a more precise location, each section is subdivided into quarter sections (may be called corners) as in the northeast quarter of sect. 7, abbreviated NE¼ sect. 7. Quarter sections may themselves be divided into quarters, as in the northwest quarter of the northeast quarter of sect. 7, abbreviated NW¼NE¼ sect. 7, T.2N., R.3E. Locations within

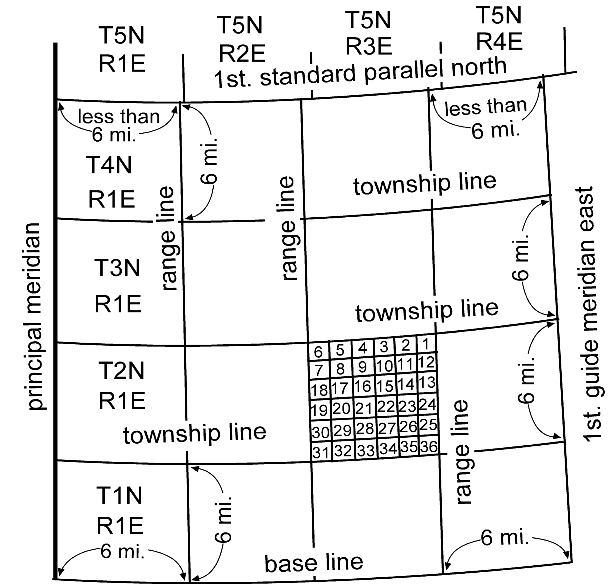


Fig. 2.2.
The Township-Range grid system. The basic unit is a 24-mile block of 6-square-mile townships. Township T2N, R3E is divided into sections. (After Greenhood 1964)

a quarter are given in feet measured from a point that is specified. The surveys were not always done perfectly and were sometimes forced into irregular shapes by the topography. It is necessary to see the local survey map to be certain of the locations.

Map scales are expressed as a ratio in which the first number is the length of one unit on the map and the second number is the number of units of the same length on the ground. The larger the scale of the map, the smaller the second number in the ratio. Geological maps suitable for detailed interpretation are typically published at scales ranging from 1 : 63 300 (1 in to the mile: 1 in = 63 300 in = 1 mile) to 1 : 24 000 (1 in = 24 000 inches, the 7.5 minute quadrangles of the USGS). Larger scales are useful for making very detailed maps. Base maps should always contain bar scales which make it easy to enlarge or reduce the map while preserving the correct scale.

Most governmentally produced base maps have an accuracy equivalent to the U.S. Class 1 map standard (Fowler 1997). This standard states that the horizontal position of 90% of the points must be within 0.5 mm of their true location at the scale of the map. For example, at the 1 : 24 000 scale, a point on the map must be within 0.5 mm × 24 000 = 12 m (39.4 ft) of its true location. The vertical position of 90% of all contours must be within half the contour interval in open areas and spot elevations must be within one-quarter of a contour of their true elevation. A 20-ft contour (characteristic of 1 : 24 000 USGS topographic maps) can be expected to be accurate to ±10 ft (3 m). For comparison, the width of the thinnest contour line is about 0.01 in on a 1 : 24 000 USGS topographic map and represents 20 ft (6.01 m) at the scale of the map. The accuracy of an enlarged (or reduced) map is no better than it was at the original scale. Ground surveys of point locations such as wells can be expected to be accurate to within 1 m and satellite surveys can be accurate to 0.1 m within the survey area (Aitken 1994), significantly greater accuracies than that of many base maps.

2.2.2 Geologic Mapping in 3-D

Widely available computer technology now makes it possible to construct and interpret geological maps in 3-D. Three-dimensional geologic outcrop mapping usually begins with a digital elevation model (DEM). A DEM is a grid of elevation values and their corresponding *xy* coordinates (Bolstad 2002). The DEM is contoured (typically by triangulation because it is computationally fast and fits the control points, see Chap. 3) to produce a computer visualization of the topographic surface (Fig. 2.3). The accuracy of the most widely available DEMs is not as great as the corresponding topographic map, which may be important in critical applications. For DEMs produced at the scale of a 7.5-minute quadrangle and derived from a photogrammetric source, 90 percent have a vertical accuracy of 7-meter root mean square error or better and 10 percent are in the 8- to 15-meter range (source USGS EROS Data Center).

Geological contact lines can be constructed as three-dimensional traces on the surface of the contoured DEM. This can be achieved by extracting lines from the 3-D topographic surface within 3-D software (Fig. 2.4), or by mapping 3-D lines in outcrop using a Global Positioning System (GPS) receiver, which provides a digital file that can be used with the computer map (Bolstad 2002). Extracting lines from the contoured DEM usually is done over a superimposed image of the hard-copy geological map. Geographic information system (GIS) software allows the image of a geological map to be draped over the DEM surface, giving a realistic view of the map surface. Images used for this purpose need to be ortho rectified, aligned and georeferenced to fit the DEM. Once the

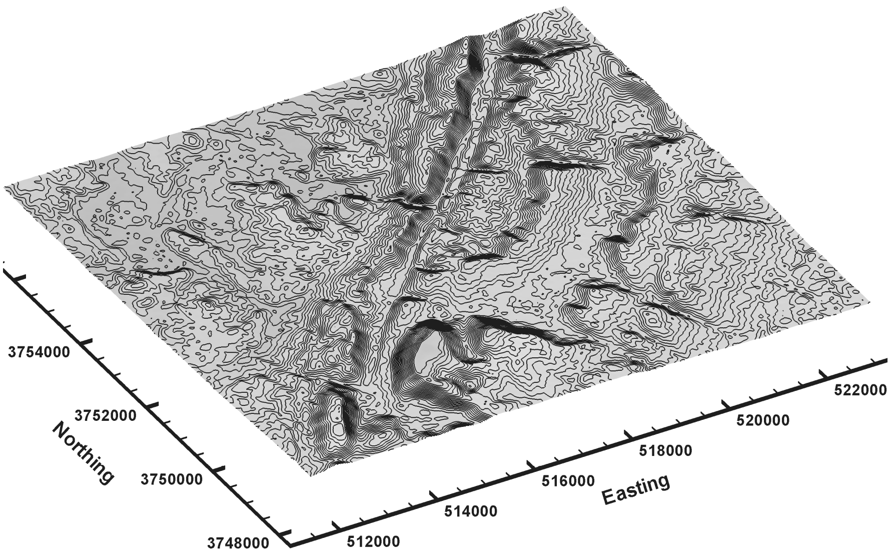


Fig. 2.3. Oblique view to the NE of the Blount Springs area, Alabama, 50 ft topographic contours derived from a 30 m Digital Elevation Model (NAD-27). Horizontal coordinates are UTMs in meters, no vertical exaggeration

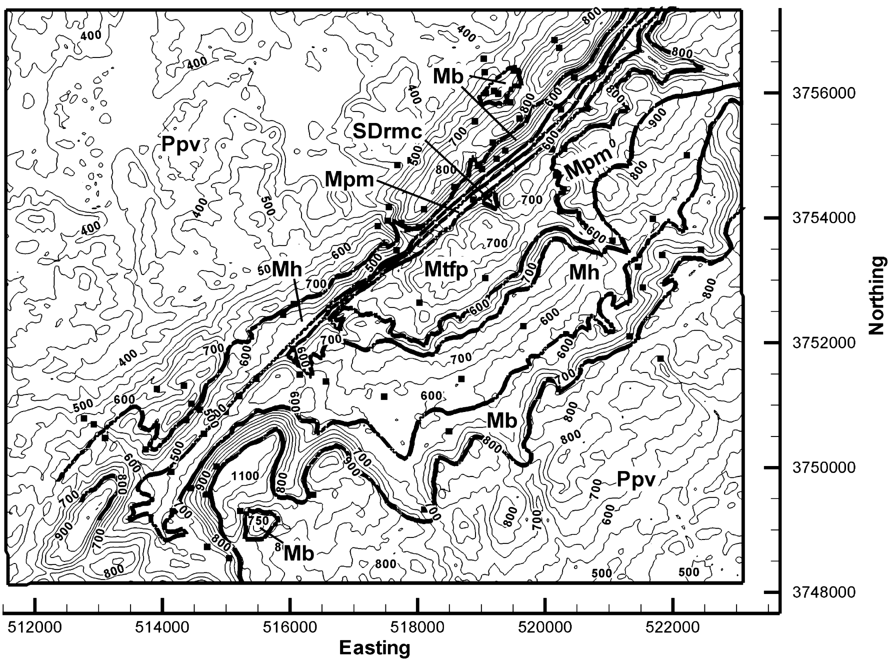


Fig. 2.4. Geologic map of a portion of the southern Sequatchie anticline in the vicinity of Blount Springs, Alabama. Base map is a DEM. The units are, from oldest to youngest, Silurian Red Mountain Formation and Devonian Chattanooga Shale (*SDrmc*), Mississippian Tuscumbia Limestone and Fort Payne Formation (*Mtfp*), Mississippian Pride Mountain Formation (*Mpm*), Mississippian Hartselle Sandstone (*Mh*), Mississippian Bangor Limestone (*Mb*), Pennsylvanian Pottsville Formation (*Ppv*). *Thick lines* are geologic contacts, *thin lines* are topographic contours (50 ft interval). Horizontal scale is UTM in meters. *Solid squares* are locations where bedding attitudes have been measured. (After Cherry 1990; Thomas 1986)

contacts are digitized, they can be visualized as part of the 3-D model (Fig. 2.5), edited and otherwise manipulated digitally.

The geologic map of the Blount Springs area (Figs. 2.4, 2.5) will provide the data for an ongoing example of the process of creating and validating a structure contour map. The map area is located along the Sequatchie anticline at the southern end of the Appalachian fold-thrust belt and is the frontal anticline of the fold-thrust belt.

