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# COMMONS GAME Made More Exciting by an Intelligent Utilization of the Two Evolutionary Algorithms

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In this paper, we suggest that Evolutionary Algorithms could be utilized in order to let the COMMONS GAME, one of the most popular environmental games, become much more exciting. In order to attain this objective, we utilize Multi-Objective Evolutionary Algorithms to generate various skilled players. Further, we suggest that Evolutionary Programming could be utilized to find out an appropriate point of each card at the COMMONS GAME. Several game playings utilizing the new rule of the COMMONS GAME confirm the effectiveness of our approach.

## 1 Introduction

Gaming is regarded by many people as a new and promising tool to deal with complex problems in which human decisions have far reaching effects on others. It has been used for various purposes such as decision-making, education, training, research, entertainment, and etc. [1]-[12]. In recent years, various approaches concerning the applications of Evolutionary Algorithms to the field of games have been proposed [13]-[17].

In this paper, we suggest that EAs could be utilized for making the COMMONS GAME [8], one of the most popular environmental games, become much more exciting. In particular, in order to attain this objective, we shall try to utilize Evolutionary Algorithms in the following steps:

- 1) First, we shall consider a new rule for assigning a point to each colored card in the COMMONS GAME which takes the environmental changes into account.

- 2) Second, we shall utilize Multi-Objective Evolutionary Algorithms (MOEA) [18][19] to generate various skilled players whose choice of each card is done in a timely fashion.
- 3) Further, we shall utilize Evolutionary Programming (EP) [20][21] to derive appropriate combinations of the rules (concerning the point of each card) which could be used to help players fully enjoy game playing.

This paper is organized as follows. In section 2, we shall introduce the original COMMONS GAME briefly and touch upon several problems involved in the original COMMONS GAME. We shall suggest that EAs could be utilized in order to let game playing become much more exciting. We shall also show several results of game playing (utilizing the new rule derived by MOEA & FEP) which confirm the effectiveness of our approach. This paper concludes with discussions concerning the contributions of this paper and future perspectives.

## 2 COMMONS GAME

### 2.1 History of Gaming

Historically speaking, gaming<sup>1</sup> has its origin in war games [4]. However, after the Second World War, it has been applied to various peaceful purposes. A large number of business games have been developed with the purpose of training students in business school [2][5][6]. Further, some environmental games have also been developed in order to help people consider seriously about the environmental state of the world [8]-[10]. Gaming has also successfully been utilized for operational purposes [7]. Depending upon the purpose of the game, gaming can be categorized into several classes such as Entertainment Gaming, Educational Gaming, Operational Gaming and etc. [1][2] Due to space, we don't go into details concerning the literature of gaming and the categorization of gaming. Interested readers are kindly asked to read the books and papers [1]-[12].

In the following subsection, we shall briefly introduce COMMONS GAME<sup>2</sup> [8]. We shall also briefly touch upon the computer gaming system of the COMMONS GAME.

### 2.2 Brief Introduction of the COMMONS GAME

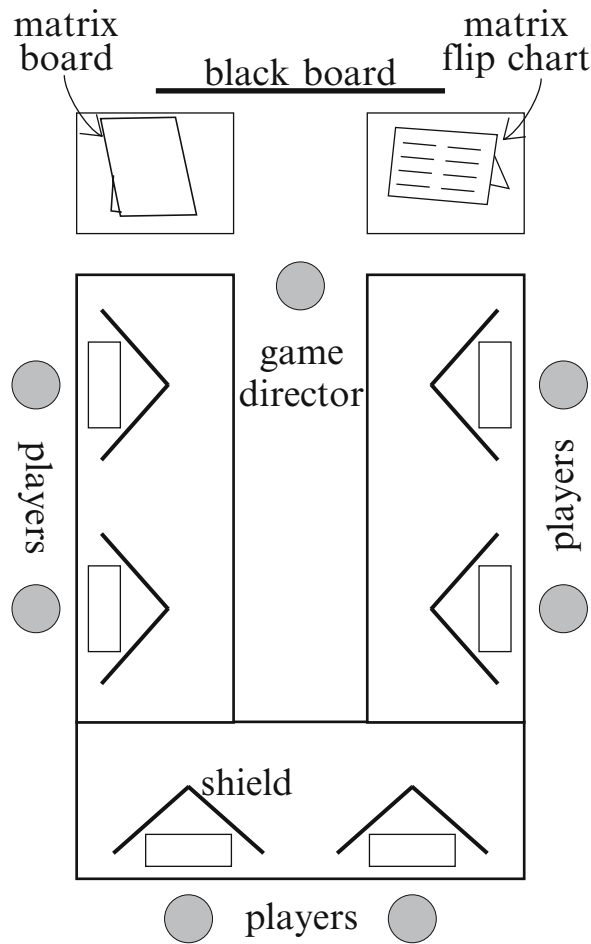
The COMMONS GAME was developed by Powers *et al.* in 1977 [8]. Since we live in a world having only finite natural resources such as fishes and forests

