Lecture Notes in Physics Monographs 61

Ray Optics, Fermat's Principle, and Applications to General Relativity

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Preface

All kind of information from distant celestial bodies comes to us in the form of electromagnetic radiation. In most cases the propagation of this radiation can be described, as a reasonable approximation, in terms of rays. This is true not only in the optical range but also in the radio range of the electromagnetic spectrum. For this reason the laws of ray optics are of fundamental importance for astronomy, astrophysics, and cosmology.

According to general relativity, light rays are the light-like geodesics of a Lorentzian metric by which the spacetime geometry is described. This, however, is true only as long as the light rays are freely propagating under the only influence of the gravitational field which is coded in the spacetime geometry. If a light ray is influenced, in addition, by an optical medium (e.g., by a plasma), then it will not follow a light-like geodesic of the spacetime metric. It is true that for electromagnetic radiation traveling through the universe usually the influence of a medium on the path of the ray and on the frequency is small. However, there are several cases in which this influence is very well measurable, in particular in the radio range. For example, the deflection of radio rays in the gravitational field of the Sun is considerably influenced by the Solar corona. Moreover, current and planned Doppler experiments with microwaves in the Solar system reach an accuracy in the frequency of $\Delta\omega/\omega \cong 10^{-15}$ which makes it necessary to take the influence of the interplanetary medium into account. Finally, even in cases where the quantitative influence of the medium is negligibly small it is interesting to ask in which way the qualitative aspects of the theory are influenced by the medium. The latter remark applies, in particular, to the intriguing theory of gravitational lensing.

Unfortunately, general-relativistic light propagation in media is not usually treated in standard textbooks, and the more specialized literature is concentrated on particular types of media and on particular applications rather than on general methodology. In this sense a comprehensive review of general-relativistic ray optics in media would fill a gap in the literature. It is the purpose of this monograph to provide such a review.

Actually, this monograph grew out of a more special idea. It was my original plan to write a review on variational principles for light rays in general relativistic media, and to give some applications to astronomy and astrophysics, in particular to the theory of gravitational lensing. However, I soon realized the necessity of precisely formulating the mathematical theory of light rays in general before I could tackle the question of whether these light rays are characterized by a variational principle. The sections on variational principles and on applications are now at the end of Part II, in which a general mathematical framework for ray optics is set up. This is written in the language of symplectic geometry, thereby elucidating the well-known analogy between ray optics and the phase-space formulation of classical mechanics.

Moreover, I found it desirable to also treat the question of how to derive ray optics as an approximation scheme from Maxwell's equations. This is the topic of Part I which serves the purpose of physically motivating the fundamental definitions of Part II. In vacuo, the passage from Maxwell's equations to ray optics is, of course, an elementary textbook matter and the generalization to isotropic and non-dispersive media is quite straightforward. However, for anisotropic and/or dispersive media this passage is more subtle. In Part I two types of media are discussed in detail, viz., an anisotropic one and a dispersive one, and the emphasis is on general methodology.

I have organized the material in such a way that it should be possible to read Part II without having read Part I. This is not recommended, of course, but the reader might wish to do so. Both parts begin with an introductory section containing a brief guide to the literature and a statement of assumptions and notations used throughout. Whenever the reader feels that a symbol needs explanation or that the underlying assumptions are not clearly stated, he or she should consult the introductory section of the respective part. Also, the index might be of help if problems of that kind occur.

Large parts of this monograph present material which, in essence, is not new. However, I hope that the formulation chosen here might give some new insight. As to Part I, our discussion of the passage from Maxwell's equations to ray optics includes several mathematical details which are difficult to find in the literature, although the general features are certainly known to experts. To mention just one example, it is certainly known to experts that in a linear but anisotropic medium on a general-relativistic spacetime the light rays are determined by two "optical Finsler metrics"; to the best of my knowledge, however, a full proof of this fact is given here for the first time. As to Part II, the basic formalism is just the 170-year-old Hamiltonian optics, rewritten in modern mathematical terminology and adapted to the framework of general relativity. However, the presentation is based on some general mathematical definitions which have not been used before. This remark applies, in particular, to Definition 5.1.1, which is the definition of what I call "ray-optical structures". This definition formalizes the widely accepted idea that all of ray optics can be derived from a "dispersion relation". (The term "ray system" is sometimes used by Vladimir Arnold and his collaborators in a similar though not quite identical sense.) It also applies, e.g., to Definition 5.4.1, on "dilation-invariant" ray optical structures, which characterizes dispersion-free media in a geometric way.

On the other hand, I want to direct the reader's attention to the fact that this monograph contains some particular results which, as far as I know, have not been known before. These include, e.g.:

- the general redshift formula for light rays in media on a general-relativistic spacetime in Sect. 6.2;
- the results on light bundles in isotropic non-dispersive media on a general-relativistic spacetime in Sect. 6.4, in particular the generalized "reciprocity theorem" (Theorem 6.4.3);
- Theorem 7.3.1, which can be viewed as a version of Fermat's principle for light rays in (possibly anisotropic and dispersive) media on general-relativistic spacetimes;
- Theorem 7.5.4, which generalizes the "Morse index theorem" of Riemannian geometry to the case of light rays in stationary media on stationary general-relativistic spacetimes.

Some of the questions raised in this monograph remain unanswered, i.e., to some extent this is an interim report on work in progress. In particular, this remark applies to the following two special issues. (a) In Part I we are able to prove that for the linear medium treated in Chap. 2 ray optics is associated with approximate solutions of Maxwell's equations, i.e., that ray optics gives a viable approximation scheme for electromagnetic radiation. Unfortunately, we are not able to prove a similar result for the plasma model of Chap. 3. This is a gap which should be filled in the future. (b) In Part II we are able to establish a Morse index theorem for light rays in stationary media. However, it is still an open question whether these results can be generalized to the non-stationary case in which, up to now, a Morse theory exists only for vacuum rays. With Fermat's principle in the form of Theorem 7.3.1 we have a starting point for setting up a Morse theory for light rays in arbitrary (non-stationary) media. This is an interesting problem to be tackled in future work.

This monograph in its present form is a slightly revised version of my *Habilitation* thesis. I would like to use this opportunity to thank the members of the Habilitation Committee, Karl-Eberhard Hellwig, Erwin Sedlmayr, Bernd Wegner, John Beem, Friedrich Wilhelm Hehl, and Gernot Neugebauer, for their interest in this work and for several useful comments. In particular, I would like to thank Bernd Wegner for paving the way to having this text published with Springer Verlag.

While working at this monograph I have profited from many discussions, in particular with my academic teacher Karl-Eberhard Hellwig and his collaborators at the Technical University in Berlin, but also with other colleagues. Special thanks are due to Wolfgang Hasse and Marcus Kriele for collaboration on various aspects of light propagation in general relativity; to Wolfgang

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