2.2.3 Wells

The location of points in a well are measured in well logs with respect to the elevation of the wellhead and are usually given as positive numbers. Depths in oil and gas wells are usually measured from the Kelly bushing (Fig. 2.6a). The elevation of the Kelly bushing (KB) is given in a surveyor's report included as part of the well-log header information. Alternatively, depths may be measured from ground level (GL) or the derrick floor (DF).

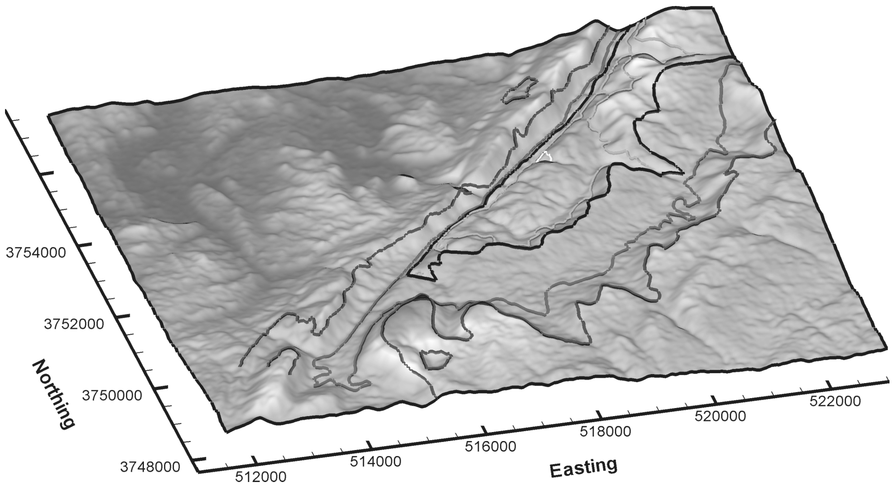


Fig. 2.5. 3-D oblique view of geologic map of Blount Springs area, from Fig. 2.4. Topography is shown as shaded relief map, without vertical exaggeration

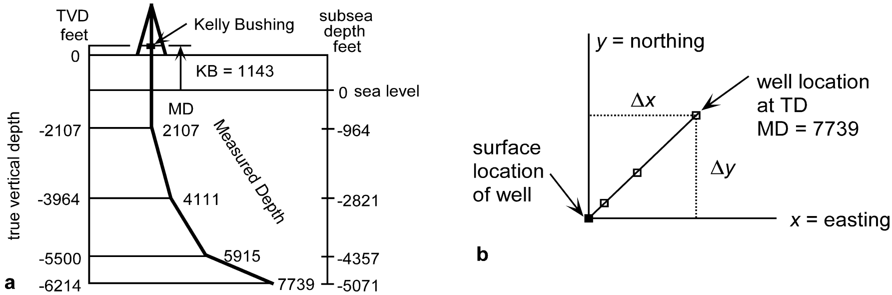


Fig. 2.6. Location in a deviated well. **a** Vertical section through a deviated well, in the northeast-southwest direction. Depths in the well are measured downward from the Kelly bushing (*KB*). True vertical depths (*TVD*) are calculated from the borehole deviation survey. **b** Map view of the deviated well. Locations of points down hole are given by their distance from the surface location. *TD* total depth; *MD* measured depth

2.2.3.1 Datum

The coordinates of points in a well need to be corrected to a common datum elevation, normally sea level. The depths should be adjusted so that they are positive above sea level and negative below. In a vertical well the log depths are converted to a sea-level datum with the following equation:

$$\mathrm{SD} = \mathrm{KB} - \mathrm{MD} \quad , \tag{2.1}$$

where SD = subsea depth, KB = elevation of Kelly bushing or other measurement of surface elevation, MD = measured depth on well log.

2.2.3.2
Deviated Well

Many wells are purposely drilled to deviate from the vertical. This means that points in the well are not directly below the surface location. The shape of the well is determined by a deviation survey, the terminology of which is given in Fig. 2.6. The primary information from a deviation survey is the azimuth and inclination of the borehole and the downhole position of the measurement, for a number of points down the well. This information is converted by some form of smoothing calculation into the *xyz* coordinates of selected points in well-log coordinates, known as true vertical depth (TVD, Fig. 2.6a) and is given in the log of the survey. The TVD must be corrected for the elevation of the Kelly bushing to give the locations of points with respect to the datum. In a deviated well the log depths are converted to a sea-level datum with the following equation:

$$SD = KB - TVD \quad , \tag{2.2}$$

where SD = subsea depth, KB = elevation of Kelly bushing or other measurement of surface elevation, TVD = true vertical depth from the deviation survey.

The calculated position of the points determined from a deviation survey depends on the spacing between the measurement points and the particular smoothing calculation used to give the TVD locations. If the measurement points are spaced tens of meters or a hundred or more feet apart, the positions of points toward the bottom of a 3 000-m (10 000-ft) well might be uncertain by tens of meters or a hundred feet or so. This is because the absolute location of a point depends on the accuracy of the location of all the points above it in the well. Small errors accumulate. The relative positions of points spaced a small distance apart along the well should be fairly accurate. Points will be accurately located if the deviation survey is based on points spaced only a meter or less apart.

Locations in a deviated well are commonly given as the *xyz* coordinates of points in the well relative to the surface location, for example, P_1 and P_2 in Fig. 2.7a. Here *z* is the vertical axis, positive upward, the subscript "1" will always denote the upper point and the subscript "2" will denote the lower point. A unit boundary is likely to be located between control points, making it necessary to calculate its location.

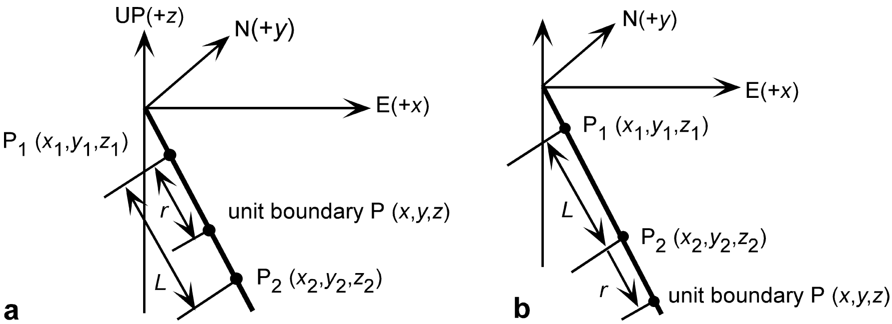


Fig. 2.7. Calculation of position of a unit boundary in a deviated well. **a** Boundary located between two control points. **b** Boundary located below the control points

To find the coordinates of an intermediate point, P, between P_1 and P_2, let r be the distance along the well from the upper control point to the point P. In a right-handed xyz coordinate system (Fig. 2.7) with z positive upward, positive x = east and positive y = north, the point P (x, y, z) between P_1 (x_1, y_1, z_1) and P_2 (x_2, y_2, z_2) at a location given by the ratio r/s (where $r + s = L$), has the coordinates (Eves 1984)

$$x = (rx_2 + sx_1) / (r + s) \quad , \tag{2.3a}$$

$$y = (ry_2 + sy_1) / (r + s) \quad , \tag{2.3b}$$

$$z = (rz_2 + sz_1) / (r + s) \quad . \tag{2.3c}$$

L is found from the Pythagorean theorem:

$$L = [(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]^{1/2} \quad . \tag{2.4}$$

Substitute $s = L - r$ into Eq. 2.3 to obtain

$$x = (rx_2 - rx_1 + Lx_1) / L \quad , \tag{2.5a}$$

$$y = (ry_2 - ry_1 + Ly_1) / L \quad , \tag{2.5b}$$

$$z = (rz_2 - rz_1 + Lz_1) / L \quad . \tag{2.5c}$$

If the well is straight between the upper and lower points, L will be equal to the log distance. If L is not equal to the log distance, then the well is not straight and the calculated value of L will give the more internally consistent answer. More precise location of the point will require definition of the curvature of the well between the control points.

It may be required to locate a contact below the last control point given by the deviation survey (Fig. 2.7b). A simple linear extrapolation can be used, based on the assumption that the well continues in a straight line below the last control point with the same orientation it had between the last two control points. The subscripts "1" and "2" again represent the upper and lower control points. Let $\Delta x = (x_2 - x_1)$, $\Delta y = (y_2 - y_1)$, and $\Delta z = (z_2 - z_1)$, then

$$x = x_2 + r\Delta x / L \quad , \tag{2.6a}$$

$$y = y_2 + r\Delta y / L \quad , \tag{2.6b}$$

$$z = z_2 + r\Delta z / L \quad , \tag{2.6c}$$

where x, y, and z = the coordinates of a point at a well-log distance r below the last control point, P_2, having the coordinates (x_2, y_2, z_2), and L (Eq. 2.4) = the straight-line distance between the last two control points.

As an example of the location of points in a deviated well, consider the following information from a well (KB at 660 ft) on which the formation boundary of interest is at a log depth of 1 225 ft. The deviation survey gives the coordinates of points above and below the boundary as 1 200 ft TVD, 50 ft northing, –1 050 ft easting and 1 400 ft TVD, 150 ft northing, –1 150 ft easting. What are the coordinates of the formation boundary? The subsea depths of the upper and lower points, found by subtracting the log depths from the elevation of the Kelly bushing are P_1: $z = -540$ ft and P_2: $z = -740$ ft, respectively. From Eq. 2.4, the straight-line distance between the two points is $L = 225$ ft. From Eq. 2.3, the coordinates of a point $r = 25$ ft down the well from the upper point are –560 ft subsea, 60 ft northing, –1 060 ft easting. Note that negative northing is to the south and negative easting is to the west.