<sup>1</sup> Occasionally, game theory has been confused with gaming. Gaming means the use of a game for one of the various purposes such as teaching, training, operations, entertainment, and etc. [1][2].

<sup>2</sup> The COMMONS GAME was designed by Powers *et al.* [8] in order to let people have a chance to consider seriously about the COMMONS. Therefore, COMMONS GAME can be categorized into the class of the Educational Gaming [1].

(commons), it is wise to consider their careful utilization. The COMMONS GAME may be quite helpful in stimulating discussion of this problem. Figure 1 shows the layout of the original COMMONS GAME.

In the following, we give a brief introduction to this game. Six players are asked to sit around a table. Following a brief introduction of the game, the game director tells the players that their objective is to maximize their own gains by choosing one card among the five colored (Green, Red, Black, Orange, Yellow) cards in each round. In each round, players hide their cards behind a cardboard shield to ensure individual privacy.



**Fig. 1.** Layout of the original COMMONS GAME

Each colored card has its own special meaning concerning the attitude toward the environmental protection, and has the following effect upon the total gains of each player:

**Green card:** A green card represents high exploitation of the commons: Players who play a green card can get a maximum reward. However, they lose 20 points if one of the other players plays a black card in the same round.

**Red card:** A red card indicates a careful utilization of the commons: Red cards are only worth about forty percent as many points as green cards.

**Black card:** This card has a punishing effect on the players with green cards: Players who have played a black card have to lose 6 points divided by the number of black cards played at the same round, but are able to punish green card players by giving them  $-20$  points.

**Orange card:** An orange card gives an encouraging effect to red card players: Players who have played this card have to lose 6 points divided by the number of orange cards played at the same round but are able to add 10 points to red card players.

**Yellow card:** A yellow card denotes a complete abstention from utilization of the commons: Players who play this card get 6 points.

Depending upon the players' strategies, the state of the commons change: If players are too eager to exploit the commons, then deterioration of the commons occurs. Players have a matrix flip chart on the board representing the payoffs for the red and green cards under different conditions of the commons. Although players are informed that there will be 60 rounds, each game ends after 50 rounds. After each 8<sup>th</sup> round, players have a three-minute conference. They can discuss everything about the game and decide every possible way to play in future rounds.

Due to space, the detail of the rules are not explained. For those interested in further details, however, we recommend reading the paper written by Powers *et al.* [8].

**Remark 2.1.** In each round, each player can choose one of the 5 cards. However, COMMONS GAME is quite different from traditional "card game" in which cards are distributed randomly to the players and losing cards is frequently occurred. In this game, each player can choose any card in each round in order to represent his (or her) attitude toward the environmental protection.

