Chapter 1

INTRODUCTION

It is now 18 years since Vedeler published his treatise entitled *Grillage Beams in Ships and Similar Structures*, and the period has seen some important developments in the elastic analysis of this type of structure. This is particularly the case for the grillages in ship structures, and the object of the present text is to describe the advances which are most important to the designer. Carefully conducted experiments have been carried out on plated grillages, leading to simple rules for determining the contribution of the plating to the strength of the beams. This period has also seen the introduction of the digital electronic computer, and the present trend is to adopt straightforward numerical methods which were previously rejected due to the prohibitively lengthy arithmetic involved. Nowadays, with very high-speed computers readily available, these methods are often the most popular of all.

Grillage is the term given to a structure of intersecting beams which is loaded normal to its surface. Grillages are particularly common in ship structures, and a typical hull construction for a frigate is shown in fig. 1. The beams are the stiffening members for the plating which is present to provide watertight integrity, and they are usually placed longitudinally and transversely forming a mesh which intersects orthogonally, at right angles. The decks, bottoms and bulkheads of ships are all examples of flat grillages. This form of construction is also commonly used in the decks of bridges. This text is therefore mainly concerned with the analysis of flat plated grillages of orthogonally intersecting beams; it should be mentioned that the term 'beam' is applied to either the longitudinal or transverse stiffening members, not particularly to the transverse beams of ships' decks, a term used in naval architecture. Some of the methods described are also applicable to non-orthogonal grillages.

A grillage is a highly redundant structure which cannot be analysed purely by statical considerations, and, to obtain a solution, recourse has to be made to the conditions for compatible deflections of the components of the structure. For many redundant structures, plastic analysis methods have been advocated as providing a simpler yet more realistic approach than elastic analysis. In this method, the load for final plastic collapse is calculated, and the working load is taken as some fraction of this. To calculate the collapse load for a grillage, the positions of the plastic 'hinges', where the beam section has become completely plastic, are postulated, and sufficient hinges are assumed to allow collapse as a mechanism. A simple application of the principle of virtual work leads to the collapse load, without any reference to the sequence of events leading up to collapse, and the mechanism giving to the smallest collapse load is the correct one. This collapse load is independent of any initial residual stresses due to fabrication.

Unfortunately, the methods of plastic analysis, which have been developed for structures involving bending action such as the portal frames of buildings, are not so obviously applicable to grillages in which the members are rigidly connected at the intersections. This is because a plane structure which is supported at all its edges cannot be laterally deformed into a surface curved in two directions without introducing a membrane stretching action. This is the case, even if the supporting structure at the edges cannot provide any membrane restraint, and the action inside the grillage is then one of tension around the centre and tangential compression at the edges. The plating of ship grillages adds appreciably to the membrane strength of these structures. Long after the structure has exhausted its bending strength, the structure will deform as a membrane, and the final failure will usually be at some weak point in the details of the connections at the intersections, but gross plastic deformation may occur long before this. For these reasons, the methods of plastic design are thought to be of more limited application in the case of grillages, and the present text is concerned with elastic methods of design. Elastic design ensures that the range of stress due to the load is kept to some fraction of the yield stress, and in this way all noticeable plastic deformation is prevented. The only yielding possible is on the first application of load, due to residual fabrication stresses.

The plating of present-day grillages is usually welded to the stiffening members, and, to calculate the maximum stresses and deflections of these members, the approach adopted is to consider the plating as an effective flange to both longitudinal and transverse beams. In this way, the analysis is reduced to that of an unplated grillage, and is merely a question of applying Euler–Bernoulli beam theory to a structure containing a large number of redundancies. The early chapters of this text are concerned with the basic equations of this analysis, and their solution by approximate methods designed to reduce the arithmetic, by a neat exact method using Fourier series, and by straightforward numerical methods using a modern digital computer. These latter methods are the most suitable for including the effects of shear deflection and torsional rigidity, which may be regarded as refinements. The problem of assigning an effective breadth for the beams in plated ship grillages is then discussed, and methods for designing simple welded connections are described, the relevant experimental data from various research projects being presented. The last chapters are concerned with design data sheets which have been worked out for certain cases of concentrated and uniform pressure loadings, and the question of minimum weight design. The design of grillages, as distinct from analysis, has so far received comparatively little attention from research workers, but we may now regard ourselves as standing at a turning point: with digital computers commonly available, the problems of analysis are completely solved, and these computers can be readily utilized to study some of the important problems of minimum weight design.