2.3 Orientations of Lines and Planes

The basic structural measurements at a point are the orientations of lines and planes. The attitude of a plane is its orientation in three dimensions. The attitude may be given as the strike and dip (Fig. 2.8). Strike is the orientation of a horizontal line on the plane and the dip is the angle between the horizontal and the plane, measured perpendicular to the strike in the downward direction. Compass directions will be given here as the *trend*, which is the azimuth on a 360° compass (Ragan 1985), and will be indicated by numbers always containing three digits. Strike and dip are written in text form as strike, dip, dip direction; for example 340, 22NE and represented by the map symbols of Fig. 2.9a–d. The degree symbol may be written after each angle (i.e., 340°, 22° NE) or may be omitted for the sake of simplicity, as will be done here (Rowland and Duebendorfer 1994). The alternative form on a quadrant compass is the *bearing* and the dip (Ragan 1985), for which the same attitude would be written as N20W, 22NE. In subsurface geology, where the frame of reference may be a vertical shaft or well bore, the inclination of a plane may be given as the hade, which is the angle from a downward-

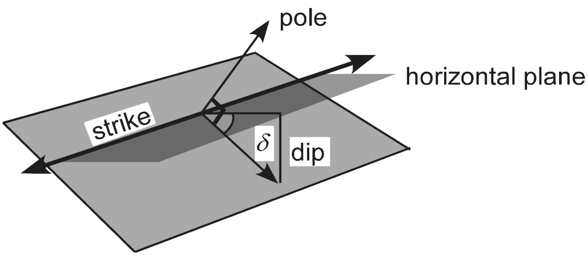


Fig. 2.8. Attitude of the *shaded* plane can be given by its strike and dip, dip vector, or pole

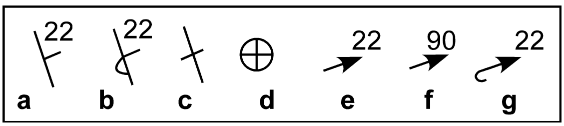


Fig. 2.9. Map symbols for the attitude of a plane. **a** Strike and dip. **b** Strike and dip of overturned bed. **c** Strike of vertical bed. **d** Horizontal bed. **e** Azimuth and plunge of dip. **f** Facing (stratigraphic up) direction of vertical bed. **g** Azimuth and plunge of dip, overturned bed

pointing vertical line and the plane. The hade angle is the complement of the dip (90° – δ). The attitude of a plane may also be given as the azimuth and plunge of the dip vector, written as dip amount, dip direction. By this convention, the previous attitude is given as 22, 070, indicated by the map symbol of Fig. 2.9e. This is the form that is usually used in this book because it is short and convenient for numerical calculations. The dip vector will occasionally be written in short form as $\boldsymbol{\delta}$, where the bold type indicates a vector. The attitude of a plane may also be represented by the orientation of its pole, a line perpendicular to the plane (Fig. 2.8).

The orientation of a line is given by its trend and plunge. The trend is the angle, β, between north and the vertical projection of the line onto a horizontal plane (Fig. 2.10). The plunge is the angle, p, in the vertical plane between the line and the horizontal. The orientation of a line is written as plunge amount, azimuth of trend, for example 30, 060 is a line plunging 30° toward the azimuth 060°. On a map the orientation of a line is represented by the same type of symbol as the azimuth and plunge of the dip (Fig. 2.9b) which is also a line. The two symbols could be differentiated on a map by means of line weight.

Given the *xyz* coordinates of two points, it is possible to calculate the trend and plunge of the line between them (Fig. 2.11). If subscript 1 represents the higher of the two points, the plunge of the line determined from the following equations will be downward. The preliminary value of the azimuth θ' and plunge δ of the line between points 1 and 2 are (from Eqs. 12.11 and 12.12)

$$\theta' = \arctan(\Delta x / \Delta y) \quad , \tag{2.7}$$

$$\delta = \arcsin[(z_2 - z_1)/L] \quad , \tag{2.8}$$

where

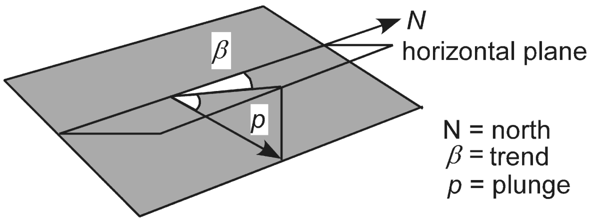


Fig. 2.10.
Trend and plunge of a line in the *shaded* surface

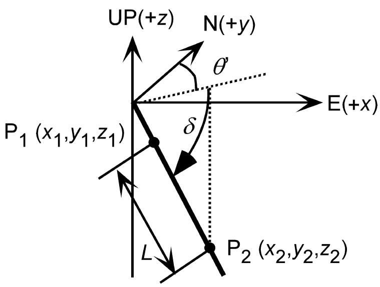


Fig. 2.11.
Trend and plunge of a line between two points

$$\Delta x = (x_2 - x_1) \quad , \tag{2.9a}$$

$$\Delta y = (y_2 - y_1) \quad , \tag{2.9b}$$

and L is given by Eq. 2.4. Division by zero is not allowed in Eq. 2.7. The preliminary azimuth, θ', calculated from Eq. 2.7, is always in the range of 000 to 090°. The angle must be converted to the true azimuth, θ, in the range of 000 to 360° using Table 2.1.

As an example of the calculation of the orientation of a line, find the trend and plunge of the well between points P_1 and P_2 from the example in Sect. 2.2.3.2. The locations of the two points are found from the deviation survey to be P_1: $z = -540$ ft, 50 ft northing, −1 050 ft easting and P_2: $z = -740$ ft, 150 ft northing, −1 150 ft easting. The preliminary plunge and trend of the line, from Eqs. 2.7 and 2.8, is 55, −045. The sign of Δx is negative ($\Delta x = (-1\,150 - (-1\,050) = -100$) and Δy is positive ($\Delta y = 150 - 50 = +100$), and so from Table 2.1, the true azimuth is $\theta = 360 + (-45) = 315$.

The orientation of a line can also be represented by its pitch or rake, r, which is the angle measured in a specific plane between the line and the strike of the plane (Fig. 2.12). Both the rake and the attitude of the plane must be specified. This form is convenient for recording lines that are attached to important planes, like striations on a fault plane. Alternatively the orientation of a line in a plane can be represented by its apparent dip and bearing (Fig. 2.12). Apparent dip, δ', is the orientation of a line that lies in a plane in some direction other than the true dip (Fig. 2.12). An apparent dip can be written as a plunge and trend or as a rake in the plane. Apparent dips are important in drawing a cross section that is in a direction oblique to the true dip direction of bedding.

It is possible to represent the three-dimensional orientations of lines and planes quantitatively on graphs called stereograms and tangent diagrams. The graphical techniques aid in the visualization of geometric relationships and allow for the rapid solution of many three-dimensional problems involving lines and planes.

Table 2.1.
True azimuth, θ, determined from preliminary azimuth, θ' based on the signs of Δx and Δy (Eq. 2.9) or $\cos\alpha$ and $\cos\beta$ (Eq. 2.14)

Azimuth (deg)	$\cos\alpha$; Δx	$\cos\beta$; Δy	θ
000+ to 090	+	+	θ'
090+ to 180	+	–	$180 + \theta'$
180+ to 270	–	–	$180 + \theta'$
270+ to 360	–	+	$360 + \theta'$

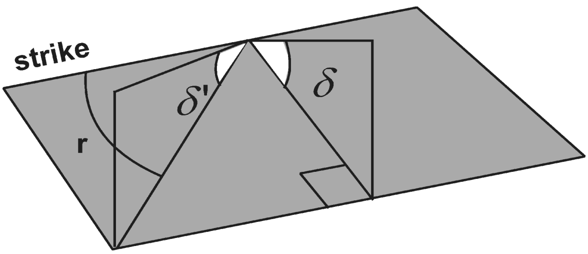


Fig. 2.12.
True dip, δ, apparent dip, δ', and rake (or pitch), r

2.3.1 Stereogram

A stereogram (Dennis 1967; Lisle and Leyshon 2004) is the projection of the latitude and longitude lines of a hemisphere onto a circular graph. Two different projections are commonly used in structural interpretation, the equal-area net (Schmidt or Lambert net) and the equal-angle stereographic net (Wulff net). The different projection techniques result in the preservation of different properties of the original sphere (Greenhood 1964). The equal-area net is required if points are to be contoured into spatially meaningful concentrations. An equal-angle net produces false concentrations from an evenly spaced distribution of points, although the angular relationships are correct. The lower-hemisphere equal-area stereograms (Fig. 2.13a) will be used throughout this book (an enlarged copy for use in working problems is given as Fig. 2.28 at the end of the chapter).

The outer or primitive circle of the equal-area stereogram is the projection of a horizontal plane (Fig. 2.13a). The compass directions are marked around the edge of the primitive circle. The projections of longitude lines form circular arcs that join the north and south poles of the graph and represent great circles. The trace of a plane is a great circle. A line plots as a point. The projections of latitude lines are elliptical curves, concentric about the north and south poles and represent small circles. The center of the graph is a vertical line. Planes and lines plotted on a stereogram can be visualized as if they were intersecting the surface of a hemispherical bowl (Fig. 2.13b).