**Remark 2.2.** Red players usually can only get about 40 % of the returns that green card players receive (assuming that no players have chosen the black card at the same round). In the original COMMONS GAME, there are 17 main environmental states  $(-8, -7, \dots, -1, 0, +1, \dots, +8)$ . (Each main environmental state (except 0 state) has 10 subordinate states (the 0 state has 21 subordinate states)). Initial state of the COMMONS GAME is 0. The state of the commons changes, depending upon the players' strategies. If players

**Table 1.** Points which can be gained by the red and the green card players

State: -8		State: -1		State: 0		State: 1		State: 8	
R	G	R	G	R	G	R	G	R	G
—	0	—	70	—	100	—	130	—	200
-10	2	25	72	40	102	55	132	90	202
-8	4	27	74	42	104	57	134	92	204
-6	6	29	76	44	106	59	136	94	206
-4	8	31	78	46	108	61	138	96	208
-2	10	33	80	48	110	63	140	98	210
0	—	35	—	50	—	65	—	100	—

are eager to exploit the commons (many players often use green cards), the deterioration of the commons occurs and the state of the commons changes to a minus state such as  $-1$ ,  $-2$ , and etc. When the deterioration of the commons has occurred, then the returns that green players and red players receive decrease. Table 1 shows the returns that green players can get when no players have chosen the black card nor the orange card. It also shows the returns that red players can receive. The second table from the left shows that green players can receive 70 point (when the state of the environment is  $-1$  and no players have chosen the red cards) which is only the 70 % of the returns which can be obtained at the initial state 0. This table also shows that the points that green players can get change depending upon the number of the red cards played in the same rounds (Remark 2.4). The points that red card players receive also decrease heavily when the environmental state becomes minus. In the  $-1$  state, red players can get only about 70 % of the returns that they could receive in the initial state 0. (On the other hand,) If almost all of the players consider seriously about the commons and execute wise utilization of the commons, then environmental state ameliorates (state of the environmental becomes positive) and the returns that green players and red players receive increase as shown in Table 1.

**Remark 2.3.** Utilization of a green card also incurs degradation of the subordinate state. Although natural amelioration of the commons occurs 8 times during 50 rounds, too much exploitation of the commons (that is to say, too much use of the green card) causes serious degradation of the environmental state (One green card corresponds to one degradation of the subordinate state).

**Remark 2.4.** Each row in the five tables in Table 1 corresponds numbers of the red cards chosen. For an example, let us consider that case that the current main state is  $+1$ , and numbers of the red and green card players are 5 and 1, respectively. Then, each red card player can get point 63 which corresponds to the point written in the 6<sup>th</sup> row and the 1<sup>st</sup> column of the table concerning the state  $+1$ . The green card player can get point 140.

### 2.3 Computer Gaming System of the COMMONS GAME

More than 20 years ago, one of the authors and his students succeeded in constructing a personal computer gaming system of the original COMMONS game [11][12]. In this gaming system, each player does not need to play one of the five cards in order to show his decision concerning the commons. Instead, he has to choose a column and a row number in a matrix written on a paper delivered by the game director.

A computer screen gives players various information such as the state of the commons and points received by the players in each round. If the state of the commons declines, the color of the waves becomes tinged with yellow. Also, the color of the waves becomes tinged with blue if the state of the commons improves. During the conference time, the computer screen provides players with a beautiful color graphic and gives information regarding the time passed.

## 3 Evolutionary Algorithms for Making Game Playing Much More Exciting

We have so far enjoyed a large number of playings of the original COMMONS GAME. Although those experiences have given us a valuable chance to consider seriously about the commons, we did find that some players lost interest, in the middle of the game, because the COMMONS GAME is comparatively monotonous. In order to make the game much more exciting [22]-[29], we have tried to find the reason why some players lost interest in the middle of the COMMONS GAME. We have come to the conclusion that the way that each player receives points when he (or she) chooses one of the five cards sometimes makes the game playing rather monotonous.

In particular, we have concluded that the following rule make the game playing monotonous: In the original COMMONS GAME, green card players receive a penalty,  $-20$  points, when some player chooses a black card. On the other hand, black card players receive a point  $-6/(\text{the number of players who have chosen a black card})$ . Orange card players receive a point  $-6/(\text{the number of players who have chosen an orange card})$ .

We consider that some change in the points  $-20$  and  $-6$  mentioned above would make the COMMONS GAME much more exciting. In order to find an appropriate point for each card, we shall try to utilize EP.

In section 3.1, we suggest that Multi-Objective Evolutionary Algorithms (MOEA) [18][19] can generate various kinds of Neural Network Players with different strategies. In section 3.2, we show that Evolutionary Programming [20][21] can be a useful tool for finding appropriate points of the cards in the COMMONS game.