Chapter 2

FORCE METHOD

The elastic analysis of a plane grillage of beams under loads normal to the surface consists of satisfying the conditions of equilibrium and compatibility at every intersection point. Thus at a typical intersection point there is a reaction on the longitudinal and an equal and opposite reaction on the transverse member. This intersection reaction is statically indeterminate, but may be derived from the condition that, at the intersection, both beams are attached and have equal deflections. In a grillage containing p longitudinal and q transverse members, there are $p \times q$ intersections, and pq unknown reactions. The deflections may be calculated in terms of these reactions, and by equating the longitudinal and transverse beam deflection at every intersection, pq linear simultaneous equations are obtained, which may be solved numerically. The bending moments are then obtained by statics. It will be assumed for the present that shear deflections and torsional rigidity of the beams may be neglected.

2.1. Numerical example 3 × 1 grillage

We will consider first the action of the grillage in fig. 2 under equal loads of magnitude P at positions P_1 and P_2. Let the upwards reaction on the longitudinal at positions 1 and 2 be R_1 and R_2 respectively. There will be equal and opposite reactions on the transverse members at these positions. The intersection deflections may now be derived in terms of forces using simple beam theory, as follows:

Transverse beam deflection at position 1 $= \dfrac{a^3}{EI_a}\dfrac{R_1}{48}$. (2.1)

Transverse beam deflection at position 2 $= \dfrac{a^3}{EI_a}\dfrac{R_2}{48}$. (2.2)

Longitudinal beam deflection at position 1

$$= \frac{b^3}{EI_b}\left(-\frac{R_1}{48}-\frac{11R_2}{768}+\frac{41P}{24\times 64}\right). \qquad (2.3)$$

Longitudinal beam deflection at position 2

$$= \frac{b^3}{EI_b}\left(-\frac{11R_1}{384}-\frac{R_2}{48}+\frac{39P}{16\times 64}\right). \qquad (2.4)$$

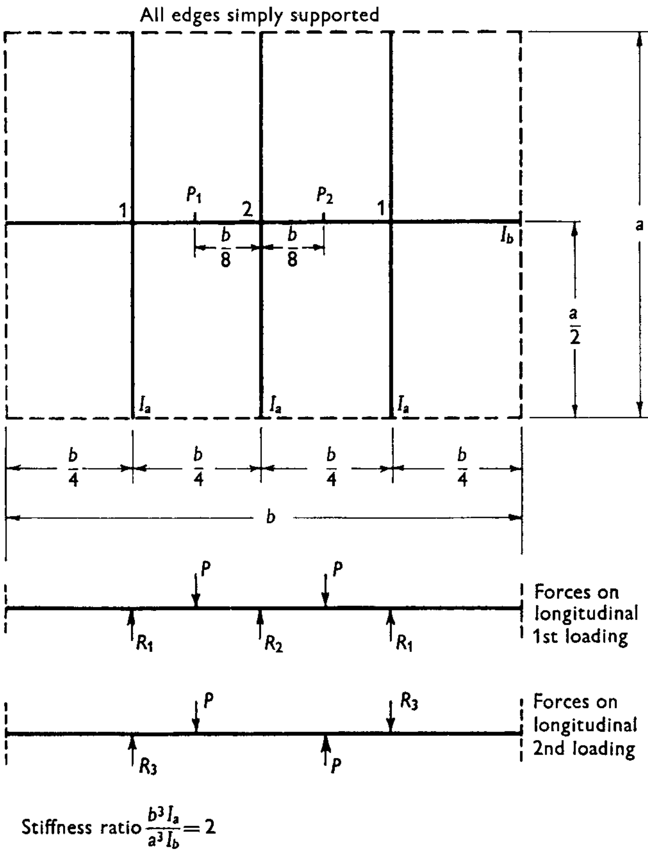

Fig. 2. 3×1 grillage example.

Equating (2.1) and (2.3), (2.2) and (2.4), we obtain two simultaneous equations in R_1 and R_2. Introducing the numerical value for the stiffness ratio $b^3 I_a / a^3 I_b = 2$, and solving,

$$R_1 = 0{\cdot}510P \quad \text{and} \quad R_2 = 0{\cdot}752P.$$

Consider, now, the action of the grillage under a downward load P at position P_1 and an upward load P at P_2. The intersection reaction at position 2 will be zero by symmetry. Let the reaction at 1 be R_3 acting upward on the left end of the longitudinal and downward on the right end.

Transverse beam deflection at position 1 (left) $= \dfrac{a^3}{EI_a} \dfrac{R_2}{48}$.