A stereogram is used with a transparent overlay on which the data are plotted and which can be rotated about the center of the graph. To begin using a stereogram, on the overlay, mark the center and the north, east, south, and west directions, and the primitive circle (Fig. 2.14a). To plot the attitude of a plane from the strike and dip (for example, 60, 32SE), mark the strike on the primitive circle. Then rotate the overlay so that the strike direction lies on the N-S axis (Fig. 2.14b), find the great circle corresponding to the dip by counting down (inward) along the E-W axis from the primitive circle (zero dip) the dip amount in the dip direction. Draw a line along the great circle and

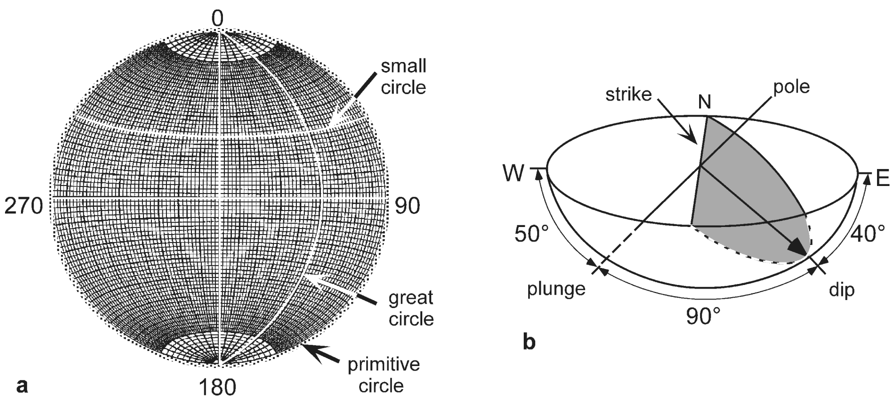


Fig. 2.13. Stereogram. **a** Equal-area stereogram (Schmidt or Lambert net), lower hemisphere projection. **b** Visualization of a plane and its pole in a lower hemisphere. (After Rowland and Duebendorfer 1994)

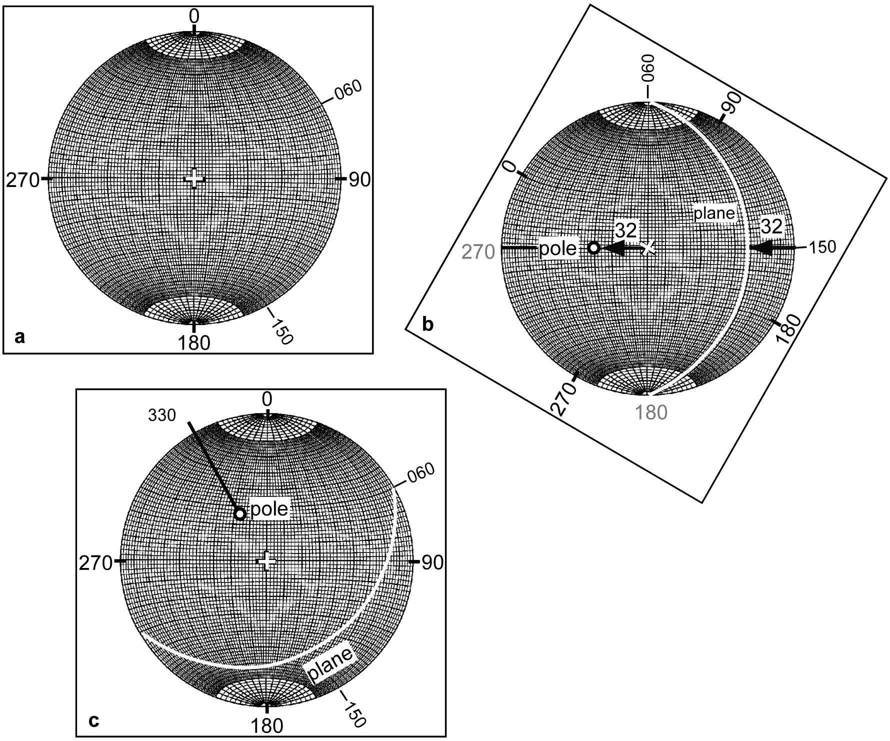


Fig. 2.14. Plotting the orientation of the plane and its pole on an equal-area lower-hemisphere stereogram. The plane has a dip of 32SE (150), and a strike of 060. **a** NESW compass directions and strike and dip directions are marked on the overlay (*square*). **b** Overlay rotated to bring the strike to north and the dip to east. Trace of plane and pole to plane marked on overlay. **c** Overlay returned to its original position. The plunge and trend of the pole is 58, 330

then return the overlay to its original position to see the plane in its correct orientation (Fig. 2.14c). Find the pole to the plane by placing the overlay in the starting position for drawing bedding, that is, with the strike direction N-S. Along the E-W axis, count in from the primitive circle an amount equal to the dip, then count another 90° to find the pole (Fig. 2.14b) or, equivalently, count the dip amount up (outward) from the center point of the diagram. Mark the position of the E-W line on the primitive circle and return the overlay to its original position to find the trend of the pole (330°). The plunge of the pole is 90° minus the dip. To plot planes and poles from the dip and dip azimuth of the plane (the plane previously plotted is $\boldsymbol{\delta} = 32, 150$) mark the dip direction on the primitive circle (Fig. 2.14a), rotate the overlay to bring this direction to E-W, and count the dip amount inward to find the point that represents the orientation of the dip vector. The great circle projection of the bed goes through this point. Plot 90° plus the dip along E-W to find the pole (Fig. 2.14b). Return the overlay to its original position to see the result (Fig. 2.14c). If the plunge and trend to be plotted are those of a line, follow the same steps as in plotting a dip vector.

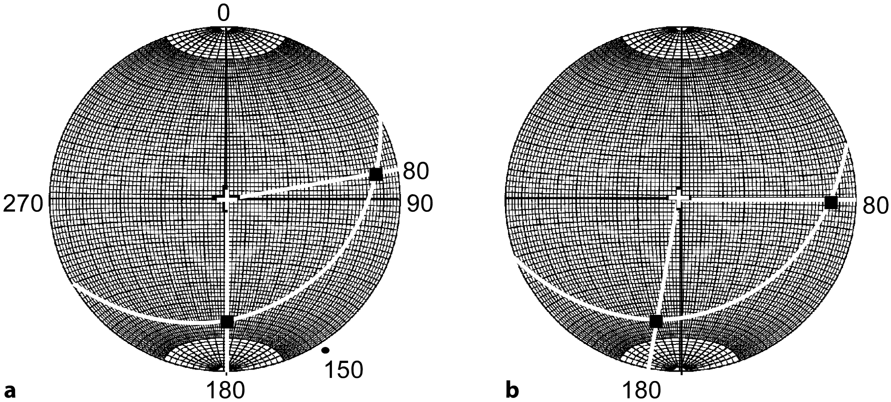


Fig. 2.15. Apparent dip in the plane given by the dip vector 32, 150 on an equal-area, lower-hemisphere stereogram. **a** Plane plotted with apparent dips shown as *black squares*. The south-trending apparent dip is 28, 180. **b** Overlay rotated 10° to bring the 080° trending apparent dip into measurement position; apparent dip is 15, 080

Plotting poles and lines can be done more quickly on a stereogram using a Biemsderfer plotter (Wise 2005), a calibrated dip scale that rotates on the center of the net to allow plotting of points without rotating the overlay. The attitudes of lines such as dip vectors and lineations are also quickly and easily plotted on a tangent diagram, described in Sect. 2.3.3.

Apparent dips are quickly determined on a stereogram as the orientation of the point of intersection between a line in the direction of the apparent dip and the great-circle trace of a plane. For example, find the apparent dips along the azimuths 080 and 180 for the plane plotted previously ($\boldsymbol{\delta}$ = 32, 150). Plot lines from the center of the graph in the azimuth directions (Fig. 2.15a). The intersections of the azimuth lines with the great circle are the apparent dips. Dips can be read from the N-S axis as well as the E-W and so the 180° azimuth is in measurement direction. The angle measured inward from the primitive circle to the intersection is the apparent dip, 28°. The overlay is rotated into measurement position for the 080° azimuth (Fig. 2.15b) to find the apparent dip of 15°.

Apparent dip problems can be worked backwards to find the true dip from two apparent dips. Plot the points representing the apparent dips, then rotate the overlay until both points fall on the same great circle. This great circle is the true dip plane.

2.3.2 Natural Variation of Dip and Measurement Error

The effect of measurement error or of the natural irregularity of the measured surface on the determination of the attitude of a plane is readily visualized on a stereogram. The plane is represented by its pole (Fig. 2.16). Irregularities of the measured surface and measurement errors should produce a circular distribution of error around this pole (Cruden and Charlesworth 1976). An error of 4° around the true dip is probably

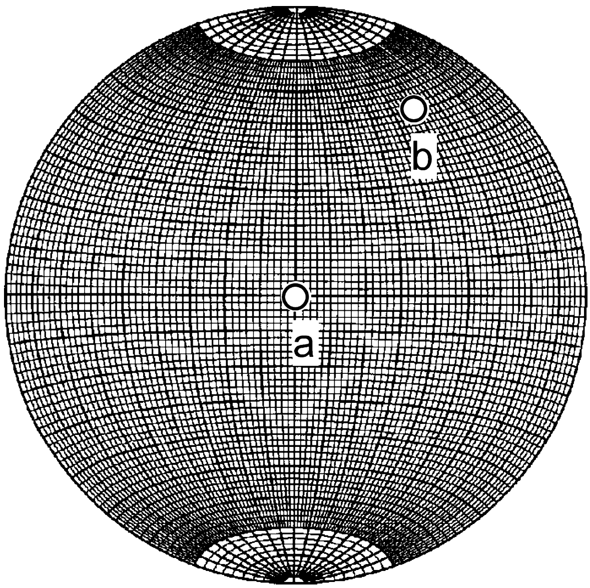


Fig. 2.16.
Equal-area, lower-hemisphere stereogram. Measurement error of 4° (radius of circles) around true bedding poles (points *a* and *b*)

the maximum expected for routine field measurements on a normally smooth bed surface. Measurements on a rough surface may show an even greater variability. A good average attitude from a rough surface can be obtained by making several measurements and then separately averaging the strikes and dips or the trends and plunges of the dips. A good field measurement procedure is to lie a flat field notebook or square of rigid plastic on a rough bed surface to average out the irregularities.