### 3.1 MOEAs for Generating Intelligent Players

Multi-Objective optimization is one of the most promising fields in Evolutionary Algorithms research. Due to the population search of EAs, Multi-Objective Evolutionary Algorithms (MOEAs) can evolve candidates of Pareto optimal solutions. Hence, in comparison with conventional EAs, MOEAs can simultaneously find out various solutions. In this paper, we employ NSGA-II [18], proposed by Deb *et al.*, to evolve game players with Neural Networks. The NSGA-II utilizing crowded tournament selection, the notion of archive, and ranking method with non-dominated sort, is one of the most famous MOEAs. Most recent studies proposing new MOEAs cite their paper [19]. (A brief introduction of the NSGA-II is given in the Appendix.)

**Neural Network Model.** Our objective is to simultaneously construct plenty of Neural Network players with different strategies. The neural network model adopted in this paper has 26 input units, one hidden layer with 30 units, and 5 output units<sup>3</sup>.

In Table 2, input variables into this neural network model are given: In order to represent the current state of the commons, two inputs, consisting of main state and subordinate state, are prepared. For inputs 6.–9., 5 different inputs corresponding to each card are prepared.

The weights of the neural network model are evolved by MOEA. That is, the number of gene in an individual is 965 (the number of weights between input layer and hidden layer:  $26 \times 30$ , the number of weights between hidden layer and output layer:  $30 \times 5$ , and the number of thresholds: 5).

**Fitness Evaluation.** In order to evaluate each individual, it is better to let him play with various game players. Fitness evaluation of individuals is carried out as follows:

**Table 2.** Input variables for Neural Network players

1.	Difference between the total points of each player and the average
2.	Rank of each player
3.	States of the environment: main state & subordinate state
4.	Changes in the environment
5.	Round Number
6.	Weighted sum of each card having been chosen by all of the players
7.	Weighted sum of each card having been chosen by each player
8.	The number of each card chosen in the previous round
9.	The card chosen by each player in the previous round

<sup>3</sup> Each output unit corresponds to each colored card. The colored card corresponding to the output unit which has emitted the highest output value is considered to be that chosen by the neural network player.

**Table 3.** Efficiency of each card

Player's card	$E_i(C)$	Situations
R	+1	No black player, but some green players
	-1	Otherwise
B	+1	No green player
	-1	Otherwise
G	+1	Some black players
	-1	Otherwise

- 1) Choose 30 individuals randomly from the parent population at each generation, where they become the opponents for 6 game runs (5 individuals are needed as opponents for a single game run).
- 2) The child population is generated from the parent population.
- 3) Each individual in the parent and child populations plays with the opponents chosen in **1**).
- 4) As a consequence of game runs, individuals are evaluated with two objective functions<sup>4</sup>  $O_v$  and  $O_e$ : Variance of the total number of each card chosen in each game run and the efficiency of the cards played, respectively. Variance of the number of the card played is calculated by the following equation:

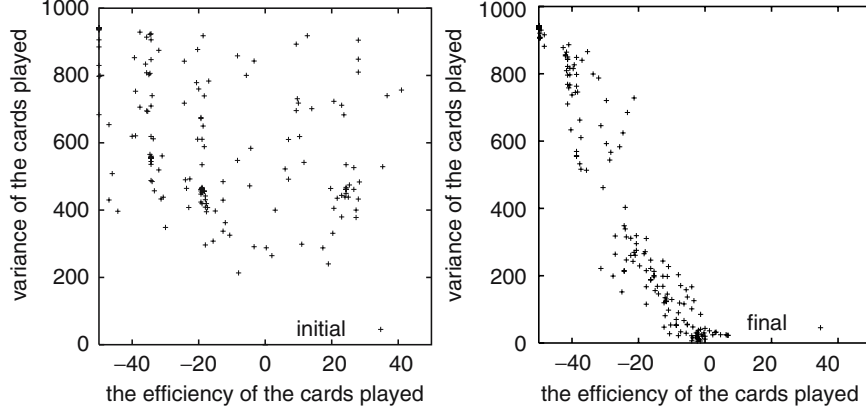
$$O_v = V_{RGB} + 20 * N_{OY},$$

where  $V_{RGB}$  is the variance of the total number of red, green, and black cards played in each game run, and  $N_{OY}$  is the total number of orange and yellow cards chosen. The efficiency of the cards used is calculated by integrating the evaluation at each round (Table 3 shows the way how the efficiency of the cards played at each round is evaluated):