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Longitudinal beam deflection at position 1 (left)

$$= \frac{b^3}{EI_b}\left(-\frac{R_3}{384}+\frac{11P}{96\times 64}\right).$$

Equating these two expressions and introducing the numerical stiffness ratio, we obtain

$$R_3 = 0{\cdot}137P.$$

2.2. Use of symmetry of structure

Applying the principle of superposition to the above example and adding the solutions for the two loadings, we may derive the solution for a load $2P$ at P_1 and zero at P_2. This unsymmetrical loading might have been discussed directly in terms of three unknown reactions at the intersection points, leading to three simultaneous equations. By making use of the symmetry of the structure, and splitting the load into symmetrical and anti-symmetrical components, we have reduced

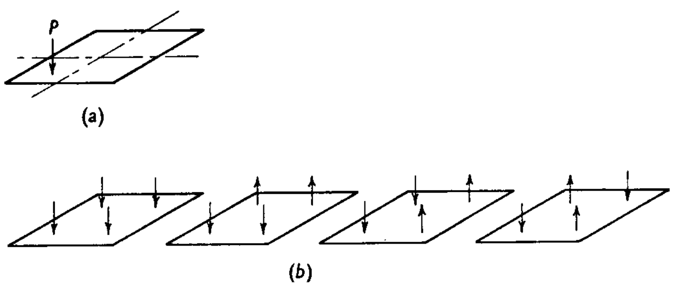

Fig. 3. Treatment of unsymmetrical loadings. (*a*) Two axes of symmetry, unsymmetrical loading. (*b*) Four symmetrical components of loading for grillage calculation.

the problem to independent sets of two and one simultaneous equations. This may seem trivial for the very small grillage considered, but it is of major importance to reduce the labour of numerical solution of the equations, in larger grillages. In general, any unsymmetrical loading on a grillage having two axes of symmetry may be resolved into four components having differing types of symmetry, according to the scheme illustrated in fig. 3. Then, only the intersection points in one quarter of the grillage appear in the analysis, the remainder being given by symmetry. In future discussions of symmetrical grillages, we will usually refer only to the number of beams in one quarter of the grillage, including the central beams where present.

2.3. Flexibility coefficients

The process of setting up equations in the forces may be largely reduced to a numerical operation, using general formulae for the flexibility of a beam under concentrated loadings. These coefficients β_{st} are defined by the relation

$$w_s = \beta_{st}\frac{a^3}{EI}R_t,$$

where w_s is the deflection at a distance x_s from the end supports under a load R_t at distance x_t from the end, a = span of beam and EI = flexural rigidity. The coefficients are derived from beam theory, and for uniform beams are given in the following table of expressions, valid for $x_s \leqslant x_t$. The coefficients for $x_s > x_t$ may be derived using Maxwell's reciprocal theorem.

TABLE 1. *Flexibility coefficients*

(*a*) Unsymmetrical load R_t, distance x_t from one end

(i) simply supported ends $\beta_{st} = \frac{1}{6}\frac{x_s}{a}\left(1-\frac{x_t}{a}\right)\left(\frac{2x_t}{a}-\frac{x_s^2+x_t^2}{a^2}\right)$

(ii) clamped ends $\beta_{st} = \frac{1}{6}\left(\frac{x_s}{a}\right)^2\left(1-\frac{x_t}{a}\right)^2\left[3\frac{x_t}{a}-\frac{x_s}{a}\left(1+\frac{2x_t}{a}\right)\right]$

(*b*) Symmetrical loading: two loads R_t, x_t from each end

(i) simply supported ends $\beta_{st} = \frac{1}{6}\frac{x_s}{a}\left[\frac{3x_t}{a}\left(1-\frac{x_t}{a}\right)-\frac{x_s^2}{a^2}\right]$

(ii) clamped ends $\beta_{st} = \frac{1}{6}\left(\frac{x_s}{a}\right)^2\left[3\frac{x_t}{a}\left(1-\frac{x_t}{a}\right)-\frac{x_s}{a}\right]$

(*c*) Anti-symmetrical loading: downwards load R_t, x_t from left end, and upwards load R_t, x_t from right end

(i) simply supported ends $\beta_{st} = \frac{1}{6}\frac{x_s}{a}\left(1-\frac{2x_t}{a}\right)\left(\frac{x_t}{a}-\frac{x_s^2+x_t^2}{a^2}\right)$