The effect of the error is related to the attitude of the plane. If the bed is horizontal, the pole is vertical (Fig. 2.16, point a) and the error means that the azimuth of the dip could be in any direction, even though the true three-dimensional orientation of the plane is rather well constrained. Small irregularities on a bed surface have the same effect (Woodcock 1976; Ragan 1985). On a steeply dipping plane (Fig. 2.16, point b), the same amount of error causes little variation in either the azimuth or the dip. Conversely, the measurement of the trend of a line on a gently dipping surface is accurate to within a few degrees, but the direction measured on a steeply dipping plane may show significantly greater error (Woodcock 1976). A precision of about 2° is about normal for calculations done using a stereogram. For greater precision, the calculations should be done analytically by methods that will be presented later in the chapter.

2.3.3 Tangent Diagram

The other useful diagram for representing the attitudes of planes is the tangent diagram (Fig. 2.17; an enlarged copy is given at the end of the chapter as Fig. 2.29). Developed by Hubbert (1931), it has been popularized by Bengtson (1980, 1981a,b) in the context of dipmeter interpretation. It is particularly valuable in the determina-

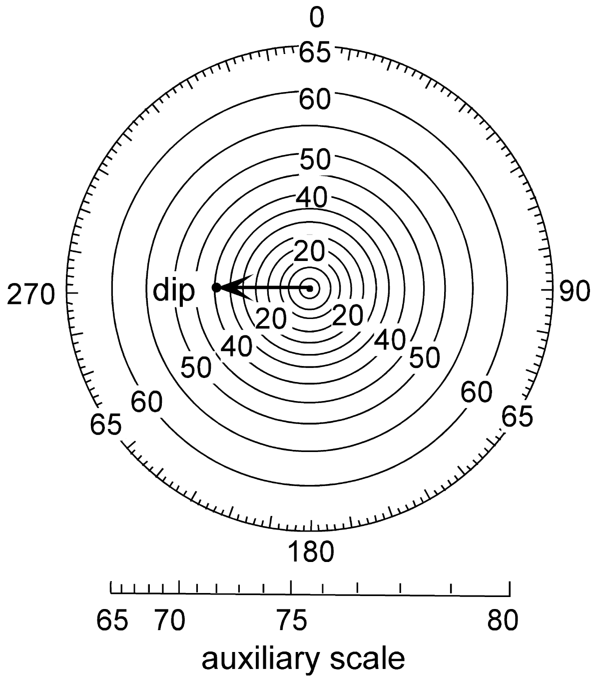


Fig. 2.17.
Tangent diagram. *Arrow* represents a plane dipping 40° to the west (40, 270). (After Bengtson 1980)

tion of the crest lines of cylindrical and conical folds (Chap. 5). The concentric circles on the diagram represent the dip magnitude and their spacing is proportional to the tangent of the dip, hence the name tangent diagram. The center of the diagram is zero dip. The azimuth is marked around the margin of the outer circle. The attitude of a plane is represented by a vector from the origin in the direction of the azimuth and having a length equal to the dip amount (Fig. 2.17). The attitude can be shown with the complete vector or as a point plotted at the location of the tip of the vector. See Sect. 2.8 for how to plot dip vector points on a tangent diagram using a spreadsheet. A major convenience of the tangent diagram is that no overlay is required and that certain problems are solved very quickly and without the rotations that are required with the stereogram. The drawback of the tangent diagram is that very steep dips require a very large diagram. The diagram in Fig. 2.17 extends to a dip of 65°. A calibrated scale that can be used to plot dips from 65° to 80° is given at the bottom of this figure. To use it, plot the vector along the appropriate radius and use the auxiliary scale to find the added length of the vector beyond the outer circle of the diagram. The diagram is not practical if a significant percentage of the dips are over 70°, for which a stereogram is more suitable. The tangent diagram can be used as a circular histogram, even for steep dips, by plotting the steep dips with their correct azimuths along the outer circle.

A tangent diagram is a convenient tool for finding the true or apparent dip. The apparent dip in a given direction (Fig. 2.18a) is the vector in the appropriate direction.

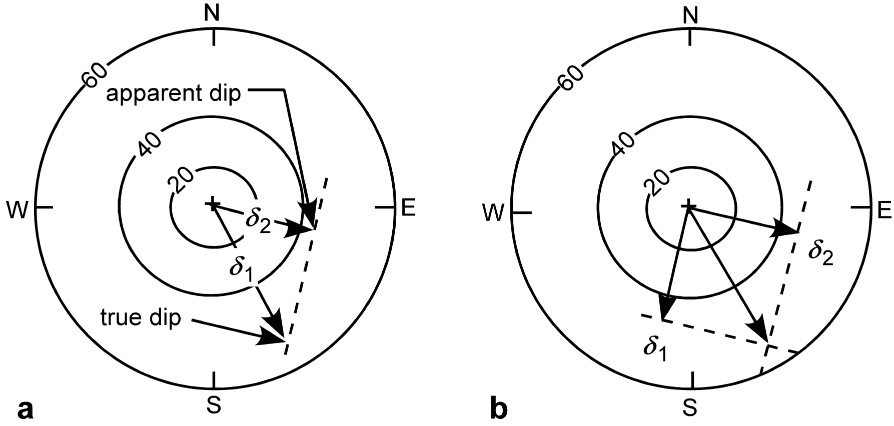


Fig. 2.18. Apparent dip on a tangent diagram. **a** Apparent dip from true dip. **b** True dip from two apparent dips. (After Bengtson 1980)

For example, given a dip vector of a bed $\boldsymbol{\delta}_1$, find the apparent dip in the direction of $\boldsymbol{\delta}_2$. Project the tip of $\boldsymbol{\delta}_2$ onto the direction of $\boldsymbol{\delta}_1$. The projection is along a line perpendicular to the apparent dip. The length of the projected bedding vector in the direction of the apparent dip is the amount of the apparent dip. The true dip can be found from two apparent dips. The perpendiculars from the apparent dips ($\boldsymbol{\delta}_1$ and $\boldsymbol{\delta}_2$) intersect at the tip of the true dip vector (Fig. 2.18b).

2.4 Finding the Orientations of Planes

The attitude of a plane measured by hand with a compass or given by a dipmeter log is effectively the value at a single point. Measured over such a small area, the attitude is very sensitive to small measurement errors, surface irregularities, and the presence of small-scale structures. The following two sections describe how to find the attitude from three points that can be widely separated and from structure contours. Both methods provide an average attitude representative of the map-scale structure.

The farther the points depart from a straight line, the more reliable the expected result because small irregularities or location errors will have less influence on the result. If more than three points are available to find the attitude of the plane, a best-fit can be determined using planar regression or moment of inertia analysis (Fernández 2005). A typical situation for this application would be a series of points along the outcrop trace of a plane (e.g., Fig. 2.4). On the other hand, the farther the points are from one another, the greater the possibility that they no longer fall on a single plane. A high-quality calculated dip will be compatible with the surrounding data as demonstrated with a structure contour map (Chap. 3) and cross section (Chap. 6). If the unit thickness is known (Chap. 4), a very powerful test of the quality of a calculated dip is to show that both the top and base of the unit fit their respective outcrop traces.

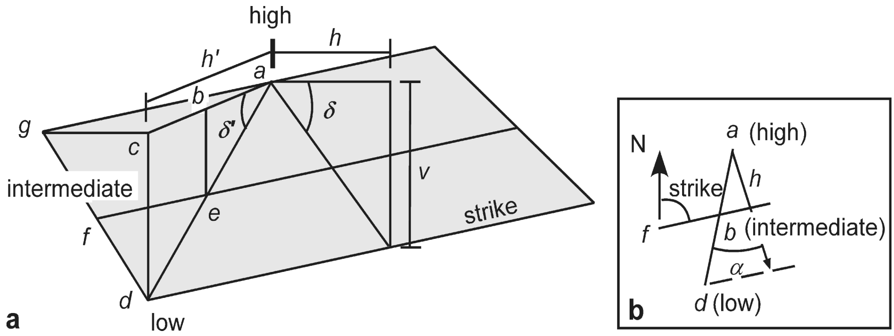


Fig. 2.19. True dip, δ, and apparent dip, δ'. **a** Perspective view. **b** Map view. *N:* north. For explanation of *a–h* and *v*, see text

2.4.1 Graphical Three-Point Problem

The attitude of a plane can be uniquely determined from three points that are not on a straight line. Let the highest elevation be point a and the lowest be point d (Fig. 2.19). The intermediate elevation, *f*, must also occur along the line joining a and d as point e. The line fe is the strike line. The horizontal (map) distance from a to e by linear interploation is ab, where

$$\text{ab} = (\text{ac} \times \text{be}) / \text{cd} \quad . \tag{2.10}$$

Plot the length ab on the map (Fig. 2.19b) and join point f and b to obtain the strike line. The dip vector lies along the perpendicular to the strike, directed from the high point to the intermediate elevation along the strike line. The dip amount is

$$\delta = \arctan\,(v / h) \quad , \tag{2.11}$$

where v = the elevation difference between the highest and the lowest points and h = the horizontal (map) distance between the highest point and a strike-parallel line through d. The azimuth of the dip is measured directly from the map direction of the dip.

A typical example of a 3-point problem is seen on the map of Fig. 2.20a. The map shows the elevations of three locations identified in the field as being on the same contact (a, f, d). These points could just as easily be the elevations of a formation boundary identified in three wells. To find the attitude of the contact, draw a line between the highest and lowest points (a–d, Fig. 2.20b) and measure its length. Use Eq. 2.10 to find the distance along the line from the high point to the level of the intermediate elevation (*e*). Connect the two intermediate elevations to find the strike line (Fig. 2.20b,c). Draw a perpendicular from the strike line (e–f) to the lowest point (d, Fig. 2.20d). The horizontal length of the line is h and the elevation change is v. Determine the dip from Eq. 2.11. The azimuth of the dip is measured from the map. The dip vector in this example is 22, 125.