$$O_e = \sum_{i=1}^{50} E_i(C).$$

**Experimental Results** Figure 2 depicts the individual distributions at the initial and final generations. In the individual distributions at the final generation, a Pareto set is constructed. Since fitness measurement in this paper is a relative one, i.e., opponents are randomly chosen at every generation, some initial individuals in which the efficiency of the use of the cards and the variance are close to  $-48$  and  $400$ , respectively, seem to have better performance. However, it is just caused by the game playings with naive players. In fact, the final individuals have become much more sophisticated compared with all of the initial individuals.

<sup>4</sup> According to the implementation of NSGA-II, the objective functions used here are to be minimized.



**Fig. 2.** Individual distributions at the initial (LEFT) and the final (RIGHT) generations

### 3.2 Fast Evolutionary Programming to Design Appropriate Game Rules

**Fast Evolutionary Programming.** Fast Evolutionary Programming (FEP) proposed by Xin *et al.* [21] is used in this paper because the Fast Evolutionary Programming is easy to implement and performs well due to the Cauchy distribution mutation. Individuals in the FEP are composed of a pair of real valued vectors  $(X, \eta)$ , where  $X$  and  $\eta$  indicate the design variables in the problems and variance parameter used in self-adaptive mutation, respectively. (A brief introduction of the FEP is give in the Appendix.)

**Utilization of the FEP for Constructing a New Rule of the COMMONS GAME.** In this paper, we employ three variables,  $W_G$ ,  $A$ , and  $W_o$  to represent new rules. The meaning of them is described as follows:

1. Penalty  $P_G$  for green players: We shall propose an appropriate way for penalizing green players which takes the environmental changes into account:  $P_G = -W_G \times (\text{Gain } G)$ , where  $W_G$  means the numerical value that is determined by the FEP, and “Gain  $G$ ” denotes the return that the green players can get if any other player does not choose the black card.
2. Point  $AOB$  that black players loose: We shall propose an appropriate way (for asking black players pay cost in trying to penalize green players) which takes the environmental changes into account:  $AOB = OB/NOB(OB = -A \times (\text{Gain } R))$ , where  $A$  means the numerical value that is determined by the FEP, and  $NOB$  means the number of players who have chosen the black cards, and “Gain  $R$ ” denotes the return that the red player can get.
3. Point  $OR$  that orange players add to the red players: We shall propose an appropriate way (for helping red players maintain the commons) which

takes the environmental changes into account:  $OR = W_o \times (\text{Gain R})$ , where  $W_o$  means the numerical value that is determined by the FEP.

**Fitness Evaluation.** In order to evaluate the rule parameters mentioned in the previous subsection, 200 games per an individual are carried out. Before runs of the FEP, six neural network players in the final population of MOEA are randomly chosen per game. Namely, 1200 neural network players are selected (including duplicated selection).

After each game, a rule parameter is evaluated by the following:

- a) A game in which black cards & orange cards are seldom chosen (almost all of the players only choose a green card or a red card) is quite monotonous. Therefore, the more black (or orange) cards are chosen in a game, the higher value of the evaluation function should be given.
- b) A game in which the ranking of each player often changes is exciting. Therefore, the more changes of the top player during a game run, the higher evaluation value should be given. A game in which there is a small difference between the total points of the top player and those of the last player is very exciting. Therefore, the small variance in the total points of each player at the end of a game, the higher the evaluation value should be given.
- c) When the environmental deterioration had occurred heavily, each player can receive only a small amount of return. Under such a state of environment, the game should become monotonous. Therefore, a small evaluation value should be given if a game has brought heavy deterioration to the environment. On the other hand, a high evaluation value should be given when a game has brought a moderate final state of the environment.