(ii) clamped ends $\beta_{st} = \frac{1}{6}\left(\frac{x_s}{a}\right)^2\left(1-\frac{2x_t}{a}\right)\left[3\frac{x_t}{a}\left(1-\frac{x_t}{a}\right)-\frac{x_s}{a}\left(1+2\frac{x_t}{a}-2\left(\frac{x_t}{a}\right)^2\right)\right]$

2.4. Range of application

Solution by the Force Method requires first the formulation of a set of linear simultaneous equations, and for uniform beams this is simple and straightforward, using algebraic formulae. For non-uniform beams, this part of the analysis would be appreciably more difficult and laborious. The equations may be solved on a desk mechanical calculating machine for up to about five unknowns. For larger numbers of unknowns, a digital electronic computer would normally be used. Since present medium sized computers have standard subroutines which can solve up to about 70 equations, the Force Method may now be regarded as a very powerful means of grillage analysis.

Chapter 3

APPROXIMATE METHODS

Although the Force Method has become very attractive recently due to the introduction of digital electronic computers, more approximate methods, designed to avoid the solution of large sets of equations, may be of interest to those who do not have ready access to a computer. Historically, these approximate methods were of considerable importance, and three methods will now be described briefly and applied numerically to the simple 9×3 beam grillage shown in fig. 4. The beams of each set are equal and evenly spaced, all edges are simply supported, and the grillage is acted on by a central concentrated load. An exact solution, derived by solving the simultaneous equations, gives the following results:

central deflection = $0{\cdot}000731 Pa^3/EI_g$,
maximum longitudinal girder bending moment = $0{\cdot}0330 Pa$,
maximum transverse stiffner bending moment = $0{\cdot}0242 Pa$,

3.1. Minimum Potential Energy Method

In this method, a deflected shape is assumed

$$w = \sum_{m=1}^{\lambda} \sum_{n=1}^{\lambda} a_{mn} X_m(x) Y_n(y). \tag{3.1}$$

The functions $X_m(x)$, $Y_n(y)$ should satisfy the boundary conditions at the edges of the grillage, and it simplifies the subsequent algebra if they also satisfy the conditions,

$$\int_0^a X''_{m_1} X''_{m_2}\, dx = 0 \quad \text{for} \quad m_1 \neq m_2,$$

$$\int_0^b Y''_{n_1} Y''_{n_2}\, dy = 0 \quad \text{for} \quad n_1 \neq n_2.$$

The numerical coefficients a_{mn} are determined by the condition that the change in potential energy due to the assumed deflections is a minimum. The potential energy may be written,

$$V = V_g + V_s - W,$$

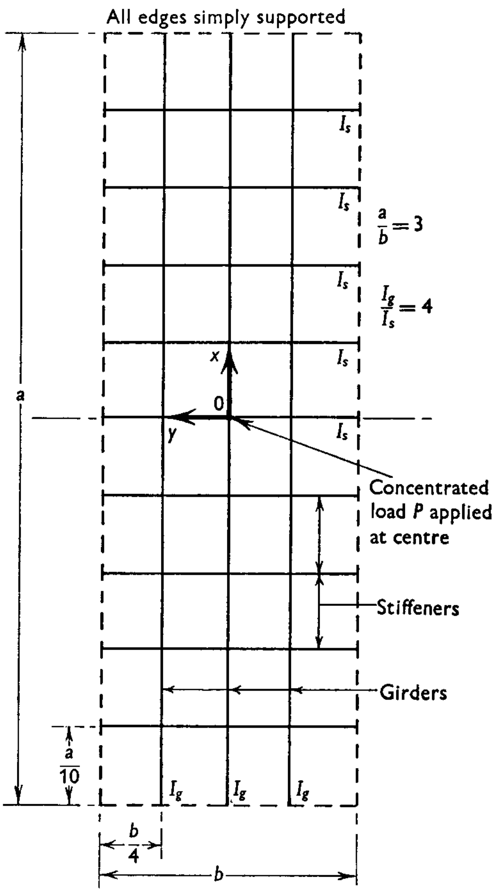

Fig. 4. 9 × 3 grillage for approximate analysis.

where V_g, V_s are the elastic strain energies of the girders and stiffeners and W is the work done by the external load. For minimum potential energy

$$\frac{\partial V}{\partial a_{mn}} = 0 \quad \text{for} \quad m = 1, 2, \ldots, \lambda \quad \text{and} \quad n = 1, 2, \ldots, \lambda, \qquad (3.2)$$

which provides λ^2 simultaneous linear equations for the λ^2 unknown coefficients a_{mn}. In contrast to the simultaneous equations of the Force Method, the energy equations can usually be iterated.