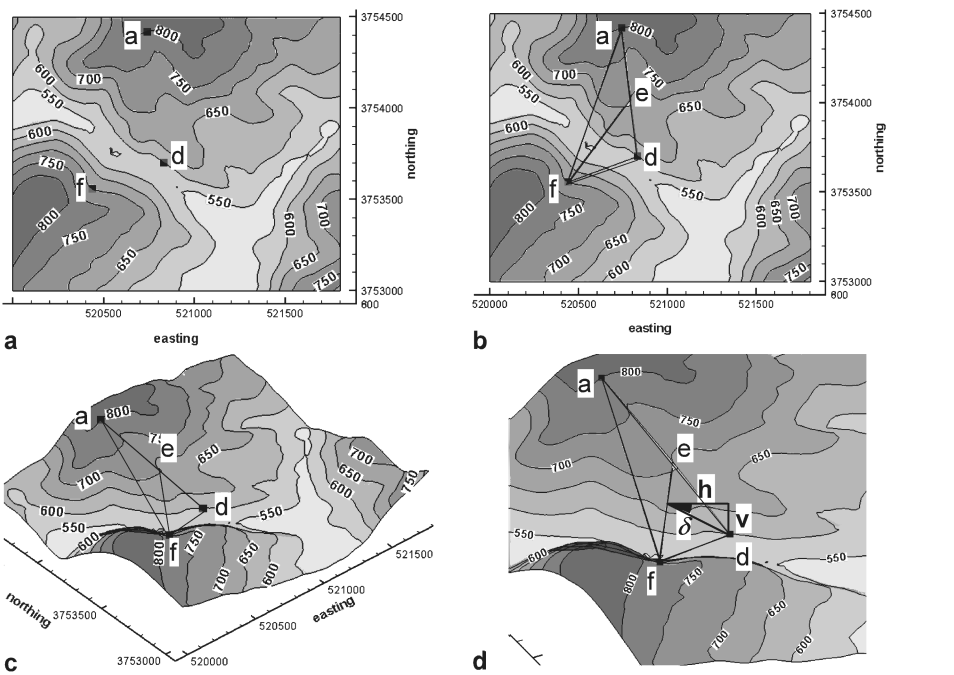
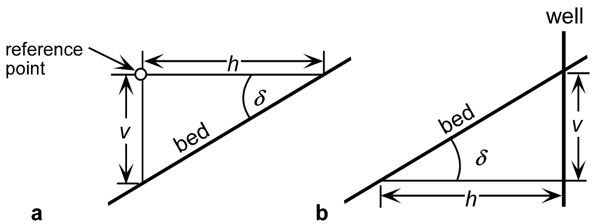


Fig. 2.20. Attitude determination from three points on a topographic map. Horizontal scale in km, vertical scale in ft. **a** Three points (*solid squares*) on a contact. **b** Determination of the strike line (e–f). **c** The same three points in oblique 3-D view to NE. **d** Enlarged 3-D view of three-point solution

Fig. 2.21.
Distance to a point on a dipping bed, in vertical cross sections in the dip direction. **a** Vertical distance from a reference point to a dipping bed. **b** Horizontal distance from a well to a dipping bed



A dip can be converted from degrees into feet/mile or meter/kilometer by solving Eq. 2.11 for v and letting h be the reference length (5 280 ft for ft/mile or 1 000 m for m/km):

$$v = h \tan \delta \quad , \tag{2.12}$$

where v = the vertical elevation change, h = reference length, and δ = dip. The same relationship can be used to determine the vertical distance from a reference point to a dipping horizon seen in a nearby outcrop (Fig. 2.21a).

Another useful application of Eq. 2.11 is to find the distance to the intersection between a horizontal plane (such as an oil-water contact) and a dipping plane (such as the top of the unit containing the contact) which are separated by a known distance in the well. Solve Eq. 2.11 for h (Fig. 2.21b):

$$h = v / \tan\delta \quad , \tag{2.13}$$

where h = distance from the well bore to the intersection with a dipping bed and v = vertical distance in the well between the intersection of the dipping horizon and the horizontal horizon.

2.4.2 Analytical Three-Point Problem

The attitude of a plane is given by the trend and plunge of the dip vector (Fig. 2.22). The dip vector can be determined analytically from the *xyz* coordinates of three non-colinear points (derived in Sect. 12.3). The preliminary trend and plunge of the dip vector is

$$\theta' = \arctan(A/B) \quad , \tag{2.14}$$

$$\delta = \arcsin\{-\cos[90 + \arccos(C/E)]\} \quad , \tag{2.15}$$

where θ' = the preliminary azimuth of the dip, δ = the amount of the dip, and

$$A = y_1z_2 + z_1y_3 + y_2z_3 - z_2y_3 - z_3y_1 - z_1y_2 \quad , \tag{2.16a}$$

$$B = z_2x_3 + z_3x_1 + z_1x_2 - x_1z_2 - z_1x_3 - x_2z_3 \quad , \tag{2.16b}$$

$$C = x_1y_2 + y_1x_3 + x_2y_3 - y_2x_3 - y_3x_1 - y_1x_2 \quad , \tag{2.16c}$$

$$D = z_1y_2x_3 + z_2y_3x_1 + z_3y_1x_2 - x_1y_2z_3 - y_1z_2x_3 - z_1x_2y_3 \quad , \tag{2.16d}$$

$$E = (A^2 + B^2 + C^2)^{1/2} \quad . \tag{2.16e}$$

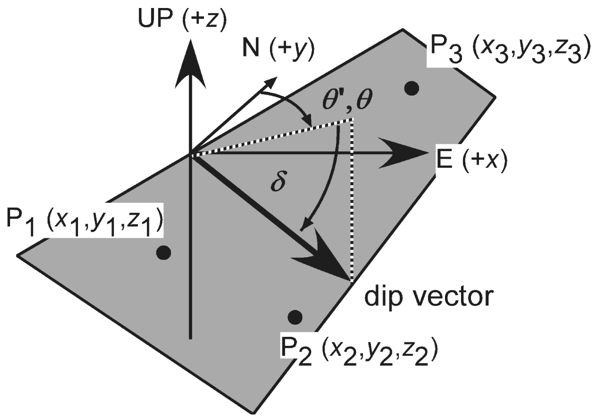


Fig. 2.22. Three points on a plane and the dip vector of the plane

The sign of $E = -\text{sign}\,D$ if $D \neq 0$; $= \text{sign}\,C$ if $D = 0$ and $C \neq 0$; $= \text{sign}\,B$ if $C = D = 0$. Division by zero is not allowed in Eq. 2.14. The value of θ' computed from Eq. 2.7 is always between the values of 000° and 090° and is equal to 90° if $B = 0$. The true azimuth, θ, of the dip in the complete range from 000° to 360° can be determined from θ' and the signs of $\cos\alpha$ and $\cos\beta$ (Eqs. 2.17 and Table 2.1):

$$\cos\alpha = A / E \quad , \tag{2.17a}$$

$$\cos\beta = B / E \quad , \tag{2.17b}$$

where A, B, and E are given by Eqs. 2.16a,b,e above.

As an example, find the analytical solution to the 3-point problem in Fig. 2.20. The three points have the coordinates, in xyz order, of 520 739, 3 754 420, 800; 520 438, 3 753 560, 700; 520 833, 3 753 700, 600. From Eqs. 2.14–2.17, the dip is 22° at an azimuth of 125°. The leading UTM digits of the x and y coordinates are identical and need not be included in the calculation.

2.5 Apparent Dip

Apparent dip, δ', is the angle in a plane between the horizontal and some direction other than the true dip (Fig. 2.19a). To find the apparent dip, let the horizontal angle between the true and apparent dip be α, then

$$\delta' = \arctan(\tan\delta \, \cos\alpha) \quad . \tag{2.18}$$

On a completed structure-contour map, the apparent dip in a given direction is found from

$$\delta' = \arctan(I / h') \quad , \tag{2.19}$$

where I = the contour interval and h' = the horizontal distance on the map between the contours in the direction of interest (Fig. 2.23). If the direction perpendicular to the contours is selected, then the apparent dip is the true dip. If the strike direction is selected, the apparent dip is zero.

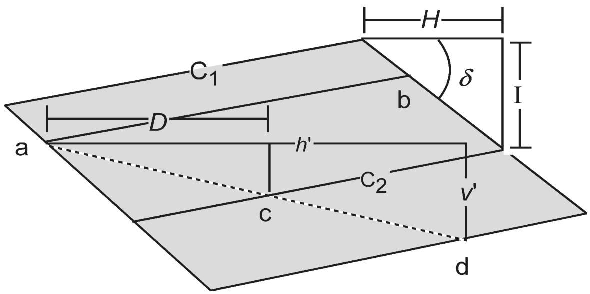


Fig. 2.23. Relationships between structure contours, point elevations, and dip. Points a and b are at the same elevation; c and d are at different elevations. C_1 and C_2 are structure contours. For explanation of other symbols, see text

2.6 Structure Contours

Structure contours provide an efficient and effective representation of the attitude of a surface. The contours are strike lines and the dip direction is perpendicular to the contours (Fig. 2.23). The simplest way to construct structure contours from a geologic map is to connect points on a contact that lie at the same elevation. If two points on a contact have the same elevation, for example a and b in Fig. 2.23, then a line joining them is a strike line or structure contour. Finished maps normally show contours at even increments of elevation.

Structure contours are the primary method for illustrating the shape of structures in the subsurface. Generated from outcrop maps, structure contours provide an effective method for validating the outcrop pattern and for smoothing out local dip variations caused by bed roughness. If a map horizon is planar, the contours on it are parallel and uniformly spaced. Folding produces curved contours that usually are not far from parallel at a local scale. Abrupt changes in direction indicate fold hinges, faults, or mistakes. Unless radical thickness changes occur, the structure contours on adjacent horizons are approximately parallel.

The following sections describe the creation of straight-line structure contours from the elevations of three points and the interpretation of contours in the vicinity of a dip measurement. The generation of curved structure contours from multiple control points is discussed in Chap. 3.