By taking into account the above points, we have constructed the following evaluation function  $T(x)$ <sup>5</sup>:

$$T(x) = f(x) + g(x) + h(x) + \alpha(x) + \beta(x),$$

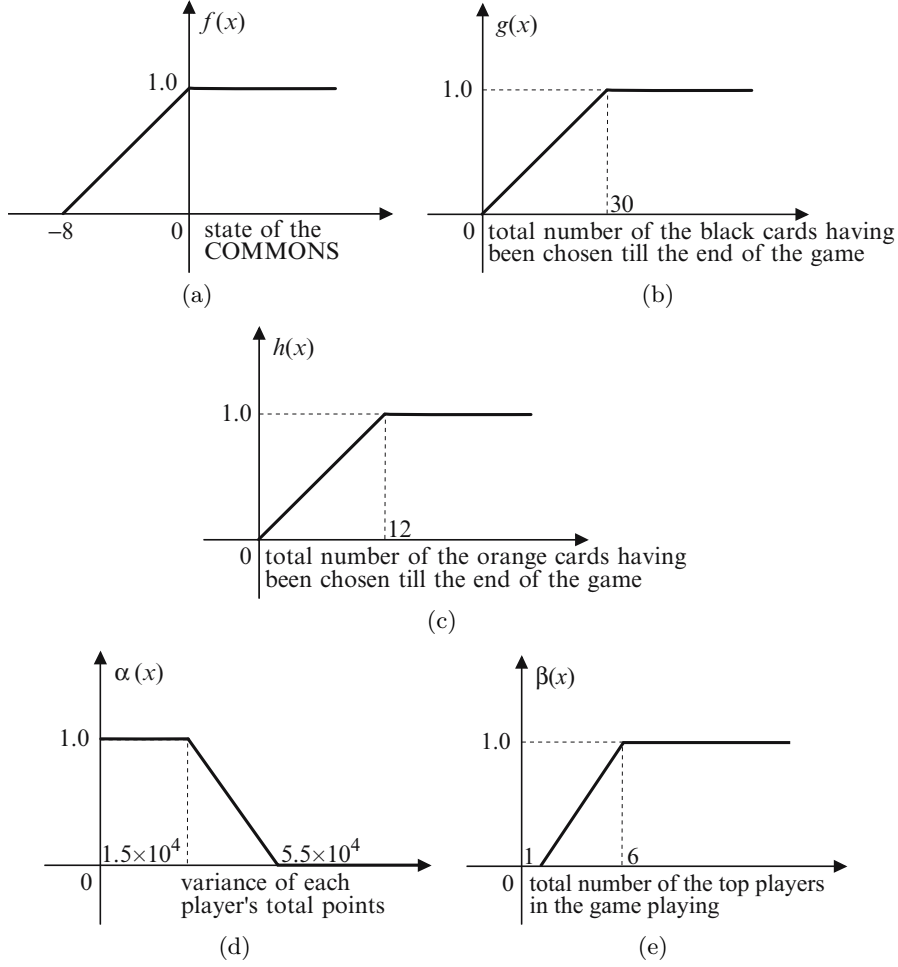
where  $x$  denotes a chromosome. The function values of  $f(x)$ ,  $g(x)$ ,  $h(x)$ ,  $\alpha(x)$ , and  $\beta(x)$ , correspond to the followings:

- $f(x)$ : The environmental state at the end of the game;
- $g(x)$ : The total number of the black card having been chosen;
- $h(x)$ : The total number of the orange card having been chosen;
- $\alpha(x)$ : The sum of the variance of the points having been gained by each player;
- $\beta(x)$ : The total number of the changes of the top player.

### 3.3 New Rules Obtained by the MOEA and the FEP

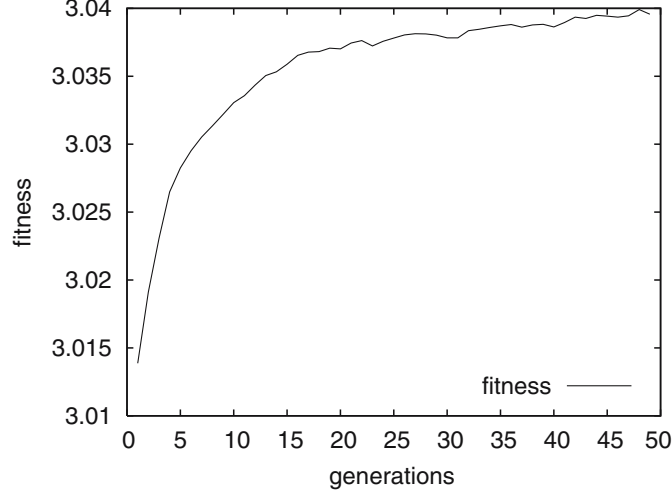
In order to find appropriate combinations of the three variables,  $W_G$ ,  $A$ , and  $W_o$ , evolutionary search by the FEP has been carried out for 50 generations.

<sup>5</sup> In Fig. 3, 5 functions which constitute the evaluation function  $T(x)$  are illustrated.



**Fig. 3.** 5 functions which constitute the evaluation function  $T(x)$

The population size of the FEP has been set to be 10. The changes of the average fitness values during the evolution are depicted in Fig. 4. This graph is plotted by averaging over 20 runs. The changes over the whole generations are not so much. However, the main contribution of these changes are caused by the changes of  $\beta(x)$ . This means that the utilization of the FEP has contributed a lot in changing top players. The reason why other sub-functions, such as  $f(x)$ ,  $g(x)$  and so on, did not affect the evolution process is that the neural network players are already sophisticated enough: They play various kinds of cards, including black and orange cards so that the final environmental state has not been deteriorated so seriously.



**Fig. 4.** Changes of averaged fitness value during evolution

We analyzed the best individuals found after the 20 runs. There are two groups in the best individuals: They are around  $W_G = 0.27$ ,  $A = 0.04$ , and  $W_o = 0.12$ , and  $W_G = 0.29$ ,  $A = 0.2$ , and  $W_o = 0.1$ , respectively. This analysis reveals that much penalization of green players causes the frequent changes of the top players.

### 3.4 Game Playings Utilizing the New Rules

In order to investigate whether the new rules having been obtained by the MOEA and the FEP are appropriate or not, one of the authors asked his 12 undergraduate students of Osaka Kyoiku University (who had experienced several game playings of the original COMMONS GAME) to play the new games<sup>6</sup> which have been modified by taking the results obtained by the use of the MOEA and the FEP into account. The authors watched the attitudes of the students who participated in the playings of the new games. They felt that almost all of the students concentrated more on the new games than before. After the games, they asked the students for their impressions of the new games.

Answers from the 12 students can be summarized as follows:

- (1) 9 students out of 12 expressed their opinions that the new games are far more exciting than the original game.

<sup>6</sup> The two new games with the parameter values  $W_g = 0.27$ ,  $A = 0.04$ , and  $W_o = 0.12$  &  $W_g = 0.29$ ,  $A = 0.2$ , and  $W_o = 0.1$ , respectively, have been played.

- (2) Some of the 9 students (who expressed positive impressions toward the new games) explained the reason why they have felt the new games exciting:
  - (2-1) In the new games, players can easily choose the black card since cost of using the black card has become rather low. Therefore, all of the players can enjoy dynamic game playing.
  - (2-2) In the original COMMONS GAME, penalty point to the green card player by a black card is always  $-20$ . However, in the new games, it depends on the environmental state. When the environmental state is 0 or  $+1$ , damage due to the use of the green card is heavy. This causes the new games rather exciting.
  - (2-3) In the new games, points concerning penalty to the green card, cost of using the black card, and etc. are not fixed. Therefore, players should manage to find a good plan for choosing one of the cards in order to adapt successfully to each environmental situation. This causes good thrill to each player.
- (3) One of the 3 students (who have shown us negative impressions on the new games) explained the reason why he prefers the original game to the new games: Players (enjoying the game playing of the original game) should be very careful in using black card since its utilization causes them considerable minus point. However, a black card gives heavy minus point to the green card players wherever the state of the environment is. This makes a good thrill to him.
- (4) He also added the following reason: In the new games, penalty point to the green card players is comparatively small when the environmental state is negative such as the state “ $-3$ ” or the state “ $-4$ .” This causes the situation that players can easily choose the green card.
- (5) Many players pointed out the problems involved in the COMMONS GAME: Players who have chosen the red card in almost all of the rounds can easily win the game.