2.6.1 Structure Contours from Point Elevations

Structure contours are usually generated from point elevations, especially in subsurface work. Except for the unlikely circumstance where all the points happen to fall on the selected contour elevations, contouring will require placing contours between control points that have different elevations. This means that contour values must

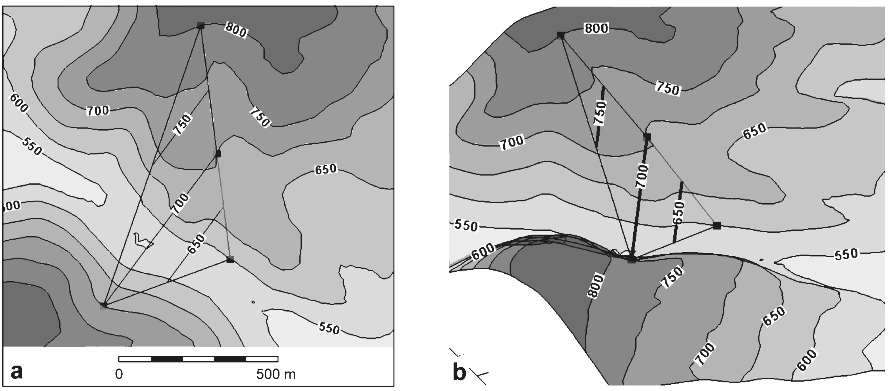


Fig. 2.24. Structure contours on a dipping plane (from Fig. 2.20). The 650-ft and 750-ft contours are interpolated between the four control points. **a** Plan view. **b** 3-D oblique view to NE

be interpolated between the control points. Using linear interpolation (more complex forms of interpolation are considered in Chap. 3), an interpolated even-elevation contour lies at point c (Fig. 2.23) on the straight line between the two points at known elevations *a* and *d*. Following the method of Eq. 2.10, the elevation of the upper control point is *a*, the horizontal distance of the desired contour from the highest control point is *D*, the elevation of the desired contour is C_2, the map distance between the two control points is *h*', and the vertical distance between the two control points is *v*', giving

$$D = h' \ (a - C_2) / v' \quad . \tag{2.20}$$

As an example, the strike line constructed in Fig. 2.20b is itself a structure contour line. Two additional structure contour lines are shown in Fig. 2.24.

2.6.2 Structure Contours from Attitude

If the dip is known, then the spacing between structure contours is found from the method of Eq. 2.11 (Fig. 2.23) to be

$$H = I / \tan \delta \quad , \tag{2.21}$$

where H = horizontal spacing between structure contours measured perpendicular to the contour strike, I = contour interval, and δ = dip. The trend of the structure contours is perpendicular to the dip direction.

2.6.3 Dip from Structure Contours

If the attitude is given by structure contours, the dip is found by solving Eq. 2.21:

$$\tan \delta = I / H \quad , \tag{2.22}$$

where δ = dip, I = contour interval, and H = spacing between contours. The dip direction is perpendicular to the contour, in the direction toward the lower contour.

2.7 Intersecting Contoured Surfaces

The trace of a contact on a geologic map represents the intersection of the topographic surface with the boundaries of the geologic units. Finding the intersection between two surfaces is a fundamental technique in three-dimensional interpretation. In Sect. 2.6.1, three points along the outcrop trace of a planar geological contact were used to generate a structure-contour map. Conversely, the outcrop trace of a contact can be determined from a structure contour map. The following procedure works for both plane and curved structure contour surfaces. The structure contour map is superimposed on the topographic map (Fig. 2.25a). All intersection points where both surfaces have

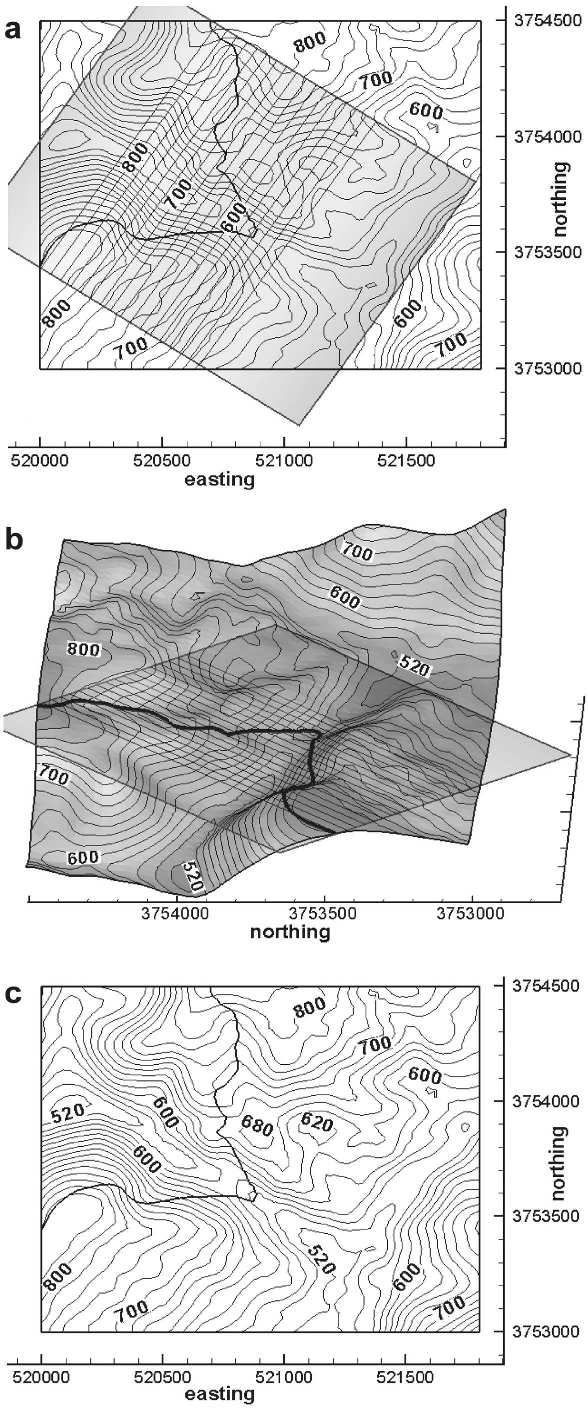


Fig. 2.25.
Intersection between topographic contours and a planar marker horizon. Contour interval 20 ft. **a** Structure contours on a geologic horizon (*within shaded rectangle*) and topographic contours. The outcrop trace of the contact is the *thick line*. **b** Oblique 3-D view to *E*. **c** Geologic map of the outcrop trace (*thick line*) with the structure contours of the plane removed

identical elevations are marked. Additional topographic and structural contours can be interpolated to add additional control. Marking the intersection points is most convenient if the topographic map is on a transparent overlay that can be placed above the structure contour map, something that is easily done in computer drafting programs. A 3-D view of the intersecting surfaces (Fig. 2.25b) helps in understanding the procedure. The intersection points are connected to obtain the outcrop pattern of the horizon (Fig. 2.25c). The inferred outcrop pattern becomes a working hypothesis for the location of the horizon. Exactly the same procedure is used to find the intersection between a fault and a stratigraphic marker or the line of intersection between two faults.

2.8 Derivation: Tangent Diagram on a Spreadsheet

To plot a tangent diagram with a spreadsheet it may be necessary to shift the origin for the azimuth (Az) from zero to the east to zero to the north using

$$\mathrm{Az} = \mathrm{Az} - 90 \quad . \tag{2.23}$$

Change the dip to the tangent of the dip,

$$r = \tan(\mathrm{dip}) \quad , \tag{2.24}$$

and change from polar to Cartesian coordinates with

$$x = r \cos(\mathrm{Az}) \quad , \tag{2.25a}$$

$$y = r \sin(\mathrm{Az}) \quad . \tag{2.25b}$$

The native angle format in spreadsheet trigonometric functions is radians and so where angles in degrees are used in trigonometric expressions they must always be converted to degrees from radians. Plot the transformed points as an *xy* graph in the spreadsheet.

2.9 Exercises

2.9.1 Interpretation of Data from an Oil Well

The Appleton oil field is located near the northern rim of the Gulf of Mexico basin. Use the data from well 4835-B in this field (Table 2.2) to solve the following problems. Find the subsea depths of the stratigraphic markers. Write a spreadsheet program using Eqs. 2.3 and 2.4 to find the coordinates of a point between two known points. Find the true vertical depths and the total rectangular coordinates of the formation contacts in the well. What is the orientation of the well where it penetrates the Smackover? Write a spreadsheet program to solve this problem based on Eq. 2.4. Find the true vertical depths and the total rectangular coordinates of the oil-water contact (OWC) and the

Table 2.2. Data from well 4835-B, Appleton field, Alabama. Kelly Bushing: 244 ft

Measured depth = log depth (ft)		True vertical depth (ft)	Depth subsea (ft)	Total rectangular coordinates (ft)	
0		0		0.00 N	0.00 E
928		928		1.72 N	1.93 E
2 308		2 308		4.53 N	9.51 W
3 282		3 282		6.37 N	6.29 W
4 150	Eutaw				
4 400	U. Tuscaloosa				
4 811		4 811		13.38 N	5.99 W
4 890	M. Tuscaloosa				
5 150	L. Tuscaloosa				
5 525	L. Cretaceous				
6 198		6 198		30.82 N	17.30 W
7 696		7 695		61.48 N	7.50 W
8 328		8 327		74.90 N	1.25 E
8 482		8 480		74.59 N	18.82 E
8 661		8 655		71.22 N	53.19 E
8 935		8 922		66.16 N	117.74 E
9 186		9 161		62.25 N	194.12 E
9 556		9 512		54.50 N	310.78 E
10 006		9 940		34.61 N	447.09 E
10 266	Cotton Valley				
10 431		10 346		7.63 N	568.29 E
11 103		10 982		55.68 S	777.65 E
11 621		11 470		116.46 S	938.37 E
11 960		11 791		142.28 S	1 045.58 E
12 370	Haynesville				
12 450		12 253		173.48 S	1 206.13 E
12 641		12 432		185.03 S	1 271.62 E
13 020	Buckner				
13 072	Smackover[a]				
13 086		12 851		210.98 S	1 418.79 E
13 190	OWC[b]				
13 286	Base Smack				

[a] Attitude from dipmeter 12, 056. [b] Oil-water contact.

base of the Smackover. Plot the locations of the stratigraphic tops relative to the surface location of the well on the map (Fig. 2.26). Connect the points to show the shape of the well in map view. How far from the well (horizontally and in which direction) would you expect to first find the intersection of the oil-water contact with the top of the Smackover Formation, assuming constant dip for the Smackover?

2.9.2 Attitude

Given the dip vector of the plane 35, 240, show on an overlay of a stereogram the trace of the plane, its dip vector, and its pole. Show the dip vector 25, 240 on a tangent diagram. What is the apparent dip along a line that trends 260?

2.9.3
Attitude from Map

Use the map of the Blount Springs area (Fig. 2.27) to answer the following questions. Find the attitude of the Mpm in its southeastern outcrop belt using the 3-point method. Draw structure contours for the Mtfp, Mpm, and Mh on the eastern side of the map. Are the upper and lower contacts of each unit parallel to each other? Do you think the contacts are mapped correctly? Determine the attitude of the eastern contact between the Mpm and the Mtfp by the 3-point method and from the structure contours. Are they the same? If they are different, discuss which answer is better. What would be the apparent dip of the Mpm in a north-south roadcut through the northwestern limb of the anticline?

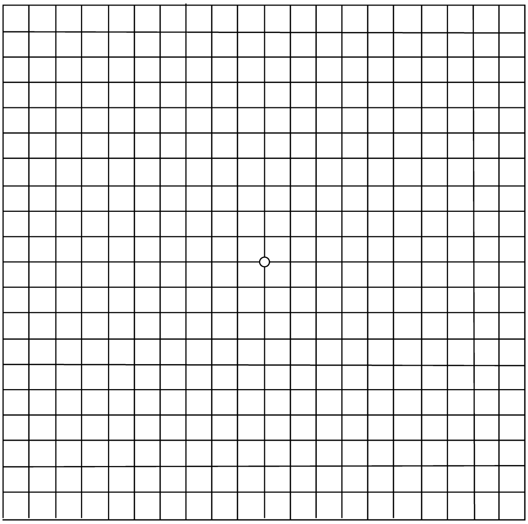


Fig. 2.26.
Map grid for plotting points in a deviated well

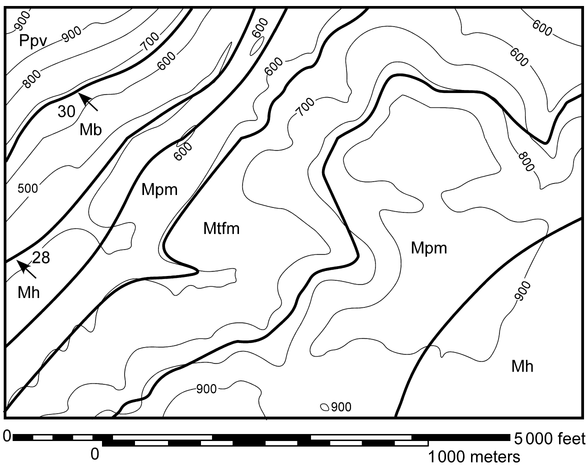


Fig. 2.27.
Geological map of the northeast corner of the Blount Springs area, southern Appalachian fold-thrust belt. *Thin lines* are topographic contours (elevations in feet). *Thick lines* are geologic contacts. *Arrows* are dip directions; *numbers* give the amount of dip

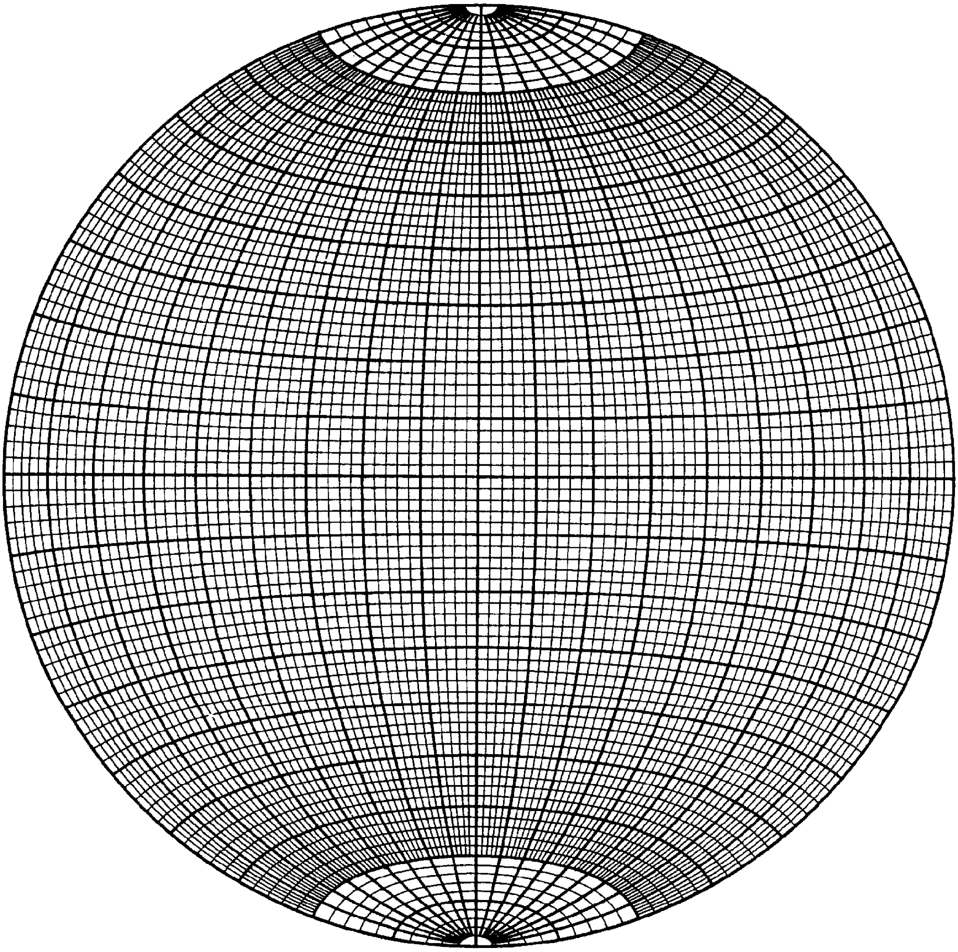


Fig. 2.28. Equal-area stereogram

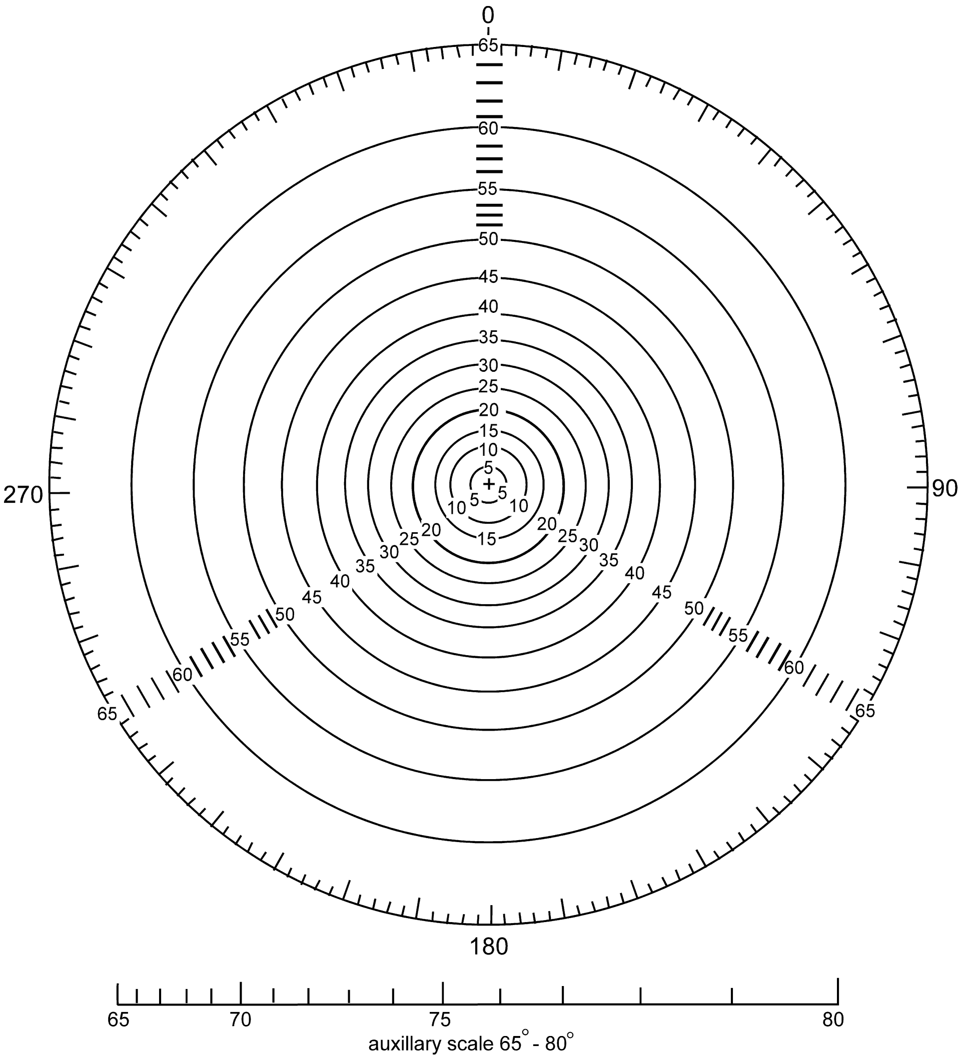


Fig. 2.29. Tangent diagram