## 4 Conclusions

In this paper, we have tried to utilize two kinds of EAs, i.e., the MOEA and the FEP for making the COMMONS GAME exciting. The MOEA has been used for generating various types of skilled players. Further, the FEP has been introduced to find out appropriate combinations of the point of each card. As shown in the Fig. 4, we have succeeded in finding highly advanced rules compared with that of the original COMMONS GAME. Several game playings of the COMMONS GAME (using the new rule (derived by using the MOEA and the FEP)) by our students suggest the effectiveness of our approach. However, this has been suggested only by several game playings done by our students. The future research is needed to carry out lots of game playings by various people for the full confirmation of our approach. Further,

we should also pay attention carefully to the impressions (concerning the new games) which were posed by our students in order to design a more advanced gaming system.

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## Appendix: Brief Introductions Concerning the NSGA-II and the FEP

### (A.1) The Nondominated Sorting Genetic Algorithm II (The NSGA-II)

Below, we briefly introduce the NSGA-II (Nondominated Sorting Genetic Algorithm II) proposed by Deb *et al.* [18]. The NSGA-II is one of the Multi-Objective Evolutionary Algorithms. It is characterized by selection mechanism: In order to select a new population for preserving diversity among solutions, two indices for the individual  $i$  are utilized: nondomination rank  $i_{rank}$  and crowding distance  $i_{distance}$ . The nondomination rank  $i_{rank}$  of an individual  $i$  indicates the number “1 + the number of individuals which dominate the individual  $i$ .” For instance, the nondomination rank  $i_{rank}$  of the individual  $i$  in the Pareto set in the combined population is 1. Individuals whose nondomination rank is 2 are dominated only by an individual in the Pareto set. The crowding distance  $i_{distance}$  denotes the average distance of the two individuals on either side of the individual  $i$  along each of the objectives.

In the selection mechanism of NSGA-II, a partial order  $\prec_n$ , called Crowded-Comparison Operator, is introduced by using the two indices  $i_{rank}$  and  $i_{distance}$ .

$$i \prec_n j \text{ if } (i_{rank} < j_{rank}) \text{ or } ((i_{rank} = j_{rank}) \text{ and } (i_{distance} > j_{distance}))$$

In order to calculate the above indices and utilize them efficiently for the selection mechanism, two sorting algorithms are proposed by Deb *et al.* For those interested in further details, we recommend reading the paper written by Deb *et al.* [18].

### (A.2) Fast Evolutionary Programming (The FEP)

Below, we show the algorithm of the FEP [21].

1. Generate the initial population consisting of  $\mu$  individuals.
2. Evaluate each individual.
3. Let each individual  $(X, \eta)$  create an offspring  $(X', \eta')$  as:

$$x'_j = x_j + \eta_j \delta_j \tag{A.2.1}$$

$$\eta'_j = \eta_j \exp(\tau' N(0, 1) + \tau N_j(0, 1)) \tag{A.2.2}$$

where  $x_j$  and  $\eta_j$  denote the  $j^{\text{th}}$  component of vectors  $X$  and  $\eta$ , respectively.  $\delta_j$ ,  $N(0, 1)$ , and  $N_j(0, 1)$  denote a Cauchy random variable, a standard Gaussian random variable, the  $j^{\text{th}}$  independent identically distributed standard Gaussian random variable, respectively. In (A.2.2), coefficients  $\tau$  and  $\tau'$  are set to be  $(\sqrt{2\sqrt{n}})^{-1}$  and  $(\sqrt{2n})^{-1}$ , respectively.

4. Evaluate each offspring.

5. Conduct pairwise comparison over all of the parents and offsprings. In order to evaluate each individual,  $q$  opponents are chosen randomly. For each comparison, each individual receives “win” when its fitness value is higher than that of the opponent.
6. Pick up  $\mu$  individuals from the set of the parents and the offsprings by taking the ranking due to the number of the winnings into account.
7. Stop if halting condition is satisfied. Otherwise go to Step 3.